TRN AU8105055

UM-P-80/30

OSCILLATION AND MIXING OF MASSLESS ELECTRON AND MUON NEUTRINOS

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Abstract

It is not necessary that the electron neutrino and muon neutrino themselves have mass for $\nu_e - \nu_\mu$ oscillations to occur. For N neutrinos, if one is massive, oscillations occur with unique oscillation length determined by the mass of the heavy neutrino, mimicing two neutrino mixing. Any observation of $\nu_e - \nu_\mu$ oscillations cannot therefore be regarded as evidence for a finite mass ν_e or ν_μ.

The conventional discussion of $\nu_e - \nu_\mu$ oscillations assumes that at least one of these two neutrinos is massive.[1] The discovery of a third generation neutrino, ν_τ, and the possibility that there may be additional neutrinos makes more complex oscillation phenomena possible.[2,3] It does not appear to have been noticed that in these circumstances $\nu_e - \nu_\mu$ oscillations can occur even if both ν_e and ν_μ are massless. All that is required is that at least one of the neutrinos has a finite mass, and that it is kinematically possible for this heavy neutrino to be produced in the reaction producing the neutrino beam. An immediate consequence of this observation is that observation of $\nu_e - \nu_\mu$ oscillations does not imply limits on the masses of $\nu_e - \nu_\mu$, contrary to popular belief, and that a reanalysis of the existing experimental limit is necessary.

We consider first the general neutrino system.[2] The eigenstates of the mass matrix will be denoted by ν_i $(i = 1, \ldots, n)$ and the neutrino states which diagonalise the weak interaction currents by N_i. The corresponding charged leptons will be labelled L_i.

$$N_i = \sum_j U_{ij} \nu_j \; ; \quad \nu_j = \sum_j N_i U_{ij}^* \tag{1}$$

is the unitary transformation connecting the weak interaction and mass eigenstates.

Suppose we start with the weak interaction production of N_2 with momentum P at time $t = 0$. Note that it must be kinematically possible to produce all of the ν_i in the reaction which provides the beam for N_2 to be the initial state. The neutrino state at a later time t is

$$\begin{aligned} |2;t\rangle &= e^{-iHt} \, |N_2\rangle \\ &= \sum_j U_{2j} \, e^{-iE_j t} \, |\nu_j\rangle \end{aligned} \tag{2}$$

$E_j = \sqrt{m_j{}^2 + P^2}$ where m_j is the mass of the jth neutrino. The probability of observing the weak production of the charged lepton L_1 at the time t is proportional to the probability $P_{21}(t)$ that the neutrino is in the state N_1 at this time

$$P_{21}(t) = | < N_1 | 2,t > |^2$$

$$= | \sum_j U^*_{1j} \ U_{2j} \ e^{-iE_j t} |^2 \qquad (3)$$

Notice that if all the neutrinos are degenerate in mass $P_{21}(t) = 0$. This confirms the fact that in these circumstances the neutrino labeling can be defined by the N_i, as these are also mass eigenstates.

In general if at least one neutrino has a different mass to the other equation (3) will contain oscillatory terms. This special neutrino need not be either ν_1 or ν_2. Since a detailed analysis of (3) would serve to confuse rather than illustrate the main point of this letter, I specialise to the case where n - 1 of the neutrinos have zero mass and one, ν_n, has a non zero mass m. Then $E_j = P$ for $j = 1, \ldots, n - 1$ and

$$P_{21}(t) = | \sum_{j=1}^{n-1} U_{2j} \ U^*_{1j} + e^{i\Delta t} \ U_{2n} \ U^*_{1n} |^2$$

where $\Delta = P - E_n$. The unitarity of U shows that

$$\sum_{j=1}^{n-1} U_{2j} \ U^*_{1j} = - U_{2n} \ U^*_{1n}$$

so in this case

$$P_{21}(t) = |2 \ U_{2n} \ U^*_{1n}|^2 \ \sin^2 \frac{\Delta t}{2} \qquad (4)$$

This reduces to the usual formula[4] for two neutrino mixing on setting $n = 2$, $U_{22} = \cos\theta$, $U_{12} = \sin\theta$. This suggests we define θ^*_{12}, an effective mixing angle, by $\sin 2\theta^*_{12} = |2\, U_{2n} U^*_{1n}|$

Then $$P_{21}(t) = \sin^2 2\theta_{12}{}^* \sin^2 \frac{\Delta t}{2}, \qquad (5)$$

which has the same form as the usual formula.

A similar calculation shows that $P_{22}(t)$, the probability of detecting type 2 neutrinos in the beam at time t may also be cast in the usual form

$$P_{22}(t) = 1 - \sin^2 2\theta^*_{22} \sin^2 \frac{\Delta t}{2} \qquad (6)$$

with a new effective mixing angle defined by

$$\sin^2 2\theta^*_{22} = 4\, U_{2n}\, U^*_{2n}\, (1 - U_{2n}\, U^*_{2n})$$

(i.e. either $\cos^2 \theta_{22} = U_{2n}\, U^*_{2n}$ or $\sin^2 \theta_{22} = U_{2n}\, U^*_{2n}$).

As long as oscillations are detected by observing one type of neutrino only, the present model mimics the standard massive $\nu_e - \nu_\mu$ mixing model. It can be distinguished if more than one type of neutrino is detected, since the effective mixing angles are not the same.

Even if a sinusoidal $\nu_e - \nu_\mu$ oscillation with a single period is detected, this phenomenon is not evidence for a $\nu_e - \nu_\mu$ mass difference. The mixing and oscillations can be the result of another neutrino having a finite mass. In the general case the oscillation will be a complicated mixture of a number of different oscillation periods.

In the case $m \ll P$, the oscillation time Δ^{-1} becomes $2P/m^2$ and (5) takes the form

$$P_{21}(t) = \sin^2 2\theta_{12}^* \sin^2 \frac{m^2 t}{4P} \tag{7}$$

This formula, with the interpretations $\theta_{12}^* = \theta$, the $\nu_e - \nu_\mu$ mixing angle, and $m = \sqrt{m_\mu^2 - m_e^2} = M_{\mu e}$, was used to analyse the attempt to observe $\nu_\mu - \nu_e$ oscillations in the CERN neutrino beam.[5] The results of that experiment were expressed as allowed regions in the $(\sin 2\theta, M_{\mu e})$ plane. The allowed region is well represented by the analytic expression

$$\sin 2\theta \, M_{\mu e}^2 < 1 \text{ eV}^2 .$$

Our analysis shows that the data can equally well be regarded as placing restrictions on the mass and mixing angles of a single heavy neutrino,

$$2 \, U_{2n} U_{1n}^* \, m^2 < 1 \text{ eV}^2$$

and that more complex interpretations are possible.

Clearly observation of neutrino oscillations, while it will demonstrate the existence of at least one massive neutrino will not provide us with unambiguous information on the neutrino masses. We will not even know which neutrino has the mass.

It is a pleasure to thank Dr. P. Herezeg for helpful discussions, and the T5 group at Los Alamos for the hospitality which made those discussions possible.

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