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Backward scattering in the one-dimensional Fermi gas

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Abstract : The Ward identity is derived for non-relativistic fermions with two-body spin-independent interaction. Using this identity for the one-dimensional Fermi gas with backward scattering the equations of the perturbation theory are solved for the effective interaction and the collective excitations of the particle density fluctuations are obtained.

I. Introduction

Recently there has been considerable interest in the one-dimensional Fermi gas model in connection to the unusual properties of the quasi-one-dimensional conductors /1/. Experimental and theoretical investigations have been devoted to the Kohn-Peierls instability and to the sudden change of their conductivity with decreasing temperature.

The one-dimensional Fermi gas model consists of spin-1/2 interacting fermions that are allowed to move on a straight line. The Fermi sea is reduced to a segment with the ends at the points $\pm k_F, k_F$ being the Fermi momentum. As the dynamics of the system is governed at low temperature mainly by low excited states we shall restrict ourselves to these states only. Their wavevectors p run within the ranges $-k_F - k_c < p < -k_F + k_c$ and $k_F - k_c < p < k_F + k_c$, where k_c is the bandwidth cut-off, much smaller than k_F. The energy levels ε_p of these single-particle states can be linearized as follows : $\varepsilon_p = \varepsilon_F + v_F(|p| - k_F)$, where ε_F is the Fermi velocity (Planck's constant has been taken equal to unit). Much theoretical work, recently reviewed by Solyom /2/, relied on this simple, linear p-dependence of the unperturbed energy levels,which is the essential feature of the model.

Mainly, there are two different approaches to the Fermi gas model. The first one is the perturbation theory approach where the fundamental quantity is the vertex part which describes the scattering of two fermions and accounts for the instabilities of the system. The perturbational treatment originates in a paper by

Bychkov et al./3/ who obtained a finite expression for the vertex part by summing up the most singular contributions (the so-called logarithmic approximation). Higher order corrections have been calculated by means of the renormalization group technique /4/. Scaling equations have been perturbationally solved for the response functions and various types of instabilities have been obtained for the ground state of the system /5/.

The second approach is a bosonization technique that can be traced back to a paper by Tomonaga /6/. Here the fundamental quantities are the operators of the particle density and spin density fluctuations that satisfy boson commutation relations. Unitary transformations have been devised to diagonalize the hamiltonian expressed in terms of these operators. The bosonization technique has been applied to the one-dimensional two-fermion model proposed by Luttinger /7/. This model differs slightly from that formulated above. The eigenvalues of the hamiltonian and the infrared behaviour of the response functions have been calculated /8/. A remarkable exact solution has been produced by Luther and Emery /9/ who allowed for a special type of spin-dependent interaction. This solution has been obtained for certain values of the coupling constants. Much subsequent work has been done within the framework of the bosonization approach /2/.

