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THE EFFECTIVE BARYON-LEPTON COUPLING CONSTANT AND THE PARITY OF LEPTONS^{+}

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Abstract

Using a phenomenological ansatz for the Lagrangian of baryon- and lepton-number violating interactions we calculate the effective baryon-lepton coupling constant within the framework of a relativistic quark model. Apart from a calculation of B-number violating cross-sections and decays this ansatz allows for a definition of the parity of leptons relative to baryons.

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I. Introduction

The most attractive feature of quark-lepton unification schemes [1,2] is the prediction of a new baryon-number violating force. Although the relative strength of about 10^{-31} seems to be of no relevance in present time, it is argued that cosmological implications are important and probably one can explain the observed baryon excess in our universe. Several selection rules for the new force have been stated. In particular, $\Delta B = \Delta L$ transitions seem to be dominant, and processes with inter-family mixing are suppressed by Cabibbo-like angles. An effective Lagrangian for transitions between three quarks and anti-leptons has been given by Weinberg [3] and Wilczek and Zee [4].

In order to compare predictions with experiments still one problem has to be solved, namely the binding of three quarks to baryonic states, since we do not observe free quarks. In the literature two different approaches have been used: SU(6) wave functions [5-10] and bag models [11-13]. It is the purpose of our paper to present a baryon-antilepton transition amplitude using a relativistic quark model proposed by Böhm and Meyer [14] and Kielanowski [15], where the baryons are described as three-quark bound states in terms of a Bethe-Salpeter amplitude. This model uses a harmonic oscillator potential in four Euclidian dimensions to describe the space part of the interaction kernel, whereas the spin part is determined by consistency requirements.

The definition of a baryon-antilepton transition amplitude gives rise to a lot of interesting questions like parity of leptons, provided the interaction conserves P. This is in general not the case, buth the strength of the P-violating transition is subject to renormalization and shown to be zero at the grand unification point in certain unified models. Even there, this is only true at the tree-graph level, loop corrections with certain Higgs mesons may change the ratio and may even violate CP, but their effect can be considered to be a small perturbation. As can be seen in Sec. II, we find that the positron behaves like a proton and the electron like an antiproton; i.e. the former is a P = +1 state.

This paper is organized as follows: We present in Sec. II an effective Lagrangian and discuss the familiar renormalization group techniques for our special case. It is further shown that B-violating cross-sections can be easily calculated using the effecting baryon-lepton coupling constant. In Sec. III the Bethe-Salpeter amplitude is used to calculate this coupling constant. Finally, in Sec. IV, we compute nucleon lifetimes and branching ratios of two-body decay channels for the grand unification groups SU(5) [2], SO(10) [16,17], and the class of unified models where the effective low-energy electroweak interaction is described by the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$, using our effective coupling constant and the empirical meson-baryon couplings.

II. The Phenomenological Lagrangian

Neglecting all the angles and phases resulting from the diagonalization of the fermion mass matrices, the general form of the low-energy effective Lagrangian for baryon- and lepton-number violating nucleon decay mediated by gauge bosons is given by [3,4,18]

$$\frac{1}{4} L = \frac{G}{\sqrt{2}} \varepsilon_{ijk}\{(\overline{u^c_{k_L}}\gamma_\mu u_{j_L})[(\overline{e^+_R}\gamma^\mu d_{i_R}) + r_e(\overline{e^+_L}\gamma^\mu d_{i_L}) + (\overline{\mu^+_R}\gamma^\mu s_{i_R}) + r_\mu(\overline{\mu^+_L}\gamma^\mu s_{i_L})] -$$

$$- (\overline{u^c_{k_L}}\gamma_\mu d_{j_L})[(\overline{\nu_{e_R}^c}\gamma^\mu d_{i_R}) + (\overline{\nu_{\mu_R}^c}\gamma^\mu s_{i_R})]\} + \text{h.c.} \tag{1}$$

where the fermion labels u, d, s, e^+, μ^+, ν_e, and ν_μ stand for the corresponding field operators. The parameters r_ℓ, ℓ = e or μ, denote the ratio of the coefficients of the effective four-fermion operators involving a left-handed charged lepton to the one involving its right-handed counterpart. They are identical with those introduced by Wilczek and Zee [4]. The values of these parameters depend on the grand unification group and the details of symmetry breaking and are furthermore subject to renormalization effects due to radiative corrections involving $SU(2) \otimes U(1)$ gauge bosons. The unrenormalized values of the r_ℓ are listed in Table I for three different unification models already considered in Refs. 4 and 13.

This Lagrangian, Eq.(1), obeys the selection rule $\Delta B = \Delta L$ as required by the invariance of the effective four-fermion operator of the low-energy theory under $SU(3) \otimes SU(2) \otimes U(1)$ transformations [3,4].

