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Stationary Yang-Mills fields with sources

L.Mathelitsch, H.Mitter and F.Widder

Institut für Theoretische Physik der Universität
A-8010 Graz

Classical, time-independent solutions of the Yang-Mills equations are studied for spherically symmetric situations. In presence of charge- and current distributions the same types of solutions are found as for purely electric sources: besides the abelian (Coulomb-Biot-Savart) solution there are two nonabelian types, one of which requires minimal source strengths and comes in two branches. The solution pattern is investigated by rough numerical calculations for a simple source model corresponding to spherical shell distributions. In absence of charge distributions an additional type is found, which has zero electric field and a magnetic field corresponding to a monopole of fixed strength. This type of solution exists for a large class of reasonable source currents. Some analytical examples are given in addition to numerical results for the shell model. Stability problems are not touched.

1. Introduction

Because of the steadily growing interest in nonabelian gauge theories on the level of quantum field theory it seems desirable to know as much as possible about classical solutions of these theories. In abelian electrodynamics one of the oldest problems is the determination of the field of a given, static charge distribution and its magnetic counterpart, a stationary current. The corresponding problem in Yang-Mills theory has been investigated for spherical charge distributions as sources [1] and revealed a host of unexpected solutions with interesting stability properties [2]. We shall follow here the program of [1] including stationary current distributions without touching the stability problem. Reasons for the study of magnetic situations are both the possibility of magnetic monopoles in nonabelian theory and the question, whether some of the multiplicity of solutions found in the electric case is reduced by the presence of magnetism.
We shall follow the notation of ref.[1] as close as possible and refer the reader to this paper for any details omitted here. In particular the basic field equations for static situations can be taken from ref.[1] equ. (2.2)-(2.5) with the only change, that we add a source current $\vec{j}_a$ on the r.h.s. of equ. (2.4).

2. Abelian solution

It is known since a long time, that the Coulomb solution of elctrodynamics is also a solution of the nonabelian field equations, if the fields and sources in the latter case point into a fixed direction in SU(2) space. If we take

$$(1) \qquad g\, j^{\mu}_{a} = \delta_{a3}\, j^{\mu}(\vec{r})$$

then the potential

$$(2) \qquad g\, A^{\mu}_{a} = \delta_{a3} \frac{1}{4\pi} \int \frac{d^3r'}{|\vec{r}-\vec{r}\,'|}\, j^{\mu}(r')$$

is a solution of the field equations. We shall consider this solution for reference purposes in spherically symmetric situations. For convenience we shall take an arbitrary length scale r_o out of the source functions and use the dimensionless variable $x=r/r_o$ instead of r. A spherically symmetric situation is then described by

$$(3) \quad g r_o^3 \rho = \delta_{a3}\, q(x) \quad , \quad g r_o^3\, j_3^k = - \varepsilon_{3kl}\, e_l\, m(x)$$

where e_k is the radial unit vector. The solution (2) has the energy

$$\mathcal{E}_c = \frac{1}{8\pi} \int \frac{d^3r\, d^3r'}{|\vec{r}-\vec{r}'|} \left(\rho(\vec{r})\rho(\vec{r}') + \vec{j}(\vec{r})\cdot\vec{j}(\vec{r}')\right) = \frac{4\pi}{g^2 r_o}\, \varepsilon_c$$

$$(4) \quad \varepsilon_c = \int_0^\infty dx \int_0^x dy\, y^2 \left[x\, q(x) q(y) + \frac{2}{9}\, y\, m(x) m(y) \right]$$

3. Nonabelian solutions: general pattern

Now we shall consider situations, in which spherical symmetry is not manifest, but can be restored by gauge transformations. A Lorentz vector V_a^μ carrying an SU(2) index a will describe such a situation, if it has the form

$$(5a) \quad V_a^0 = e_a V_0(r)$$

$$(5b) \quad V_a^k = e_a e_k V_\parallel(r) + (e_a e_k - \delta_{ak}) V_\perp^{(1)}(r) - \varepsilon_{akl}\, e_l\, V_\perp^{(2)}(r)$$

The suffices $\parallel$ resp $\perp$ refer to radial resp. angular components. The form (5) refers to a class of gauges, which we shall call the radial gauge. Within this class we may still perform gauge transformations, which change only the coefficients V_o, $V_\parallel$ etc. by radial functions. We shall start with an ansatz (5) for the current vector j_a^μ describing the (stationary) sources. For the coefficients of j_a^μ we write

$$
(6)\quad \begin{aligned} & g r_0^3\, j_0 = q(x)\,, \quad g r_0^3\, j_\parallel = v(x) \\ & g r_0^3\, j_\perp^{(1)} = \nu(x)\,, \quad g r_0^3\, j_\perp^{(2)} = \mu(x) \end{aligned}
$$

Note that v corresponds to a radial flow and cannot be removed by gauge transformations within the radial gauge. Because of the continuity equation such a flow is not possible in an abelian theory, unless the source is singular at x=0. In a nonabelian theory the situation may be different, since source and field contributions can compensate each other. For the Yang-Mills potentials A_a^μ we shall use again an ansatz of type (5). Here we may start with $A_\parallel = 0$, since this can be achieved by a gauge transformations within the radial gauge. Thus we write

$$
(7)\quad g r A_0 = f(x)\,,\ g r A_\perp^{(1)} = \beta(x)\,,\ g r A_\perp^{(2)} = 1-\alpha(x)
$$