In the Fermi gas model as formulated above there are two types of spin-independent interaction processes. The first one is the forward scattering process that involves a small momentum transfer. This process excites one particle-hole pair in the neighbourhood of $+k_F$ and another one in the neighbourhood of $-k_F$. The second one is the backward scattering process, with momentum transfer near $2k_F$, that excites two particle-hole pairs across the Fermi sea. Let us suppose that a particle with momentum p_1 and a hole with momentum p_2 are excited near $+k_F$ and a particle with momentum p_3 and a hole with momentum p_4 are excited near to the opposite end $-k_F$. In the forward scattering process the momentum transfer is $k = p_1-p_2 \doteq p_4-p_3 \sim 0$ and the excitation energies of the two particle-hole pairs are $\Delta\varepsilon_1 = v_F(p_1-p_2)$ and $\Delta\varepsilon_2 = v_F(p_4-p_3)$, corresponding to the two Fermi ends, respectively. It appears that $\Delta\varepsilon_1 = \Delta\varepsilon_2 = v_F k$. In the backward scattering process the momentum transfer is $k = p_1-p_4 = p_2-p_3 \sim 2k_F$ and the excitation energies are $\Delta\varepsilon_1 = v_F(p_1+p_4)$ and $\Delta\varepsilon_2 = v_F(-p_2-p_3)$, whence one can see that $\Delta\varepsilon_1 \neq \Delta\varepsilon_2$. Due to this fact the density of states available in the two processes is different and this gives rise to different kinematics of the two processes. Indeed, assume that an excited state with energy ε and momentum zero is achieved by creating particle-hole pairs with momentum transfer k. By straightforward calculation we obtain that the density of states in the forward scattering process ($0 < k < 2k_c$, $\varepsilon = \Delta\varepsilon_1 + \Delta\varepsilon_2 = 2v_F k$) is $(k/\pi)^2 = (\varepsilon/2\pi v_F)^2$ for $0 < k < k_c$ and $(2k_c-k)^2/\pi^2 = (4k_c v_F-\varepsilon)^2/(2\pi v_F)^2$ for $k_c < k < 2k_c$, while in the backscattering process ($2k_F-k_c < k < 2k_F + k_c$, $\varepsilon = \Delta\varepsilon_1 + \Delta\varepsilon_2 = v_F(p_1+p_4- p_2-p_3)$) the density of states is $(k_c+k-2k_F)/\pi^2$ for $2k_F-k_c < k < 2k_F$ and $(k_c-k+2k_F)/\pi^2$ for $2k_F < k < 2k_F + k_c$ (a unit length of one-dimensional space available to the system is supposed). It is shown in the body of the present paper that this differerce in the kinematics of the two interaction processes produces a completely different dynamical behaviour of the system.

The forward scattering interaction has been treated within the Tomonaga-Luttinger model /7,8,11/. The backscattering interaction has been studied by means of both bosonization technique /9/ and renormalization group approach /4,5/. However, as Haldane /12/ pointed out recently, the particle - and spin-density degrees of freedom are not completely decoupled in the bosonization technique and, consequently, this method cannot be used for treating the backscattering interaction. Instead, the very interesting solution given by Luther and Emery applies to a more general model with spin-flip forward scattering interaction. As concerned the renormalization group approach the vertex part (scattering amplitude) is approximately calculated here for a particular choice of the external variables (see, for instance, Ref.2). With our notations this means either $p_1=k_F$, $p_3=-k_F$ for the Cooper pair diagrams or $p_2=k_F$, $p_3=-k_F$ for the zero sound channel. When the system is excited by creating two particle-hole pairs coupled to a given momentum transfer the backscattering process allowed by this particular choice of the vertex part leads to a density of states equal to 4,a figure which comes from the spin degrees of freedom only. Therefore, when one restricts oneself to this particular form of the vertex part the kinematics of the backscattering process is completely distorted.

It is the aim of this paper to give an adequate treatment of the backscattering process in the one-dimensional Fermi gas model with two-body spin-independent interaction. We should mention here that backscattering effects have been calculated within the Tomonaga-Luttinger model with forward scattering when the response of this system has been studied to an external field with momentum transfer near $2k_F$ /13/.

Our approach relies upon the Ward identity which is derived for the general case of non-relativistic fermions interacting through a two-body spin-independent force. In the one-dimensional case this identity enables us to obtain the irreducible polarizations and the effective interactions both for the forward and backward scattering processes in the limit of weak coupling strenghts. The dispersion relations of the particle-density excitations are readily obtained. Our perturbation theory follows the general lines of Dzialoshinsky and Larkin /11/. The perturbation theory is outlined in Section 2. In Section 3 the Ward identity is derived. Results are given in Section 4 and conclusions in Section 5.

