ANL-HEP-PR-80-48
CONF-800724--64

MASTER

MAGNETIC MOMENTS OF COMPOSITE QUARKS AND LEPTONS--FURTHER DIFFICULTIES

Harry J. Lipkin

DISCLAIMER

Prepared for

XX International Conference

on

High Energy Physics

Madison, Wisconsin

July 17-23, 1980

U of C-AUA-USDOE

ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

Operated under Contract W-31-109-Eng-38 for the

U. S. DEPARTMENT OF ENERGY

ANL-HEP-PR-80-48
May, 1980

MAGNETIC MOMENTS OF COMPOSITE QUARKS AND LEPTONS--FURTHER DIFFICULTIES*

Harry J. Lipkin**
Argonne National Laboratory
Argonne, Illinois 60439

and

Fermi National Accelerator Laboratory
Batavia, Illinois 60510

ABSTRACT

The previously noted difficulty of obtaining Dirac magnetic moments in composite models with two basic building blocks having different charges is combined with the observation by Shaw et al. that a "light" bound fermion state built from heavy constituents must have the Dirac moment in a renormalizable theory. The new constraint on any model which builds leptons from two fundamental fields bound by non-electromagnetic forces is that the ratio of the magnetic moment to the total charge of the bound state is independent of the values of the charges of the constituents; e.g. such a bound state of a spin-½ fermion and a scalar boson will have the same magnetic moment if the fermion is neutral and the boson has charge −e or vice versa.

*Work performed under the auspices of the United States Department of Energy.

**On leave from Department of Physics, Weizmann Institute of Science, Rehovot, Israel

The difficulty of obtaining Dirac magnetic moments for nonrelativistic composite systems has recently been pointed out.[1,2] More recently Shaw et al. suggested that the essential problem is to obtain "light" bound fermions from heavy constituents, and that once this is achieved there is no separate magnetic moment difficulty.[3] However, this argument gives no clue towards finding a model which has these properties. The purpose of this comment is to combine the two approaches to give very stringent constraints on possible models. These constraints might be useful to model builders in enabling them to reject unsuitable models with a minimum of wasted effort. They might also lead to no-go theorems showing that the problem cannot be solved either in general or with a wide class of models.

The nature of the new difficulty is illustrated by the following simple example. Consider a model for the electron as a composite of a scalar boson with charge $-e$ and a neutral fermion. The naive nonrelativistic model for such a state has zero magnetic moment since the charged constituent has no angular momentum and the constituent with spin has no charge. In order to obtain a Dirac moment, the charged boson must have just the right peculiar value of orbital angular momentum so that it contributes the exact value of the Dirac magnetic moment for the combined system.

The argument of Shaw et al[3] suggests that this miracle must occur automatically if it is possible to construct a light bound state from a heavy scalar boson and a heavy fermion. The essential peculiar feature of the bound state is that the scale defined by its size (or the masses of the constituents) is much smaller than the scale defined by its Compton wave length (or the mass of the bound state). They show that the anomalous magnetic moment and the excitation spectrum are determined by the scale of the size of the system, whereas the Dirac moment is determined by the mass or Compton wave length.

But this argument has one very remarkable feature. If the super strong forces producing the bound state are not electromagnetic, there is no reference to the precise coupling of the individual constituents to the electromagnetic field; e.g. their electric charges. Thus the magnetic moment of such a low mass bound state must be very close to the Dirac moment regardless of the electric charges of the constituents. If the argument holds for a neutral fermion and a charged boson, it must also hold, with the same wave function for the composite system, for a charged fermion and a neutral boson, or for a fermion with charge xe and a boson with charge $-(1+x)e$, where x can have any arbitrary value. This puts extreme conditions on the model, and suggests that any composite model made from two different elementary fields cannot have a simple description in terms of constituents, like the constituent quark model for hadrons.

This point can also be studied more precisely and relativistically. Consider a composite model constructed from two basic fields, denoted by Ψ_1 and Ψ_2. These may be either Bose or fermi fields, but at least one fermi field is necessary to make a composite fermion. We assume that the electromagnetic current is additive in the two fields,

$$J^{\mu} = q_1 \bar{J}_1^{\mu}(\Psi_1) + q_2 \bar{J}_2^{\mu}(\Psi_2) \tag{1}$$

where $\bar{J}_1^{\mu}$ and $\bar{J}_2^{\mu}$ are "reduced" currents, depending respectively only on Ψ_1 and Ψ_2 respectively and the coupling constants q_1 and q_2 for each field are factored out.