For the electric resp. magnetic fields we obtain again form (5b) with

$$
(8a)\quad g r^2 E_\parallel = f - x f'\,,\ g r^2 E_\perp^{(1)} = \alpha f\,,\ g r^2 E_\perp^{(2)} = \beta f
$$

$$
(8b)\quad g r^2 B_\parallel = 1-\alpha^2-\beta^2\,,\ \ g r^2 B_\perp^{(1)} = x\alpha'\,,\ \ g r^2 B_\perp^{(2)} = x\beta'
$$

Here the prime denotes the derivative with respect to x. The field equations amount to a coupled system of ordinary differential equations, which reads

$$
(9)\quad \begin{aligned} & (f - x f')' + \frac{2}{x} f (\alpha^2+\beta^2) = x^2 q \\ & \alpha'' + \frac{\alpha}{x^2}\left(f^2+1-\alpha^2-\beta^2\right) = x\mu \\ & \beta'' + \frac{\beta}{x^2}\left(f^2+1-\alpha^2-\beta^2\right) = -x\nu \\ & 2(\alpha\beta' - \beta\alpha') = x^2 v \end{aligned}
$$

The covariant divergence relation

$$D_\mu j^\mu_a \equiv \partial_\mu j^\mu_a + q\,\varepsilon_{abc} A_{b,\mu} j^\mu_c = 0$$

for the source current amounts to

$$x v' + 2v + 2(\mu\beta + \nu\alpha) = 0 \tag{10}$$

It is observed, that one may determine f, α, β from the first three equations (9) for given q, μ, ν and then compute v from the last equation. Equ. (10) is fulfilled, since it is a consequence of (9). The obvious symmetries of equ. (9) allow for another choice, which is simpler. If we introduce

$$a(x) = (\alpha^2 + \beta^2)^{1/2} \qquad \varphi(x) = \arctan(\beta/\alpha)$$

$$m(x) = \mu\cos\varphi - \nu\sin\varphi\,, \quad n(x) = \mu\sin\varphi + \nu\cos\varphi \tag{11}$$

we obtain the systems

$$\begin{aligned} &(f - x f')' + \frac{2}{x} f a^2 = x^2 q \\ &a'' + \frac{a}{x^2}(f^2 + 1 - a^2) = x m + \frac{x^4 v^2}{4a^3} \end{aligned} \tag{12}$$

and

$$\begin{aligned} &2a^2\varphi' = x^2 v \\ &x v' + 2v = -2an \end{aligned} \tag{13}$$

Thus we may start with sources q, m, v to solve the system (12) and determine the remaining pieces from system (13). For vanishing sources m and v we recover the equations (4.4ab) of ref. [1].

The field energy can be written as

$$\begin{aligned} E = \frac{q^2 r_0}{4\pi}\,\mathcal{E} &= \frac{1}{2}\int_0^\infty \frac{dx}{x^2}\Big[(f - x f')^2 + 2f^2a^2 + \\ &\quad + (a^2-1)^2 + 2x^2(a')^2 + \frac{x^6 v^2}{2a^2}\Big] = \\ &= \int_0^\infty dx\, x\left[q(f - x f') - 2x\, m a' + \frac{n x^3 v}{a}\right] \end{aligned} \tag{14}$$

The last form is obtained using the continuity equation for the energy-momentum tensor and requires partial integrations, if. [1].
Next we shall consider the behavior of the solutions at small resp. large values of x. We shall start with

$$f(0) = 0, \quad a(0) = 1 \tag{15}$$

These conditions are necessary, if the energy is required to be finite; in fixing the sign of a(o) we have used a residual gauge freedom, cf.[1]. For the source functions we shall require, that q, μ, ν vanish at the origin and decrease faster than x^{-4} at infinity. If φ is regular at zero and infinity and a does not increase for large x, we find from equs. (11) and (13), that v and n behave as μ and ν at the origin and infinity, whereas φ vanishes stronger than x^3 at the origin and falls off faster than x^{-1} for large x. From the differential equations (12) we obtain for small x

$$f = f_2 x^2 + f_4 x^4 + \cdots \quad , \quad a = 1 + a_2 x^2 + a_4 x^4 + \cdots \tag{16}$$

For the field strengths (8) we obtain

$$\begin{aligned} g r_0^2 E_\parallel(0) &= -f_2 , \quad g r_0^2 E_\perp^{(1)}(0) = f_2 , \quad E_\perp^{(2)}(0) = 0 \\ g r_0^2 B_\parallel(0) &= -2a_2 , \quad g r_0^2 B_\perp^{(1)} = 2a_2 , \quad B_\perp^{(2)}(0) = 0 \end{aligned} \tag{17}$$

For the behaviour at large x it is crucial, whether the electric source term is present or not. We shall first consider the case $q \neq 0$, $f \neq 0$. Then we find from the differential equations, that we may have two types (I,II) of solutions (as for vanishing magnetic sources), which differ in their asymptotic behavior

$$f = f_{-1}^{I} x^{-1} + \cdots , \quad a = 1 + a_{-1}^{I} x^{-1} + \cdots \tag{18I}$$

$$f = f_{-1}^{II} x^{-1} + \cdots , \quad a = -1 + a_{-1}^{II} x^{-1} + \cdots \tag{18II}$$

