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NEUTRINO OSCILLATION AND RARE MUON DECAYS IN THE PRESENCE OF A HEAVY NEUTRINO

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Abstract

Heavy neutrinos have been recognised as providing a possible mechanism for muon number violating processes. Rates for these processes depend on neutrino mixing angles and masses. If the heavy neutrino mass is less than 400 MeV I show that the present limit on $\nu_\mu - \nu_e$ oscillations provides very stringent limits on the contribution of these processes to the rates for muon number violating processes.

Muon number non-conserving processes have attracted wide experimental and theoretical attention in recent years.[1,2] One mechanism which could be responsible for such processes is a possible heavy neutrino. In the presence of a heavy neutrino, the eigenstates of the neutrino mass matrix ν_e, ν_μ, ν_τ, ν_x .. etc are no longer identical with the states which diagonalise the weak currents ν_e', ν_μ', ν_τ', ν_x', ... The weak eigenstates may be obtained from the mass eigenstates by a unitary transformation.[3] For n neutrinos

$$\nu_i' = \sum_{j=1}^{n} U_{ij}\, \nu_j . \tag{1}$$

(n = 1,2,3 will correspond to e, μ and τ neutrino respectively.)
For the special case n = 3, and $m(\nu_e) = m(\nu_\mu)$ a convenient parameterisation of the matrix U in terms of small mixing angles β and γ has been given by Altarelli et al.[4] I will treat the analogous n neutrino case, with n-1 neutrinos having equal mass m_1 which may be zero, and the n th neutrino having a mass $m_n > m_1$. The processes of interest are $\mu \to e\gamma$, $\mu \to 3e$ and (μ^-, e^-) reactions on nuclei. In the presence of the heavy neutrino ν_n and the neutrino mixing parameterised by the matrix U_{ij}, these processes can occur through the graphs of figure 1. The branching ratios and relative rates for these processes are [5]

$$B(\mu \to e\gamma) = \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu_e\nu_\mu)$$

$$\simeq \frac{3\alpha}{32\pi} \left|U_{\mu n} U_{en}^*\right|^2 \left(\frac{m_n}{m_W}\right)^4 \tag{2}$$

$$B(\mu \to 3e) = \Gamma(\mu \to 3e)/\Gamma(\mu \to e\nu_e\nu_\mu)$$

$$\simeq \frac{3\alpha^2}{16\pi^2} \left|U_{\mu n} U_{en}^*\right|^2 \left(\frac{m_n}{m_W}\right)^4 \ell n \left(\frac{m_n}{m_W}\right)^2 \tag{3}$$

$$B(\mu^- Z \to e^- Z) = \Gamma(\mu^- Z \to e^- Z)/\Gamma(\mu^- Z \to \nu_\mu ;\ Z-1)$$

$$\simeq C_v(A,Z)\ |U_{\mu n}\ U^*_{en}|^2 \left[\frac{3\alpha}{8\pi} \left(\frac{m_n}{m_w} \sin\theta_w \right)^2 \ln \frac{m_w^2}{m_n^2} \right]^2 \tag{4}$$

Where the function $C_v(A,Z)$ has been tabulated by Shankar.[6]

The conventional approach to estimation of these branching ratios has relied on the value

$$|U_{\mu n}\ U^*_{en}|^2 < 2 \times 10^{-3} \tag{5}$$

obtained from the CERN experiments on non-detection of electron neutrinos in the muon neutrino beam.[7] This limit follows from the data only if it is assured that $m_m > 400$ MeV. In these circumstances ν_n cannot be produced in the K and π decays that generate the ν_μ beam. The neutrino state produced in $K,\pi \to \mu\nu_\mu$" is

$$|\nu_\mu"> = \sum_{j=1}^{n-1} U_{\mu j}\ |\nu_j>$$

and the probability of weak electron production occuring at some later time is

$$P_{\mu e} = |<\nu_e|\nu_\mu"\ >|^2$$

$$= |\sum_{j=1}^{n-1} U_{\mu j}\ U^*_{ej}|^2$$

$$= |U_{\mu n}\ U^*_{en}|^2 \tag{6}$$

where the last step uses the unitarity of U_{ij}. The limit (5) follows from ref. 7.

If however m_n < 400 MeV ν_n can be present in the beam. In this case $\nu_e - \nu_\mu$ oscillations occur even if the ν_e and ν_μ both have zero mass. The state ν_μ' produced in $K \to \mu\nu_\mu'$ is

$$|\nu_\mu'\rangle = \sum_{j=1}^{n} U_{\mu j} |\nu_j\rangle = |\nu_\mu'; t = 0\rangle \qquad (7)$$

which evolves in time t to the state

$$|\nu_\mu'; t\rangle = \sum_{j=1}^{n} U_{\mu j} e^{-iE_j t} |\nu_j\rangle \qquad (8)$$

where $E_j = \sqrt{m_j^2 + P^2}$ and P is the neutrino momentum. Now the E_j are not all identical and $|\nu_\mu'; t\rangle$ is not the same state as $|\nu_\mu', t = 0\rangle$ $P_{\mu e}$ is now time dependent, and is given by

$$P_{\mu e}(t) = |2\, U_{\mu n} U_{en}^*|^2 \sin^2 \frac{\Delta t}{2} \qquad (9)$$

where $\Delta = E_n - E_e \simeq \frac{m_n^2}{2P}$ for $P >> m_n$ and $m_1 = 0$.