II. Perturbation theory

Let us assume that the system consists of n fermions on the unit length ($k_F = \pi n/2$) interacting through a two-body spin-independent potential $v(|x-y|)$, x and y being spatial coordinates. Using a plane wave representation for the field operators,

$$\psi(x) = \sum_p c_p \, e^{ipx} , \qquad (1)$$

the hamiltonian of the system can be expressed as

$$\begin{aligned} H &= H_o + H_1 , \\ H_o &= \sum_p \varepsilon_p \, c_p^+ \, c_p , \\ H_1 &= \frac{1}{2} \sum_{kp_1p_2} V(k) c^+_{p_1+k} c^+_{p_2-k} c_{p_2} c_{p_1} , \end{aligned} \qquad (2)$$

where $c_p^+(c_p)$ is the creation (annihilation) operator of the p-fermion state, $\varepsilon_p = p^2/2m$ (m being the fermion mass) are the unperturbed single-particle energy levels and $v(k)$ is the spatial Fourier transform of the potential (the spin index is omitted for simplicity). The time dependence of the field operators in the interaction picture will be taken as

$$\psi(x,t) = \exp\left[i(H_o-\mu N)t\right] \psi(x)\exp\left[-i(H_o-\mu N)t\right] , \tag{3}$$

N being the operator of the total number of particles and μ- the chemical potential. Using Eq.(3) and the linearized form of the energy levels given in Sec.I (the Fermi velocity being taken equal to unit) the free Green function in the momentum space can be directly written down /11/ :

$$\begin{aligned} \overset{o}{G}_{+}(p,\varepsilon) &= \left[\varepsilon-p+k_F+i\eta\,\mathrm{sgn}(p-k_F)\right]^{-1} , \quad k_F-k_c < p < k_F + k_c , \\ \overset{o}{G}_{-}(p,\varepsilon) &= \left[\varepsilon+p+k_F+i\eta\,\mathrm{sgn}(-p-k_F)\right]^{-1} , \quad -k_F-k_c < p < -k_F + k_c , \end{aligned} \tag{4}$$

where $\eta = 0^+$ is a convergence factor and the subscripts + and - stand for the fermion states near $+k_F$ and $-k_F$, respectively. Throughout this paper the subscripts + and - of the Green functions will mean that the momentum variable p of these functions is restricted to either $+k_F - k_c < p < +k_F + k_c$ or $-k_F-k_c<p<-k_F+k_c$, respectively.

The Dyson equations for the Green function $G(p,\varepsilon)$ of the interacting system and for the effective interaction $V(k,\omega)$ are

$$\begin{aligned} G(p,\varepsilon) &= G^o(p,\varepsilon) + G^o(p,\varepsilon)\, \Sigma\,(p,\varepsilon)\, G(p,\varepsilon) , \\ V(k,\omega) &= v(k) + v(k)\, \Pi\,(k,\omega)\, V(k,\omega) , \end{aligned} \tag{5}$$

where $\Sigma(p,\varepsilon)$ and $\Pi(k,\omega)$ denote the proper self energy part and the irreducible polarization, respectively. The diagrammatic structure of $\Sigma(p,\varepsilon)$ and $\Pi(k,\omega)$ is shown in Figure 1 where the three-legged vertex function $\Gamma(p,\varepsilon;\ k,\omega)$ is introduced (the long-range component k=0 of the interaction is taken equal to zero so that the tadpole diagrams are excluded). The vertex function $\Gamma(p,\varepsilon;k,\omega)$ represents all irreducible diagrams with three external legs. According to the perturbation theory rules the analytic expressions of the diagrams shown in Figure 1 are

$$\Sigma\ (p,\varepsilon) = i(2\pi)^{-2} \int dk d\omega\ V(k,\omega)\ G(p-k,\ \varepsilon-\omega)\ \Gamma(p,\varepsilon;\ k,\omega)\ ,$$

$$\Pi\ (k,\omega) = -2i(2\pi)^{-2} \int dp d\varepsilon\ G(p,\varepsilon)\ G(p-k,\varepsilon-\omega)\ \Gamma(p,\varepsilon;\ k,\omega)\ . \tag{6}$$

Looking at Eqs.(5) and (6) one can see that there are five unknown quantities but four equations only. As for fith one the Ward identity, as derived in Sec.III, will be used.

III. Ward identity

As known from quantum electrodynamics the Ward identity relates the vertex function to the Green function. We shall derive here the Ward identity for non-relativistic fermions interacting through two-body spin-independent potential making use of the gauge invariance of the system /14/.