The vector potential tends asymptotically towards a pure gauge potential in the second case, as discussed in [1]. The leading components of the field strengths at large x are

$$
g r_0^2 E_\parallel = 2 f_{-1}^{I} x^{-3} + \cdots , \quad g r_0^2 E_\perp^{(1)} = f_{-1}^{I} x^{-3} + \cdots
$$

$$
g r_0^2 B_\parallel = -2 a_{-1}^{I} x^{-3} + \cdots , \quad g r_0^2 B_\perp^{(1)} = - a_{-1}^{I} x^{-3} + \cdots \tag{19I}
$$

$$
g r_0^2 E_\parallel = 2 f_{-1}^{II} x^{-3} + \cdots , \quad g r_0^2 E_\perp^{(1)} = - f_{-1}^{II} x^{-3} + \cdots
$$

$$
g r_0^2 B_\parallel = 2 a_{-1}^{II} x^{-3} + \cdots , \quad g r_0^2 B_\perp^{(1)} = - a_{-1}^{II} x^{-3} + \cdots \tag{19II}
$$

The components $E_\perp^{(2)}$, $B_\perp^{(2)}$ decrease faster than x^{-4} in both cases. It is not easily possible to distinguish the types of solutions by considering local characteristics at zero resp. large x. If we take, for instance, Roskie's classification [3] based upon invariants, we find both types of solutions in the most general class (III) on both ends. The same fact happens, if we use a recent group-theoretical classification scheme [4]: here both types are in class G for small and large x. One difference can at least be recognized: it is observed, that the sign of the products $E_\parallel E_\perp^{(1)}$ and $B_\parallel B_\perp^{(1)}$ has opposite values at the origin and infinity for type I, whereas it has the same value for type II. This sign corresponds to the helicity (or handedness) defined by the two (orthogonal) directions, in which $E_\parallel$ resp. $E_\perp^{(1)}$ have the same sign.

Now we shall consider the case of vanishing electric source q=0. From the first equation (12) it can be read off, that f must vanish; the second derivative of f has the same sign as f in this case; if we start at the origin with growing (resp. falling) f, this behavior will continue for all values of x; the energy can therefore be finite only for vanishing f. Thus we have only a magnetic field in this case. The behavior of a for small x is still given by equ.

(16). At large x we can still have solutions of type I resp. II behaving according to equ. (19I,II). If, however, $\varphi=v=n=0$, $\mu=m$, we encounter a third type, for which a decreases as x^{-2} or faster at large distances. As a consequence, we have a long range radial field with fixed strength

$$g r_0^2 B_\eta = x^{-2} + \cdots \tag{19III}$$

whereas the (only)angular component $B_\perp^{(1)}$ decreases as x^{-4} or faster. For appropriate source this configuration can even be realized with an exponential fall-off of a and $B_\perp^{(1)}$ (we shall give some examples below). It has to be noted, that this monopole type field (19III) is not a gauge artefact. This can be seen either directly by considering the action of a gauge transformation or with the aid of the classification scheme of ref.[3]: solutions of type (I,II) belong still to class (III), whereas the monopole asymptotic field is in class (II).

4. Special solutions with electric and magnetic sources

It is, of course, always possible to start from an ansatz for f, a, φ (with the right behaviour at small and large x) and to calculate the corresponding source functions, which turn out "reasonable". It is very hard, however, to see in this way, whether there are several solutions for the same sources, which is an important result for purely electric situations [1]. We have therefore performed numerical calculations with a special model, which is the simplest extension of the one [1] studied for the electrical case. We assume, that there is no "radial" current

$$v = n = 0 \tag{21}$$

and that the remaining sources are concentrated at $x=1$

$$(22) \qquad q = Q\,\delta(x-1)\ , \qquad m = M\,\delta(x-1)$$

where Q>O and M are parameters. The numerical integration of equs. (12) can be done as in [1] , with the only difference that now also a' has a discontinuity (which is M) at x=1. The energy (14) becomes

$$(23) \quad \mathcal{E} = Q\left(f(1) - \tfrac{1}{2}f'(1^+) - \tfrac{1}{2}f'(1^-)\right) - M\left(a'(1^+) + a'(1^-)\right)$$

and is to be compared with the energy (4) of the abelian solution with the same distributions (22)

$$(24) \qquad \mathcal{E}_c = \frac{Q^2}{2} + \frac{M^2}{9}$$

We have considered only a limited number of values for (Q,M), since it was only our intention to obtain a rough feeling of how the presence of magnetism may influence the situation. The calculation was performed as follows. The source-free equations (12) were integrated numerically from x=O to x=1, starting from the behavior (16) with a given set (f_2, a_2). At x=1 the discontinuities (-Q,M) were added to the computed values of (f',a'). The results (together with f(1), a(1)) were used for further numerical integration for x>1. The initial set (f_2,a_2) was discarded, whenever the computed functions (f,a) did not show the corresponding asymptotic behavior (18) for sufficiently large x. The procedure has turned out inaccurate for larger values of Q and/or $|M|$.

As far as type I is concerned, we have observed no drastic change in comparison with M=O. In general f becomes larger a smaller with increasing M. Typical curves are shown in figs. 1 and 2 for Q=10.

The type II solution was known to exist only for $Q>Q_0 \approx 5{\cdot}8$ for M=O, whereby Q_0 was a bifurcation point. We have controlled the M-dependence of this pattern for Q=10. In this case increasing M tends to increase the spacing between the two branches (i.e. the distance between the corresponding curves for f resp. a). Some curves are shown in figs. 3 and 4. Therefore one expects, that the two branches will meet for sufficiently small M. For Q=10 this happens indeed at $M=M_0\approx-2\cdot65$. Below this value we have not found solutions of type II. The point M_0 is therefore a bifurcation point. The same pattern occurs for Q=5: for M=0 no type II solution has been found, but there are two branches for positive values of M, which start at $M=M_0\approx0\cdot35$. Thus we have found three points on a "phase diagram", i.e. a curve in the (Q,M) plane marking the onset of bifurcation. A smooth curve through these points (including results at Q=0 to be discussed below) is displayed in fig.5.

