IRN AU8105062

UM-P-80/47

MASSIVE NEUTRINOS, THE GOLDBERGER-TREIMAN RELATION, AND THE $\pi \to e\nu$ BRANCHING RATIO

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Abstract

A neutrino of mass greater than 1 MeV provides a simple explanation of the residual Goldberger-Treiman discrepancy. However this reduces the $\pi \to e\nu$ branching ratio below the classical V-A value, in contradiction with the present experimental observation.

Recent suggestions of neutrino oscillations (De Rujula et al 1979, Barger et al 1980, Reines, Sobel and Pasieb, 1980) force us to consider the possibility of massive neutrinos seriously. In the note I emphasise that the existence of a neutrino with a mass greater than 1 MeV would provide an explanation of the remaining Goldberger-Treiman discrepancy. A similar explanation of the residual discrepancy involving a neutral heavy lepton was proposed by Bailin and Dombey (1976).

One can regard the Goldberger-Treiman relation (Goldberger & Treiman 1958) as a calculation of the decay constant f_π for $\pi \to \mu + \nu$ in terms of the axial vector form factor of the nucleon G_A and the strong πNN coupling constant g and form factor $F_{\pi NN}$

$$f_\pi = \frac{g_A(0)}{\sqrt{2}\, g} \frac{(m_n + m_p)}{F_{\pi NN}(0)} \qquad (1)$$

Estimates of $F_{\pi NN}(0)$ have been made in specific models (Jones and Scadron 1975) and using dispersion theory (Cass and McKellar 1980, Cass 1980). These are mutually consistent and lead to a value for f_π which is 3±1% <u>less than</u> the observed value. The failure of many attempts to correct this discrepancy using strong interactions suggests that perhaps one should inspect the weak interactions more closely.

Such a residual discrepancy is to be expected in models in which heavy neutrinos are present. In such models neutrino mixing and neutrino oscillations occur. In particular the electron and muon neutrinos ν_e', ν_μ' which couple to e and μ in the weak interactions will be linear combinations of the eigenstates ν_e, ν_μ, ν_τ, ν_α ..... of the neutrino mass matrix.

$$\nu_e' = \sum_i \nu_i \langle \nu_i | \nu_e' \rangle \qquad (2)$$

$$\nu'_\mu = \sum_i \nu_i \langle \nu_i | \nu'_\mu \rangle \tag{3}$$

If all neutrinos are sufficiently light to be emitted in neutron β decay and in π decay the calculation of f_π is not affected. However if at least one neutrino cannot be emitted in β decay, then the state $\bar{\nu}_e$ emitted in β decay has a norm less than unity. Write $|\langle\bar{\nu}_e|\bar{\nu}_e\rangle| = 1 - \frac{1}{2}\gamma^2$.

The state $\bar{\nu}_\mu$ emitted in $\pi \to \nu\mu$ may or may not have norm 1, depending on the mass of the heavy neutrinos. I will write $|\langle\bar{\nu}_\mu|\bar{\nu}_\mu\rangle| = 1 - \frac{1}{2}\beta^2$ with the understanding that β may be zero.

Nambu's (1960) pole model derivation of equation (1) makes it clear that the calculated f_π is $f_\pi^{(e)}$, the coupling constant appropriate to $\pi \to e\nu_e$. However the observed f_π is calculated from the decay rate for $\pi \to \mu\nu_\mu$, so it is $f_\pi^{(\mu)}$, the coupling constant appropriate to μ decay. Defining $f_\pi^{(0)}$ in terms of matrix elements of the axial vector current.

$$\langle 0 | A^i_\mu(0) | \pi^j \rangle = i\,\delta^{ij}\, q_\mu f_\pi^{(0)} \tag{4}$$

It is easy to see that

$$f_\pi^{(e)} = f_\pi^{(0)}\left(1 - \frac{1}{2}\gamma^2\right) \tag{5}$$

and

$$f_\pi^{(\mu)} = f_\pi^{(0)}\left(1 - \frac{1}{2}\beta^2\right)$$

Thus

$$f_\pi^{(e)} = f_\pi^{(\mu)} \frac{\left(1 - \frac{1}{2}\gamma^2\right)}{\left(1 - \frac{1}{2}\beta^2\right)} \tag{6}$$

Thus, if we identify $f_\pi^{(e)}$ with the value calculated from eqn.(1), and $f_\pi^{(\mu)}$ with the value calculated from the $\pi \to \mu\nu$ rate (i.e. the observed value), we can interpret the empirical result that

$$\frac{f_\pi^{(\mu)}}{f_\pi^{(e)}} = 1.03 \pm 0.01$$

as a statement that

$$\frac{1 - \frac{1}{2}\gamma^2}{1 - \frac{1}{2}\beta^2} = 0.97 \pm 0.01$$

This result requires $\gamma \neq 0$, and $0 \leq \beta < \gamma$. This could be regarded as weak evidence for a neutrino of mass greater than 1 MeV. The present experimental situation regarding ν_e oscillations requires some of the massive neutrinos to be lighter than 1 MeV so they may be emitted in β decay. These experiments do not require neutrinos heavier then 1 MeV, nor do they exclude them.

Finally I remark that this interpretation of the Goldberger-Treiman relation predicts a deviation of the branching ratio for $\pi \to e\nu$ from the cannonical value. (compare Bailin and Dombey 1976, Kim and Kim 1978). In the present model

$$B(\pi \to e\nu) = \left(\frac{1 - \frac{1}{2}\gamma^2}{1 - \frac{1}{2}\beta^2}\right)\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2$$

Using the above value for the first factor on the right, we predict

$$B(\pi \to e\nu) = (1.2 \pm 0.02) \times 10^{-4}$$

which is just compatible with the experimental value of $(1.26 \pm 0.02) \times 10^{-4}$. Of course to this level of accuracy radiative corrections must be included in the calculation, and the total rate for non radiative and radiative decays compared. When this is done adapting the results of Goldman and Wilson (1977) we find

$$R_\gamma = \frac{\Gamma(\pi \to e\nu + \pi \to e\nu\gamma)}{\Gamma(\pi \to \mu\nu + \pi \to \mu\nu\gamma)} = (1.16 \pm 0.02) \times 10^{-4}$$

This is significantly different from the present experimental value (Bryman and Picciotto, 1975).

$$R_\gamma^{exp} = (1.274 \pm 0.024) \times 10^{-4},$$

We would strongly urge the remeasurement of R_γ, noting that it is also slightly in disagreement with the standard theory result $R_\gamma^{std} = (1.23 \pm 0.02) \times 10^{-4}$.

In conclusion we emphasise that a residual Goldberger-Treiman discrepancy can be explained by the introduction of a heavy neutrino. As a consequence R_γ is reduced from the standard value. Reconciling $R_\gamma^{exp} > R_\gamma^{std}$ to a positive residual Goldberger-Treiman discrepancy will require some other mechanism.

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