Let us perform a gauge transformation of the field operators

$$\psi(x) \to \tilde{\psi}(x,t) = \psi(x)\ e^{i\delta\chi(x,t)} \simeq \psi(x)\left[1+i\delta\chi(x,t)\right]\ ,$$

$$\psi^{+}(x) \to \tilde{\psi}^{+}(x,t) = \psi^{+}(x)\ e^{-i\delta\chi(x,t)} \simeq \psi^{+}(x)\left[1-i\delta\chi(x,t)\right], \tag{7}$$

where $\delta\chi(x,t)$ is a real, infinitesimal function of space-time variables which generates the gauge transformation. This transformation destroys the space-time homogeneity of the system so that the Green function in the momentum space will depend on two momentum variables. Starting from the definition of the Green function it is easy to see that the gauge transformation given by Eqs.(7) leads to the following first-order variation of the Green function :

$$\delta G(p+k,\varepsilon+\omega;p,\varepsilon) = i\delta\chi(k,\omega)\left[G(p,\varepsilon) - G(p+k,\varepsilon+\omega)\right] \tag{8}$$

where $\delta\chi(k,\omega)$ is the space-time Fourier transform of the function $\delta\chi(x,t)$. The Ward identity will be derived by requiring that the variation of the Green function given by Eq.(8) be equal to that obtained from the perturbation theory.

Under the gauge transformation the creation and annihilation operators of the one-fermion states acquire the form

$$\begin{aligned} \tilde{c}_p &= c_p + i\sum_k \delta\chi(k,t)\, c_{p-k} , \\ c_p^+ &= c_p^+ - i\sum_k \delta\chi(k,t)\, c_{p+k}^+ , \end{aligned} \tag{9}$$

$\delta\chi(k,t)$ being the space Fourier transform of the $\delta\chi(x,t)$.
Up to the first order in $\delta\chi(k,t)$ the original operators c_p and c_p^+ can be obtained from Eqs.(9) as

$$\begin{aligned} c_p &= \tilde{c}_p - i\sum_k \delta\chi(k,t)\, \tilde{c}_{p-k} , \\ c_p^+ &= \tilde{c}_p^+ + i\sum_k \delta\chi(k,t)\, \tilde{c}_{p+k}^+ . \end{aligned} \tag{10}$$

Using these expressions of the creation and annihilation operators one can see that the form of the interaction hamiltonian H_1 given by Eqs.(2) and the form of the operator N of the total number of particles are left unchanged under the gauge transformation while the kinetic hamiltonian H_o becomes

$$H_o = \sum_p \varepsilon_p \tilde{c}_p^+ \tilde{c}_p - i \sum_{p\,k} (\varepsilon_p - \varepsilon_{p-k}) \delta\chi(k,t) \tilde{c}_p^+ \tilde{c}_{p-k} \quad . \tag{11}$$

Obviously, the new operator $\tilde{c}_p$ and $\tilde{c}_p^+$ given by Eqs.(9) depends on time through the $\delta\chi(k,t)$ function. It is convenient for the perturbational approach to assign this time-dependence to the hamiltonian and to consider the creation and annihilation operators $\tilde{c}_p^+$ and $\tilde{c}_p$ as time-independent. One can think of this time-dependence as arising from an external field given by a term of the form

$$- \sum_{p\,k} \frac{\partial}{\partial t} \delta\chi(k,t) \; \tilde{c}_p^+ \tilde{c}_{p-k} \quad . \tag{12}$$

It follows from Eqs.(11) and (12) that the gauge transformation procedures an additional term in the hamiltonian which, with the original notations, can be written as

$$\delta H = - i \sum_{p\,k} (\varepsilon_p - \varepsilon_{p-k} - i \frac{\partial}{\partial t}) \delta\chi(k,t) \; c_p^+ c_{p-k} \quad . \tag{13}$$