Finally we shall discuss briefly the spacing of the solutions with respect to the field energy. At Q=10 type I (which has the lowest energy) is well separated in energy from both type II-branches, which are in train well separated from the abelian solution (which has the highest energy). Starting from $M_0\approx-2\cdot65$ the separation of the two branches of type II in energy increases up to M=0, but decreases again for growing positive values of M (whereas the spacing between the f's and a's increases!). Due to the limited precision of our numerical program we are, however, not able to make any precise statement about what happens in this respect above $M\approx3$.

For smaller values of Q the sequence (I,II, abelian solution) persists for M=0, cf. [1]. For increasing $|M|$ the energetic separation between nonabelian and abelian solutions decreases: e.g. for Q=5 the value of ϵ_C is about 25 % larger than ϵ for the upper branch of type II for $M=0\cdot5$, whereas this difference amounts only 2 % for M=4. For Q=1 the abelian solution may even compete in energy with type I: for M=0 we have $\epsilon_C=0\cdot5$, $\epsilon_I=0\cdot16$, for M=-1 we have $\epsilon_C=0\cdot6$, $\epsilon_I=0\cdot44$.
It can therefore be expected, that the abelian solution has the lowest energy for sufficiently small Q, if $|M|$ is large enough (this is corroborated by the results for Q=0 to be discussed below).

5. Special solutions without electric source

Next we consider a situation (21) without radial current and without electric source and field q=f=0. Then we have to solve

$$a'' + \frac{a}{x^2}\,(1-a^2) = x\,m \tag{25}$$

It is easy to find analytical solutions for type III. A notorious example of such a monopole solution is [5]

$$a = x/\sinh x \tag{26a}$$

which corresponds to

$$m = (1 - x\coth x)^2 / x^2 \sinh x \tag{26b}$$

The field energy of this solution resp. its abelian counterpart is

$$\varepsilon = 1/2 \qquad \varepsilon_c = 0.045$$

Another simple example is

$$a = P(x)\exp(-x) \qquad P(x) = 1 + x + x^2/3 \tag{27a}$$

which corresponds to

$$m = \left[R(x) - P^3 \exp(-2x)\right] x^{-3} \exp(-x)\,, \quad R = 1 + x - \frac{x^3(1-x)}{3} \tag{27b}$$

Here we have

$$\varepsilon = 0.465 \qquad \varepsilon_c = 0.042$$

Further examples with exponential decrease can be found adding higher powers to P and corresponding terms to R. There are of course many more possibilities as e.g. gaussian or rational forms for a. Monopole fields are apparently quite natural phenomena in this context. One should not forget, however, that they get "killed" by electric sources and fields.

Usually monopoles are found in models with scalar-isovector Higgs fields ϕ_a coupled to the Yang-Mills fields [6]. This implies, however, a rather detailed dynamics for the sources, which we have instead considered as an input. Even within our more general framework one may ask, however, whether the magnetic field can be written as the gradient of a scalar-isovector potential

$$B^k_a = D^k \phi_a$$

In a Higgs model this is Bogomolny's condition [7]. With

$$g r_0 \phi_a = e_a \psi(x)$$

we obtain for such a field

$$g r B_\parallel = -x\psi' , \quad g r B_\perp^{(1)} = a\psi , \quad B_\perp^{(2)} = 0$$

whereas our ansatz (8b) reduces for vanishing electric field to

$$g r^2 B_\parallel = 1-a^2 , \quad g r^2 B_\perp^{(1)} = x a' , \quad B_\perp^{(2)} = 0$$

Thus the representation by magnetic scalar potential is only possible, if

$$\psi = a'/a \quad \text{and} \quad x^2\psi' = a^2-1$$

which can be valid only in special situations: in fact these relations hold for solution (26a), but not for (27a). Therefore this representation is not possible in general.

Finally we discuss the shell source model (22) with Q=0. In this model we must have

$$\mathrm{sgn}\, a'(1^-) = -\,\mathrm{sgn}\, M , \qquad 2\,|a'(1^-)| \geq |M|$$

since the field energy (23) is positive. Furthermore the differential equation (25) implies, that any type I solution must tend to its asymptotic limit +1 from above ($a \to 1^+$), whereas any type II solution must tend to -1 from below: this is easily recognized by expansion of a in powers of $y = 1/x$, use of the differential equation (25) and determination of the second derivative of a with respect to y. Taking into account the curvature of the solution (which changes sign, whenever a reaches the values $0, \pm 1$) one may discuss all possibilities (for instance graphically). The results can be described as follows. First of all, there is no possibility for bifurcation or "coexistence" of types I, II, III: for any value of M we may have at most one solution belonging to one of these types (and of course the abelian solution). Type I can be found for $M<0$ whereby $a_2>0$ must be such that

$-2a'(1^-) \leq M < -a'(1-)$

Two numerical examples are

$M=-2 \quad \varepsilon = 0{\cdot}961 \quad \varepsilon_c = 0{\cdot}444$

$M=-4 \quad \varepsilon = 3{\cdot}16 \quad \varepsilon_c = 1{\cdot}77$

There is only one monopole solution for a fixed value $M=M_0>0$. The numerical result is

$M_0 = 1{\cdot}438 \quad \varepsilon = M_0^2 = 2{\cdot}126 \quad \varepsilon_c = 0{\cdot}236$

For this solution a vanishes for $x\geq 1$. Type II solutions can be found only for

$M\geq M_1>0$

The numerical value M_1 is characterized by the fact, that for the corresponding solution $a=-1$ for all $x\geq 1$. By numerical analysis we have found for this solution

$M_1 = 2{\cdot}33 \quad \varepsilon = M_1^2 = 5{\cdot}43 \quad \varepsilon_c = 0{\cdot}603$

A further numerical example for type II is

$M = 4{\cdot}5 \quad \varepsilon \approx 10 \quad \varepsilon_c \approx 2$

There can be no (nonsingular, nonabelian) solutions for $0<M<M_0$ and $M_0<M<M_1$.

