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THE INFLUENCE OF MIXING OF FINITE MASS NEUTRINOS ON BETA DECAY SPECTRA

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Abstract

It is shown that neutrino mixing, of the type required for neutrino oscillations, will complicate efforts to determine the electron neutrino mass from β decay spectra. The shape of the spectra will depend on the masses and mixing angles of all neutrinos which couple to the electron neutrino.

The traditional method of measuring the mass of the electron neutrino has been to determine deviations from a straight line Kurie plot for the β decay of tritium. The possible observation of neutrino mixing and oscillations[2,3] has rekindled interest in the measurements of the neutrino mass, in that such oscillations and mixing can occur in vacuum only if at least one of the neutrinos has a mass[4].

In this letter I point out that the Kurie plot for β decay in the presence of mixing of massive neutrinos depends on the mixing angles and masses of all the neutrinos which couple to the electron. In these circumstances it is not possible to interpret the spectrum of β decay in terms of a single mass parameter. The spectrum end point is determined by the mass of the lightest neutrino, but the shape is determined by all of the neutrinos.

To see this we note the usual expression for the Kurie function $K = \frac{\sqrt{dN/d|p|}}{p^2 F}$ where $\frac{dN}{d|p|}$ is the electron spectrum, which when a neutrino of mass m is emitted in the decay takes the form[5]

$$K_\nu^2 = |M|^2 \, \delta(\delta^2 - m_\nu^2)^{\frac{1}{2}} \, \theta(\delta - m_\nu) \qquad (1)$$

where $\delta = E_{\beta max} - E_\beta$, $E_{\beta max}$ being the end point energy when the neutrino has zero mass. M is a matrix element which for ^3H decay is unity in the V-A theory.

If there is neutrino mixing of the type required by neutrino oscillations then the neutrino N_e which couples to the electron is a linear combination of neutrinos ν_i ($i = 1, \ldots, n$) which are mass eigenstates.

$$N_e = \Sigma \, \alpha_i \nu_i \qquad (2)$$

The Kurie function for β decay of tritium will then be given in a V-A theory by

$$K^2 = \sum_i P_i K_{\nu i}^2 = \sum_i P_i \, \delta(\delta^2 - m_{\nu i}^2)^{\frac{1}{2}} \, \theta(\delta - m_{\nu i}) \qquad (3)$$

where $P_i = |\alpha_i|^2$ is the probability that the neutrino ν_i is emitted in β decay.

Clearly the Kurie plot will be a complicated function of the P_i and $m_{\nu i}$. It will be very difficult to obtain clean information about neutrino masses from such a plot without prior knowledge of the P_i. This is in spite of the fact that the Kurie plot end point is determined by the lightest neutrino mass, since the very poor statistics at the end point of the spectrum and the finite resolution of any experiment make it very difficult to determine the position of the end point. More information is obtained from the spectrum shape near the end point in the usual analysis!

The spectrum given in equation (3) has a vertical slope at $\delta = m_{\nu i}$, for each i. These slope changes in the spectrum will again be smeared by the experimental resolution and will be difficult to detect. Another general feature of the modified Kurie spectrum is the form which it takes at large δ. Then

$$K = P_o^{\frac{1}{2}} \, \delta\{1 - \frac{\bar{M}^2}{4\delta^2} + \dots\}$$

where $P_o = \sum_i P_i$ and the sum is over those ν_i which are light enough to be emitted in the decay. Kurie plots are still asymptotically linear, but the slope is not necessarily unity. However in this paper I will concern myself with the case where all neutrinos have masses much less than $E_{\beta max}$, when $P_o = 1$. Some possible consequences of $P_o \neq 1$ are discussed elsewhere[6]. Deviations from straight line behaviour in the large δ region of the plot provide a measure of the mean squared mass $\bar{M}^2$ defined by

$$P_o \bar{M}^2 = \sum_i P_i m_{\nu i}^2$$

It is difficult to reanalyse the large δ data of ref.1 in terms of $\bar{M}^2$. One can however ask what range of values of P_i and $m_{\nu i}$ are compatible with the data near the end point. The large range of possibilities precludes a detailed analysis. But for example the "large mass" solution of Barger et al[7], realised by $m_{\nu e} = 0$, $m_{\nu_\mu} = 1eV$ $m_{\nu_\tau} = 7eV$ is compatible with the data, as is for example $m_{\nu_e} = 1eV$, $m_{\nu_\mu} = 100eV$, $P_{\nu_e} = .999$ $P_{\nu_\mu} = 0.001$.