The effect of this term on the Green function will be evaluated by means of the perturbation theory. Using the interaction picture given by Eq.(3) the first-order variation of the free Green function is

$$i\delta G^{\rho}(p_1,t_1;p_2,t_2) = -i \int_{-\infty}^{+\infty} dt < 0|T\left[\delta H(t) c_{p_1}(t_1) c_{p_2}^+(t_2)\right]|0>_c \quad , \tag{14}$$

where $|0>$ denotes the ground state of the non-interacting system, T is the time-ordering operator and the subscript c stands for the connected diagrams. By Fourier transforming both sides of Eq.(14) we get

$$\delta G^{o}(p+k,\varepsilon+\omega;p,\varepsilon) = -i(\varepsilon_{p+k}-\varepsilon_{p}-\omega)\delta\chi(k,\omega)G^{o}(p+k,\varepsilon+\omega)\, G^{o}(p,\varepsilon). \quad (15)$$

Equation (14) represents the first order of the perturbation theory. Switching on the full interaction and evaluating the contributions of all the terms of the perturbation series to the Green function amounts to dressing up the free Green functions in Eq.(15) and to introducing here the vertex function. This process is shown in Figure 2. Then Eq.(15) becomes

$$\delta G(p+k,\varepsilon+\omega;\, p,\varepsilon) = \quad (16)$$

$$= -i(\varepsilon_{p+k}-\varepsilon_{p}-\omega)\delta\chi(k,\omega)\, \Gamma(p+k,\varepsilon+\omega;k,\omega)\, G(p+k,\varepsilon+\omega)\, G(p,\varepsilon)$$

Comparing this result with that given by Eq.(8) we obtain immediately the Ward identity

$$\Gamma(p,\varepsilon;k,\omega) = \frac{G^{-1}(p,\varepsilon) - G^{-1}(p-k,\varepsilon-\omega)}{\varepsilon_{p-k} - \varepsilon_{p} + \omega} \quad (17)$$

We emphasize here that this is an exact result in the quantum field theory of the many-fermion systems with two-body spin-independent interaction. This identity will be used here for treating the forward and backward scattering processes in the one-dimensional Fermi gas model.

In the case of forward scattering processes there are two vertex functions, $\Gamma_{1+}(p,\varepsilon;\, k,\omega)$ and $\Gamma_{1-}(p,\varepsilon;\, k,\omega)$, corresponding to $+k_F - k_c < p < +k_F + k_c$ and $-k_F - k_c < p < -k_F + k_c$, respectively. In this case the Ward identity can be written as

$$\Gamma_{1+}(p,\varepsilon;k,\omega) = \frac{G_+^{-1}(p,\varepsilon) - G_+^{-1}(p-k,\varepsilon-\omega)}{\omega - k},$$
$$\Gamma_{1-}(p,\varepsilon;k,\omega) = \frac{G_-^{-1}(p,\varepsilon) - G_-^{-1}(p-k,\varepsilon-\omega)}{\omega + k}, \tag{18}$$

where the linearized form of the unperturbed energy levels has been used. These relations have been derived in Ref./11/ by diagrammatic methods. They have been also obtained using the equations of motion of the vertex function /15/. The Ward identity given by Eq.(17) allows the backscattering vertex functions to be written as

$$\Gamma_{2+}(p,\varepsilon;k,\omega) = \frac{G_+^{-1}(p,\varepsilon) - G_-^{-1}(p-2k_F-k,\varepsilon-\omega)}{2(k_F-p) + k + \omega},$$
$$\Gamma_{2-}(p,\varepsilon;k,\omega) = \frac{G_-^{-1}(p,\varepsilon) - G_+^{-1}(p+2k_F+k,\varepsilon-\omega)}{2(k_F+p) + k + \omega}, \tag{19}$$

where the momentum transfer k occurring in Eq.(17) has been replaced by $2k_F + k$ and $-2k_F - k$, respectively. This is convenient for keeping the variable k in the range $-2k_c < k < 2k_c$, as in the case of Eqs. (18).