The data of reference 7 were analysed for oscillations using the formula

$$P_{\mu e}(t) = \sin^2 \frac{M^2 t}{4P} \sin^2 2\alpha \qquad (10)$$

appropriate to $\nu_\mu - \nu_e$ mixing. α is the mixing angle and $M = \sqrt{m^2(\nu_\mu) - m^2(\nu_e)}$ << P. Blietschen et al give their results for the limits imposed by the non-observation of ν_e induced events as allowed regions in the $(\sin 2\alpha, M)$ plane. It turns out that these limits can be adequately expressed by the approximate formula

$$\sin 2\alpha\, M^2 < 1 eV^2. \qquad (11)$$

Comparing equations (9) and (10) we see that the result (11) can be directly reinterpreted in terms of our model as requiring

$$|2\, U_{\mu n}\, U_{en}^{*}|\, M_{n}^{2} \;<\; 1\mathrm{eV}^2 \qquad (12)$$

as long as $P \gg m_n$. Equation (19) is much more restrictive than equation (5) with $m_n \sim 250$ MeV which would give $|2U_{\mu n}\, U_{en}^{*}| m_n^{2} < 500\ \mathrm{MeV}^2$.

Equation (12) limits just the combination of parameters required to evaluate $B(\mu \to e\gamma)$. Thus we see that, if the massive neutrino has a mass less than 400 MeV,

$$B(\mu \to e\gamma) \;<\; 6.9 \times 10^{-49}.$$

The logarithmic terms in $B(\mu \to 3e)$ and $B(\mu^{-}Z \to e^{-}Z)$ preclude exact limits without recourse to additional assumptions, but it is clear that restrictions from the neutrino oscillation experiments lead to exceedingly small estimates for these quantities also.

In conclusion I reiterate that, if muon number nonconserving processes occur through neutrino mixing because of a neutrino of mass less than 400 MeV the present estimates of branching ratios are reduced by a factor of about 10^{20} because of experimental limits on neutrino oscillations. The analysis used heretofore applies only in the case that the heavy neutrino has a mass greater than 400 MeV, which is in any case necessary for muon number nonconservation to be explained by this mechanism if the branching ratios are observed near the present experimental limits.

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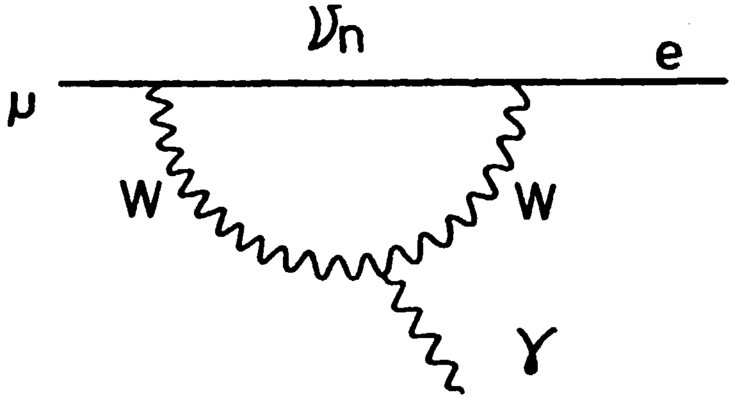
Figure Caption

Heavy neutrino contributions to

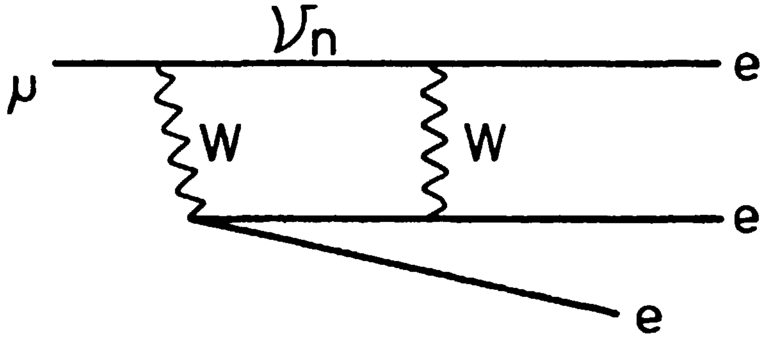
(a) $\mu \to e\gamma$

(b) $\mu \to eee$

(c) $\mu + (Z,A) \to e + (Z,A)$



(a)



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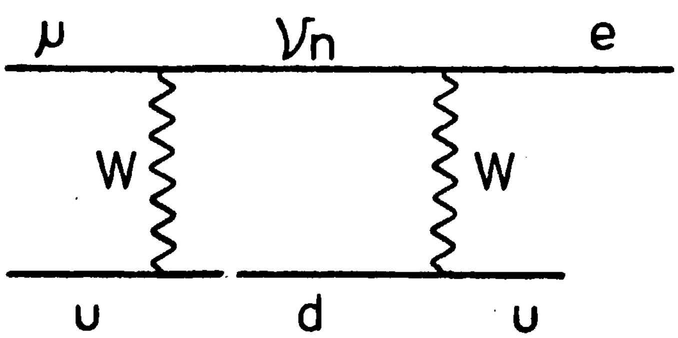

(c)