It is observed, that for all of our examples the field energy is larger than the one of the corresponding abelian solution. This is in accordance with the trends found for $Q\neq 0$ in the preceding section, but we have not found a general proof. Another observation concerns the value $M=M_1$. If the solutions are smooth in Q and M, this point should lie on the bifurcation line as drawn in fig.5. Since there is no bifurcation at $Q=0$, this point is an end point. Thus one should expect, that there exists another line marking the onset of bifurcation, which starts at M_1 and lies above the curve drawn in fig.5. We were not able to determine points on this curve (which could also coincide with the M axis above $M=M_1$) with our procedure. It is also not clear at present, how the region without nonabelian solutions continues for $Q\neq 0$. There might be a curve in fig. 5 above the Q-axis, above which no type I solution exists.

6. Conclusion

It has been demonstrated, that the multiplicity of static, spherically symmetric solutions of the Yang-Mills equations for given charge distributions is not increased by additional stationary source currents, if the charge density is different from zero. The source currents may, however, influence the bifurcation pattern of those nonabelian solutions, which require a minimal source strength for their existence. This is demonstrated by numerical calculations with a special model describing spherical shell distributions. For vanishing electric source we encounter an additional type of solution, if the current does not contain a radial flow. This type corresponds to a magnetic monopole field with fixed strength and can be realized with a large class of reasonable source currents. Also the shell model allows for such a solution, as well as for the other types. In this case we have, however, neither coexistence of different types of nonabelian solutions nor any bifurcation (in contrast to the situation, in which electric sources are present). Instead there are no nonabelian solutions for a certain range of the magnetic source strength. It is not known at present, whether such a range exists also in presence of electric sources.

The abelian solution (which exists in any case) has a higher energy than the nonabelian solutions, if the electric sources are strong enough. For sufficiently weak electric sources this pattern changes, if magnetism is present. For vanishing electric sources the abelian solution has the lowest energy in all examples, which we have considered.

With respect to monopoles the situation is therefore completely different with and without electric fields. The monopole type is only possible in the latter case. It is not clear at present, whether this remains also true for nonspherical situations in a multipole expansion. The situation suggests also to look for more general solutions of the field equations with $\vec{E}_a=0$. The most important problem left open is of course the question, which of the solutions presented above are stable. This problem is hard enough to solve in absence of magnetism [2] and has therefore not been attacked in this work.

Acknowledgement.

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Figure captions

Fig.1. Profiles of the function of f type I solutions with Q=10. Dotted line: M=-2, solid line: M=0, broken line: M=+2

Fig.2. Profiles of the function a for type I solutions with Q=10. Dotted line: M=-2, solid line: M=0, broken line:M=+2.

Fig.3. Profiles of the function f for type II solutions with Q=10. Dotted lines: M=-2, solid lines: M=0, broken lines: M=+2. The upper three curves correspond to the branch II/1 with lower energy.

Fig.4. Profiles of the function a for type II solutions with Q=10. Dotted lines: M=-2, solid lines: M=0, broken lines: M=+2. The upper three curves correspond to the branch II/1 with lower energy.

Fig.5. Region in the Q-M plane searched for solutions. Open circles: type II, full circles: bifurcation points, crosses: type I. The broken line on the M axis denotes the region without nonabelian solutions. The monopole is denoted by M_o.