We should mention here that a generalized Ward identity has been recently derived by Solyom /16/ for the Fermi gas model with forward scattering subject to an external field with momentum transfer near $2k_F$. This identity relates a three-legged vertex function to a four-legged vertex function, both of them having an interaction line which corresponds to the external field. Therefore, there is no relation between the three-legged vertex function introduced by Solyom and that used by us in the present paper.

IV. Results

Using the vertex functions given by Eqs.(18) for the forward scattering we get from Eqs.(6) the polarization parts for this process

$$\Pi_{1\pm}(k,\omega) = -2i(2\pi)^{-2}\frac{1}{\omega \pm k}\int dp d\varepsilon \left[G_{\pm}(p-k,\varepsilon-\omega)-G_{\pm}(p,\varepsilon)\right] =$$

$$= \pi^{-1}\frac{1}{\omega \pm k}\int dp\,(n^{\pm}_{p-k} - n^{\pm}_{p})\ , \qquad (20)$$

where $n^{\pm}_{p}$ is the momentum distribution near $+k_F$ and $-k_F$, respectively. The Dyson equation for the effective interaction of the forward scattering (Eqs.(5)) reads as

$$V_1(k,\omega) = v(k) + v(k)\left[\Pi_{1+}(k,\omega) + \Pi_{1-}(k,\omega)\right]V_1(k,\omega)\ . \qquad (21)$$

Assuming in the first approximation a step form of the momentum distribution corresponding to the non-interacting system /17/ we get

$$V_1(k,\omega) = \begin{cases} v(k)\left[1-2v(k)k^2/\pi(\omega^2-k^2)\right]^{-1}\ , & |k|< k_c\ , \\ v(k)\left[1-2v(k)|k|(2k_c-|k|)/\pi(\omega^2-k^2)\right]^{-1}\ , & -2k_c<k<-k_c, \\ & \text{and}\ \ k_c<k<2k_c. \end{cases} \qquad (22)$$

The singularities of the effective interaction provide us with the dispersion relation of the collective excitations of the density fluctuations :

$$\omega(k) = k\left[1 + 2v(k)/\pi\right]^{\frac{1}{2}} \quad , \quad 0 < k < k_c \quad , \qquad (23a)$$

$$\omega(k) = \left[k^2 + 2v(k)k(2k_c-k)/\pi\right]^{\frac{1}{2}} \; , \quad k_c < k < 2k_c \; . \qquad (23b)$$

These relations are symmetric with respect to $k \to -k$. In the limit of small k_c (as compared to k_F) the interaction $v(k)$ may be taken as constant, $v(k) = v$. The relations given by Eqs.(23a,b) hold for $v > -\pi/2$. Equation (23a) represents the well-known dispersion relation of the density fluctuations obtained for the first time by Tomonaga /6/.

Using the Ward identity given by Eqs. (19) for the backscattering interaction we get the polarization parts

$$\Pi_{2\pm}(k,\omega) = -2i(2\pi)^{-2} \int dp d\varepsilon \; \frac{G_{\mp}(p\mp 2k_F\mp k, \varepsilon-\omega) - G_{\pm}(p,\varepsilon)}{2(k_F \mp p) + k + \omega} =$$

$$= \frac{1}{\pi} \int dp \; \frac{n^{\mp}_{p\mp 2k_F\mp k} - n^{\pm}_{p}}{2(k_F\mp p) + k + \omega} \; . \qquad (24)$$

Let us calculate explicitly $\Pi_{2+}(k,\omega)$. As the momentum variables are restricted to $+k_F-k_c < p < +k_F+k_c$ and $-k_F-k_c < p < -2k_F-k <$ $< -k_F+k_c$ we obtain for $0 < k < k_c$

$$\Pi^{-}_{2+}(k,\omega) = \frac{1}{\pi} \int_{k_F-k_c+k}^{k_F+k_c} dp \; \frac{n^{-}_{p-2k_F-k} - n^{+}_{p}}{2(k_F-p) + k + \omega} \qquad (25)$$

Using the step form of the momentum distribution we get

$$\Pi_{2+}(k,\omega) = -\frac{1}{2\pi}\ln\left|\frac{(2k_c-|k|)^2-\omega^2}{k^2-\omega^2}\right| \qquad |k| < k_c . \quad (26)$$