Some characteristic spectra are illustrated in figure 1 and figure 2.

If neutrino oscillations are unequivocally established, then it will be necessary to use information on mixing angles and mass differences from the oscillation experiments in an analysis of beta decay spectra to determine the neutrino masses. Beta decay experiments undertaken to measure the electron neutrino mass should be analysed with the fact that they may be measuring not one mass but several kept in mind.

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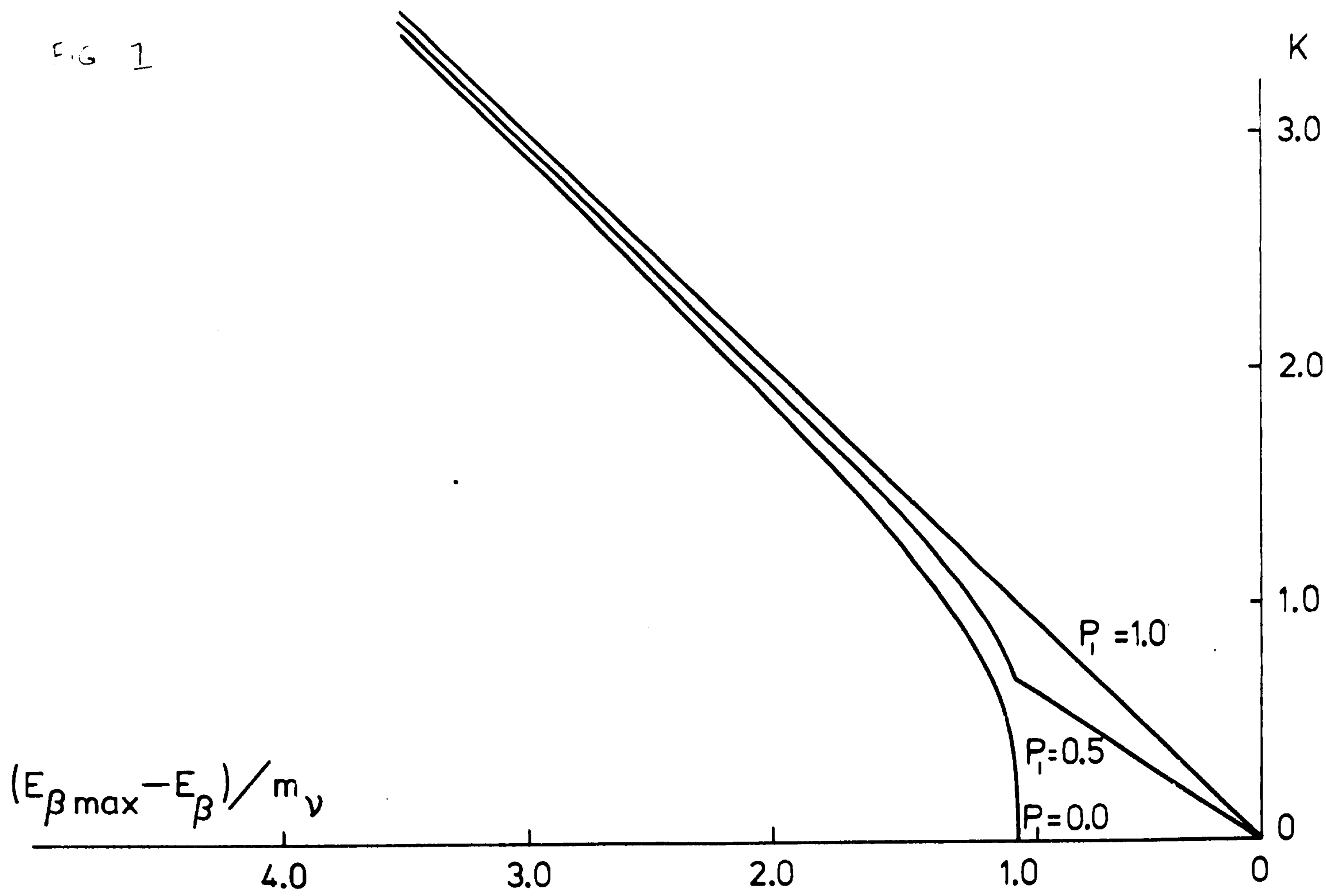
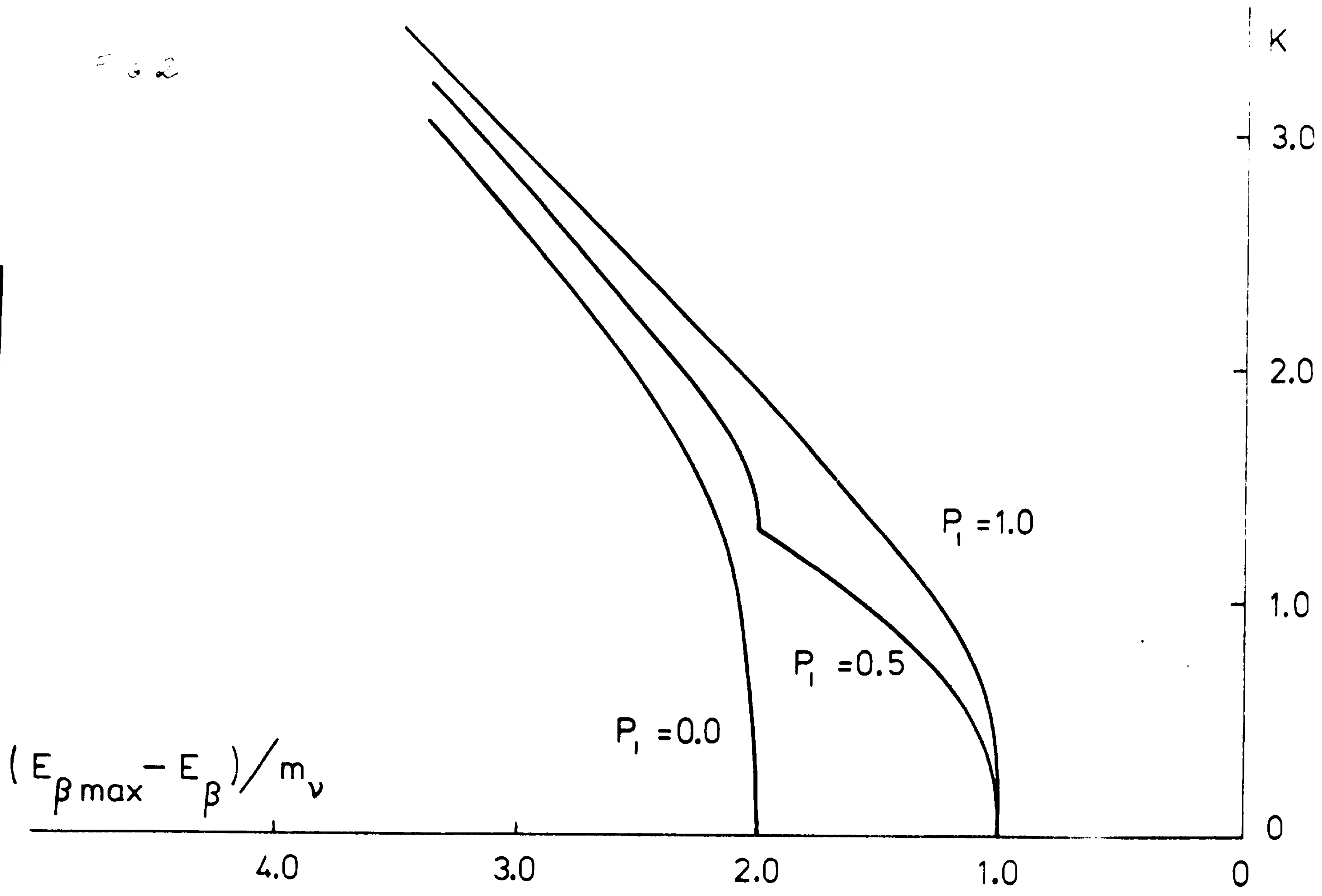
Figure Captions

1. Kurie plot for mixing of neutrino of mass zero and m_ν for different mixing probabilities.

2. Kurie plot for mixing of neutrinos of mass m_ν and $2m_\nu$ for different mixing probabilities.

FIG 1





K
3.0
2.0
1.0
0
$P_1 = 1.0$
$P_1 = 0.5$
$P_1 = 0.0$
$(E_{\beta max} - E_{\beta})/m_{\nu}$
4.0
3.0
2.0
1.0
0