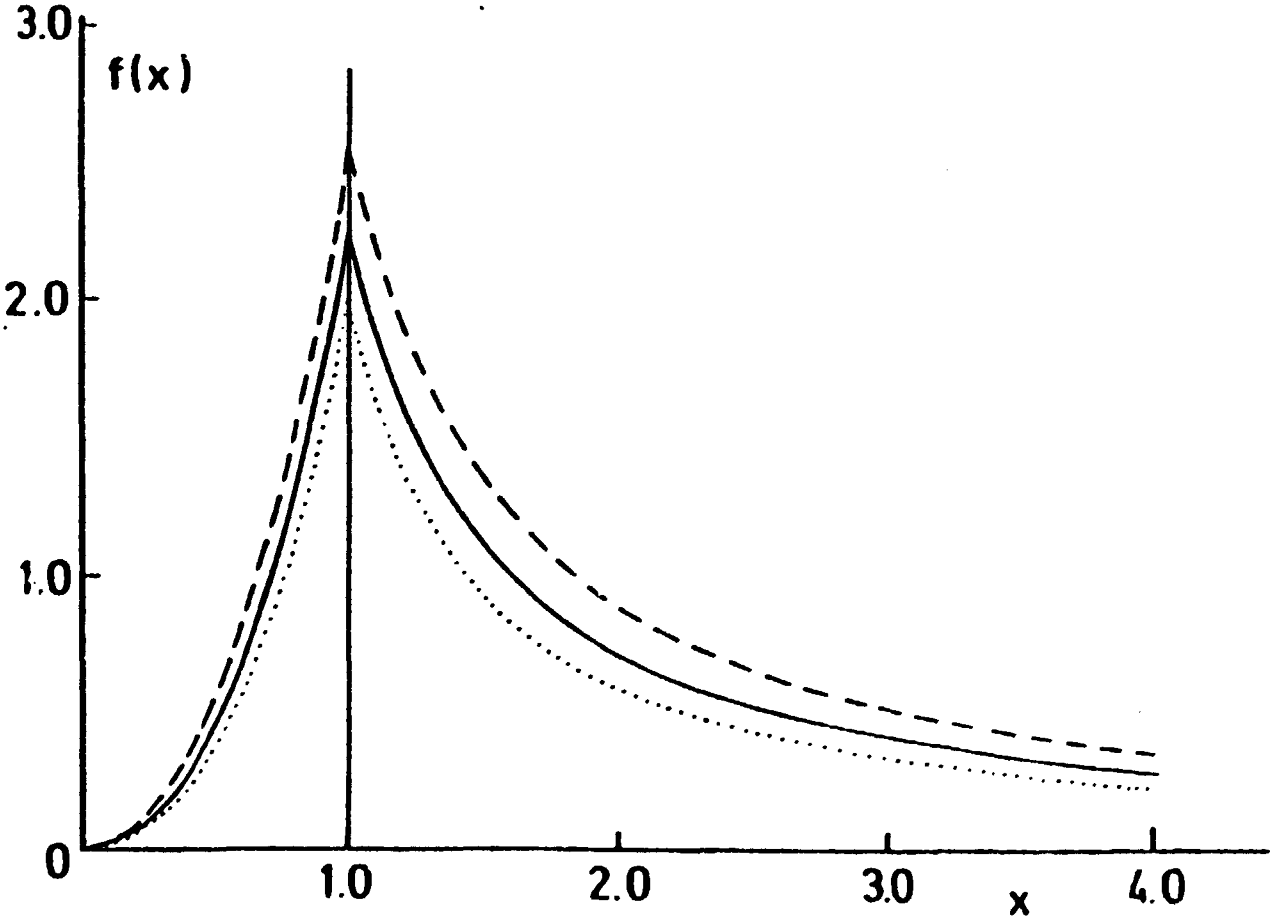

Fig. 1

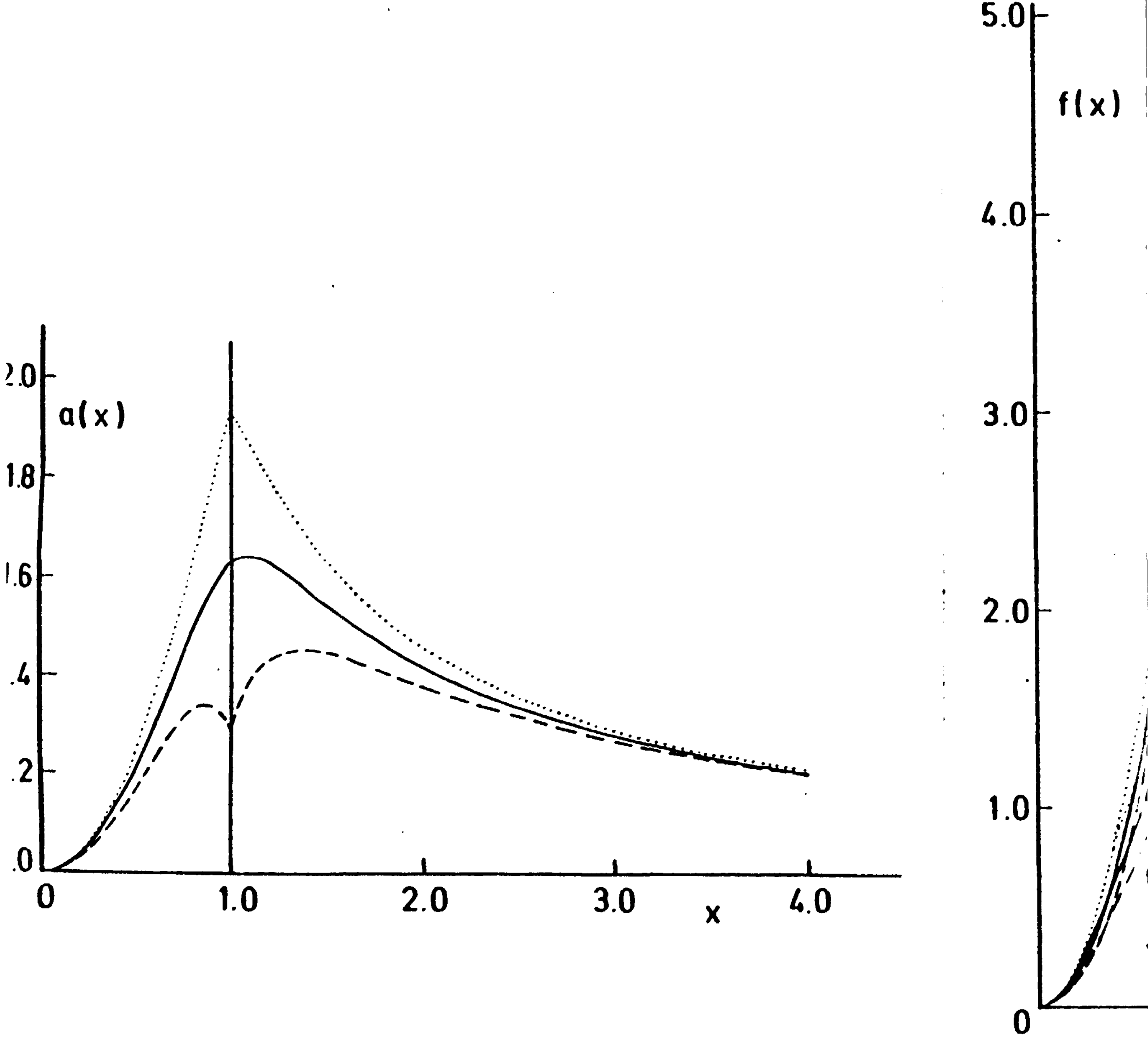