The angular momentum carried by each field can be defined relativistically by observing the behavior of each field separately under rotations. Thus we can define the total angular momentum $\vec{J}$ as the sum of the contributions from the two fields,

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \ . \tag{2}$$

The magnetic moment operator for each field can be defined separately using the specific form of the electromagnetic current. We can thus write the magnetic moment operator for the system as

$$\vec{\mu} = q_1 \vec{\mu}_1 + q_2 \vec{\mu}_2 \ . \tag{3}$$

where $\vec{\mu}_1$ and $\vec{\mu}_2$ are reduced magnetic moment operators with the coupling constants factored out.

Let us now assume that a bound state exists, formed from these constituent fields Ψ_1 and Ψ_2. The total electric charge of this bound state is given by

$$Q = \langle n_1 \rangle q_1 + \langle n_2 \rangle q_2 \ , \tag{4}$$

where n_1 and n_2 are the "reduced charges" of each field.

In the simple constituent model these are just the number of constituents of type 1 and type 2 in the bound state. In the Harari Rishon model,[4] for example, n_1 and n_2 are the numbers of V and T particles in the state and take on integral values from -3 to +3 for the quarks and leptons. Note that the wave function can contain an arbitrary number of particle-antiparticle pairs in addition to the n_1 of type 1 and the n_2 of type 2, and may not be eigenfunctions of n_1 and n_2 if charge exchange is possible between the two fields.

The magnetic moment of this state is given by the expectation value of the operator (3) in this state,

$$\langle \vec{\mu} \rangle = q_1 \langle \vec{\mu}_1 \rangle + q_2 \langle \vec{\mu}_2 \rangle \tag{5}$$

Assuming that the state has a well defined angular momentum, e.g. $J = 1/2$ for quarks and leptons, Eq. (5) can be rewritten

$$\langle \vec{\mu} \rangle = \left\{ q_1 \langle \mu_1 \cdot \vec{J} \rangle + q_2 \langle \vec{\mu} \cdot \vec{J} \rangle \right\} \langle \vec{J} \rangle / \left[J(J+1) \right] \tag{6}$$

From the argument of Shaw et al, if the bound state has a much lower mass than the constituents, the magnetic moment (6) must be the Dirac moment, independent of the values of q_1 and q_2. The ratio of the magnetic moment (6) to the total charge Q must then be independent of Q_1 and Q_2. This gives the following condition,

$$\langle \vec{\mu}_1 \cdot \vec{J} \rangle / \langle n_1 \rangle = \langle \vec{\mu}_2 \cdot \vec{J} \rangle / \langle n_2 \rangle \ . \tag{7}$$

Thus the reduced magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ carried by the two fields in the bound state wave function must satisfy the condition (7). This result (7) is a precise quantitative constraint which must be satisfied by any model which makes a light bound state out of two heavy fields with a non-electromagnetic superstrong interaction.

Note that if $\langle\vec{\mu}_1\cdot\vec{J}_1\rangle$ and $\langle\vec{\mu}_2\cdot\vec{J}_2\rangle$ have the same sign, as is the case in all simple models, where the magnetic moment of a positively charged field is parallel to the direction of the angular momentum, then $\langle\vec{J}_1\cdot\vec{J}\rangle$ and $\langle\vec{J}_2\cdot\vec{J}\rangle$ must also have the same sign. This means that for $J = 1/2$ the projections of $\vec{J}_1$ and $\vec{J}_2$ in the direction of the total angular momentum are <u>parallel</u> and are both less than 1/2. This proves that the state cannot be an eigenfunction of both J_1 and J_2 and must have components both with $J_1 = J_2 + 1/2$ and $J_1 = J_2 - 1/2$.

The wave function defined in Ref.1, Eq.(6) satisfies these constraints, since it was constructed to give a Dirac moment for all values of the charges of the constituents. However, as noted there, it can only be achieved with a peculiar relation between spin and statistics for the fundamental fields. More realistic models, if they exist, must have wave functions very different from those of simple constituent models; e.g. they could contain additional particle-antiparticle pairs with non-trivial angular momenta and significant contributions to the magnetic moment.

An alternative way out of this difficulty is to assume that electromagnetism plays an important role in the superstrong binding force and that therefore the bound state wave function depends upon the values of q_1 and q_2. In that case the condition (7) does not hold.

REFERENCES

1. H. J. Lipkin, Phys. Lett. 89B (1980) 358.

2. M. Glück, Phys. Lett. 87B (1979) 247.

3. G. L. Shaw, D. Silverman and R. Slansky, LA-UR-80-588.

4. H. Harari, Phys. Lett. 86B (1979) 83.