In the same way we get $\Pi_{2+}(k,\omega)= 0$ for $k_c < |k| < 2k_c$. By straightforward calculation we obtain also that $\Pi_{2-}(k,\omega) = \Pi_{2+}(k,\omega)$. It results that the effective interaction in the back-scattering process is

$$V_2(k,\omega) = u(k)\left[1 + \frac{u(k)}{2\pi}\ln\left|\frac{(2k_c-|k|)^2-\omega^2}{k^2-\omega^2}\right|\right]^{-2}, \quad |k|< k_c , \quad (27)$$

where $u(k) = v(2k_F+k) = v(-2k_F-k)$ is the backscattering coupling strength that can be taken as constant, $u(k) = u$. It results immediately from Eq.(27) the dispersion relation of the collective excitations induced by the backscattering interaction :

$$\omega(k) = \left[k^2 + 4k_c(k_c-k) / (1 \mp a)\right]^{\frac{1}{2}}, \quad 0 < k < k_c , \qquad (28)$$

where $a = \exp(-2\pi/u) > 0$. For repulsive interaction $u > 0$ the a parameter is smaller than unit and the frequency given by Eq.(25) exhibits a gap at k=0 of magnitude $2k_c(1\mp a)^{-\frac{1}{2}}$. One can see that this gap is proportional to k_c, a fact that is suggestive of the finite density of states available in the backscattering interaction with momentum transfer $2k_F$ (see Sec.I). In the case of attractive interaction, $u < 0$, a exceeds the unit and the branch of the frequency given by Eq.(25) containing $(1-a)$ becomes imaginary at wavevectors smaller than $2k_c(\sqrt{a}-1)/(a-1)$. This result points out an instability of the ground state of the system against attractive backscattering interaction. Therefore, one sees that the backward-scattering interaction produces a completely different behaviour of the system as compared to the forward scattering interaction.

V. Conclusions

The Ward identity has been derived for non-relativistic many-fermion systems with two-body spin-independent interaction. Using this identity the backscattering interaction has been treated in the one-dimensional Fermi gas model. The dispersion relation of the density fluctuations in the case of backward scattering (Eq.(28)) exhibits some interesting features. Among these we mention the occurrence of a gap at wavevector $2k_F$ ($k=0$ in Eq. (28)) for repulsive interaction and the imaginary values taken by the frequency at wavevectors smaller than a finite value for attractive interaction. This is an indication of an instability occurring in the system with attractive backward interaction. The nature of this instability and its connection to a possible phase transition requires further investigation.

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/17/ The calculation of the momentum distribution in the interacting ground state of the system with backward scattering involves an amount of algebra too length to be given here, and will be published elsewhere. Actually, this calculation shows that, in case of weak strengths of interaction, a negligible error is made when replacing the exact form of the momentum distribution in Eq.(20) by the step form corresponding to the non-interacting system.