Fig. 2

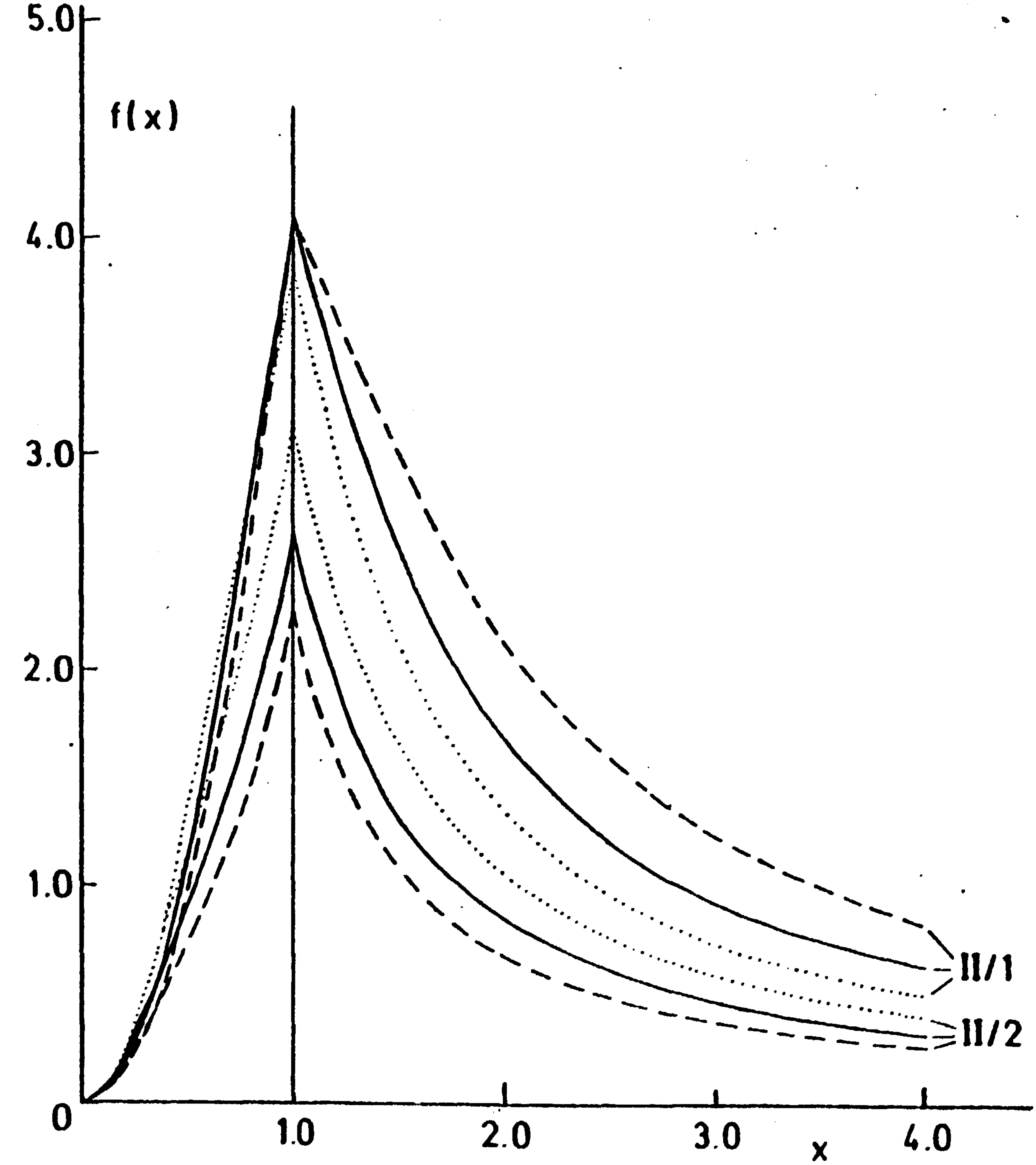

Fig. 3