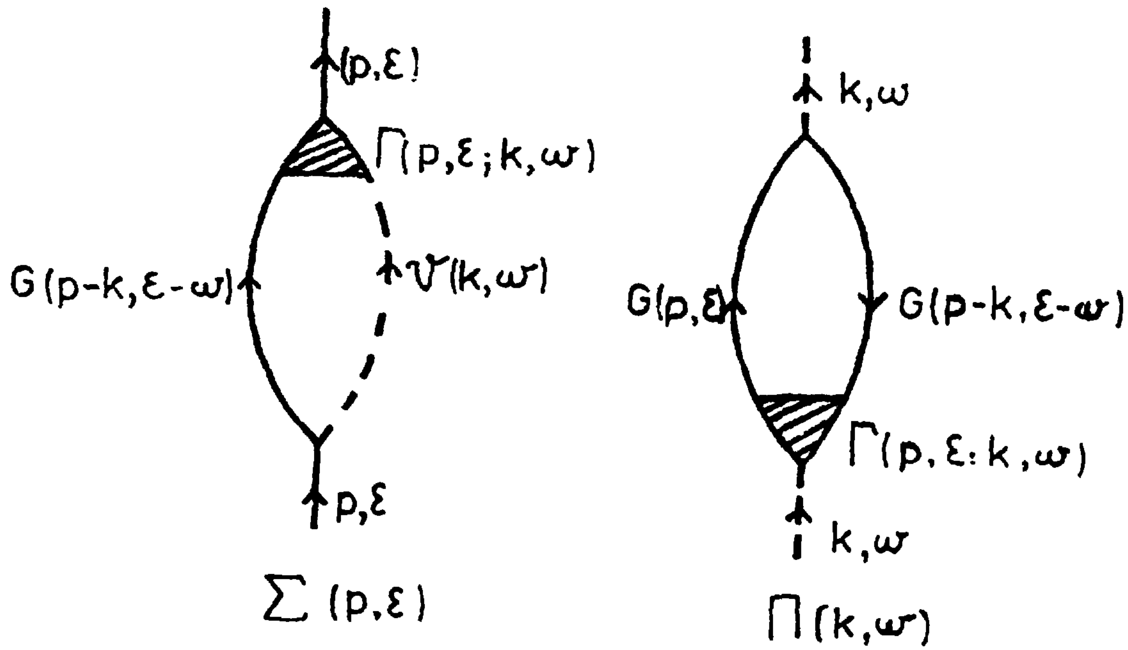

Figure 1. The diagrammatic structure of $\Sigma(p,\varepsilon)$ and $\Pi(k,\omega)$. The three-legged vertex function is denoted by $\Gamma(p,\varepsilon;\ k,\omega)$.

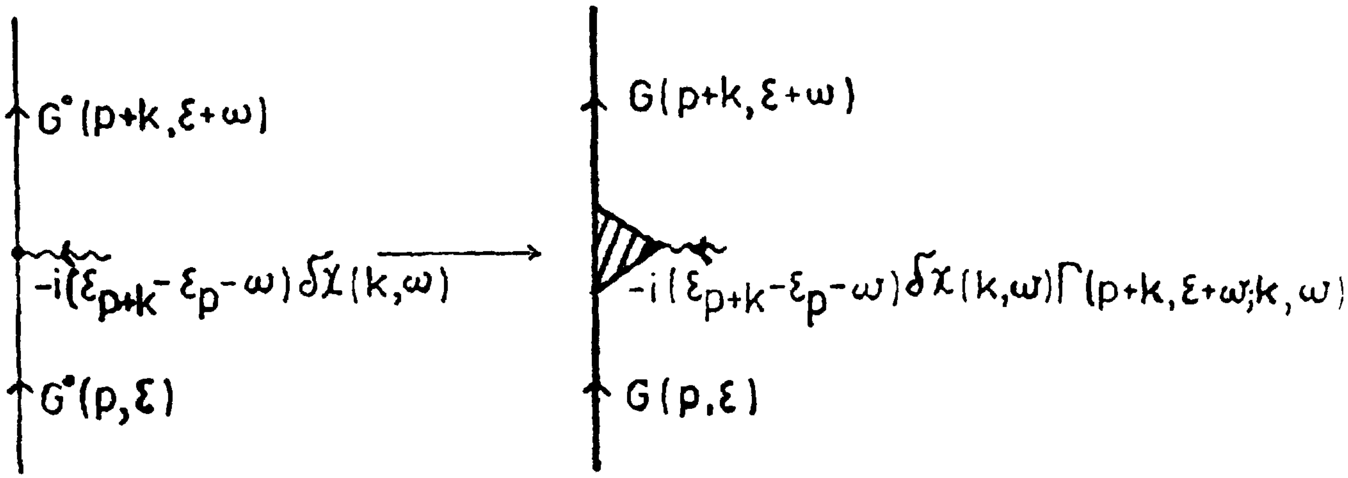

Figure 2. Dressing up the first-order variation of the Green function with interaction. The light lines represent free Green functions.

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