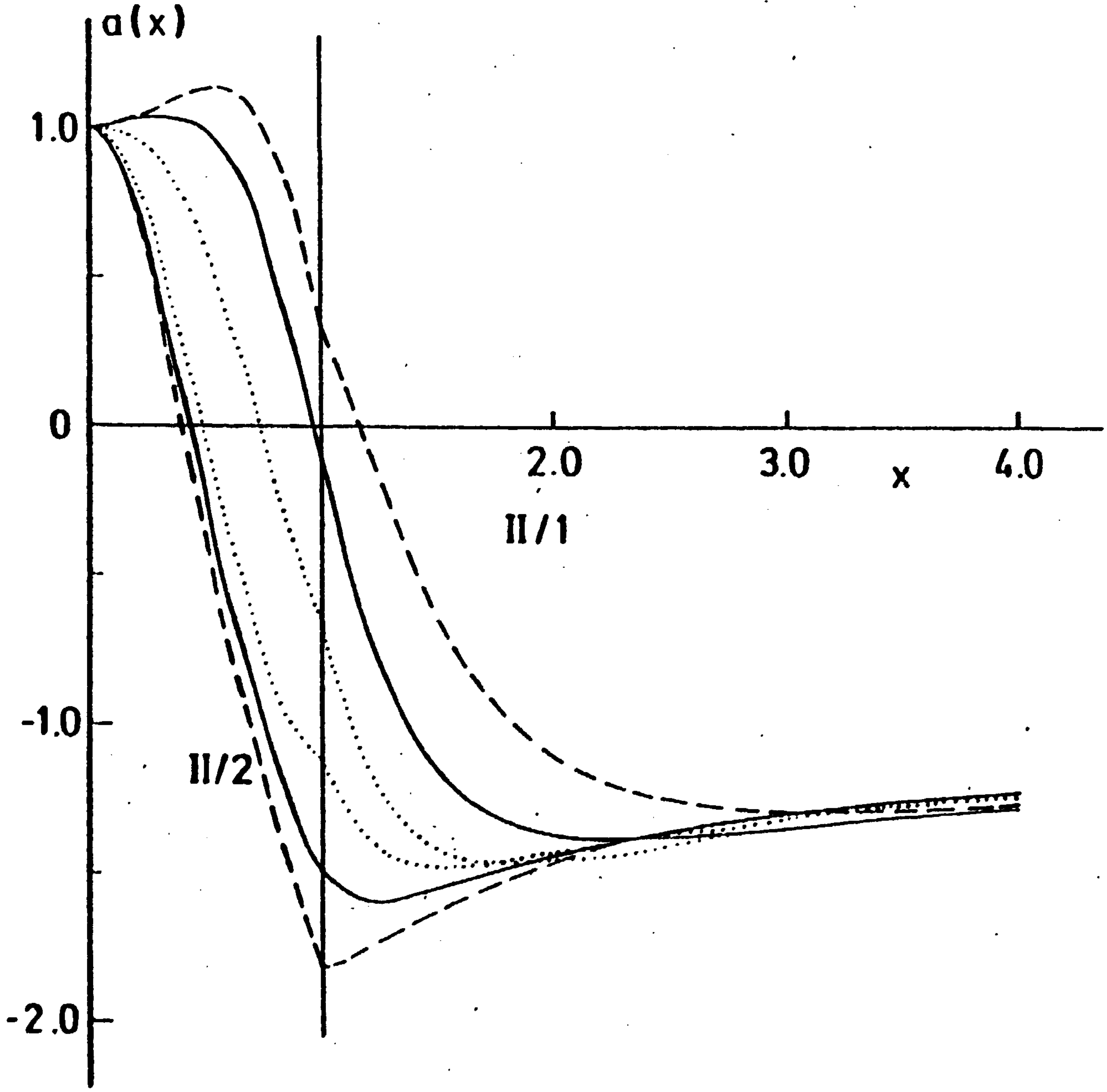

Fig. 4

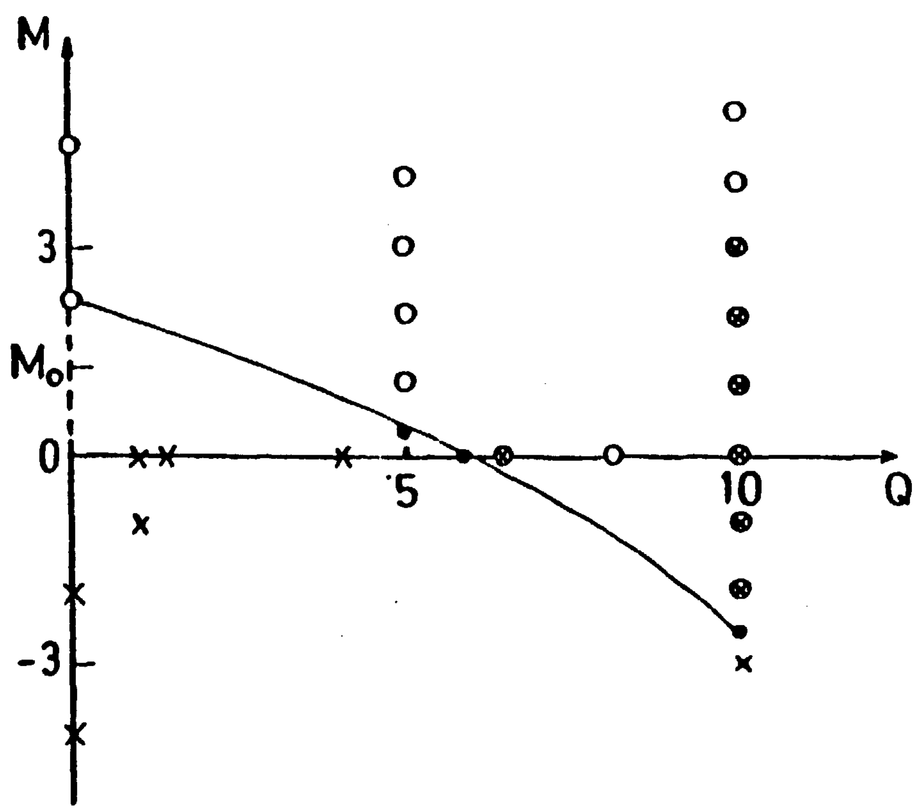

Fig. 5