In analogy to the Lagrangian on the quark level, Eq.(1), we write down an effective Lagrangian, which describes the transition of any antilepton ℓ^c, $\ell^c = e^+, \mu^+, \nu_e{}^c$ or $\nu_\mu{}^c$, into the corresponding baryon B linked to it by the kinship hypothesis of Wilczek and Zee [4]:

$$L_{eff} = f\, \overline{\psi_\ell{}^c}\, \left(\frac{1+\gamma_5}{2} \mp r\, \frac{1-\gamma_5}{2}\right) \psi_B + \text{h.c.} \tag{2}$$

Here f denotes the coupling strength of the right-handed antilepton ℓ^c_R to the baryon B. The left- to right-handed lepton ratios r are the same as in Eq.(1) for charged leptons. For massless neutrinos the ratio r is equal to zero. Again, this ansatz is consistent with the selection rule $\Delta B = \Delta L$.

Finally, the effective baryon-lepton coupling constant $\bar{f}$ is defined by rewriting the Lagrangian of Eq.(2) in the form

$$L_{eff} = \bar{f}\, \overline{\psi_{\ell}{}^{c}}\, (1 + a\gamma_5)\, \psi_B + \text{h.c.} \tag{3}$$

where $\bar{f}$ and a expressed in terms of f and r are given by

$$\begin{aligned} \bar{f} &= \frac{f}{2}(1 + r) \;, \\ a &= \frac{1 - r}{1 + r} \;. \end{aligned} \tag{4}$$

The parameter a measures the relative strength of parity violation in the transition $\ell^c \leftrightarrow B$. The value $a = 0$, corresponding to $r = 1$, means parity conservation. Like r, a depends strongly on the employed unification group and on the scale of external momenta at which matrix elements of L_{eff} are evaluated. The coupling constant $\bar{f}$ depends on the unification scheme as well as on the three-quark wave function inside the baryon.

The effective four-fermion operators which build up the Lagrangian of Eq.(1) result from the exchange of superheavy vector bosons whereas the matrix elements of these operators are calculated on a mass scale e.g. for nucleon decay of O(1) GeV. Therefore these operators have to be renormalized by exchange of SU(3), SU(2) and U(1) gauge bosons using standard renormalization group techniques.

A closer inspection shows that the radiative corrections due to SU(3) and SU(2) gauge bosons lead to an identical renormalization for all operators appearing in Eq.(1) whereas the U(1) boson exchange distinguishes between the operators containing a left-handed lepton field and those containing a right-handed one. The effects of these radiative corrections can be expressed in terms of short-distance enhancement factors A_L and A_R. The necessary computations have been performed independently by three groups [4,18,19] who agreed within their results:

$$A_L = \left[\frac{\alpha_3(\mu)}{\alpha}\right]^{\frac{6}{33-4n}} \left[\frac{\alpha_2(m_W)}{\alpha}\right]^{\frac{27}{86-16n}} \left[\frac{\alpha_1(m_W)}{\alpha}\right]^{-\frac{69}{6+80n}} \;; \tag{5}$$

$$A_R = \left[\frac{\alpha_3(\mu)}{\alpha}\right]^{\frac{6}{33-4n}} \left[\frac{\alpha_2(m_W)}{\alpha}\right]^{\frac{27}{86-16n}} \left[\frac{\alpha_1(m_W)}{\alpha}\right]^{-\frac{33}{6+80n}} .$$

Here α_3, α_2, α_1, and α are the coupling constants of SU(3), SU(2), U(1), and the grand unification group. The typical mass of O(1) GeV at which the operators are renormalized is denoted by μ, m_W stands for the mass of the W-boson and n is the number of fermion generations. Assuming $n = 3$ and using as input [9]

$$\begin{aligned} \alpha_{e.m.}(m_W) &= \frac{1}{128.6} \quad , \\ \sin^2\theta_W(m_W) &= 0.21 \quad , \\ \alpha &= 0.0244 \quad , \end{aligned} \tag{6}$$

the renormalization factors A_L and A_R obtain the following numerical values:

$$\begin{aligned} A_L &= 3.7 \quad , \\ A_R &= 3.5 \quad . \end{aligned} \tag{7}$$

Taking into account the renormalization of the coefficients of the effective operators showing up in the Lagrangian by the short-distance enhancement factors, the relations (4) between the renormalized quantities $\bar{f}$ and a on the one hand and the bare constants f and r on the other hand are modified to

$$\begin{aligned} \bar{f} &= \frac{f}{2}\,(A_R + r\,A_L) \quad , \\ a &= \frac{A_R - r\,A_L}{A_R + r\,A_L} \quad . \end{aligned} \tag{8}$$

The values of a resulting from Eqs.(7) and (8) are also given in Table I. It is obvious that the renormalized value of a would agree with the bare one if A_L and A_R were equal, i.e. if the renormalization procedure made no difference for operators with right- and left-handed leptons. This is the case if the effective electroweak interaction theory is described by the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$, which is broken to $SU(2)_L \otimes U(1)$ at an energy scale well below the grand unification mass. Of course, the breaking of the $SU(2)_R$-symmetry induces a small amount of parity violation in Eq.(3). Assuming the gauge bosons of $SU(2)_R$ to be 5 times heavier than those of $SU(2)_L$, $m_{W_R} \approx 5\, m_{W_L}$ [35], we obtain $a = 1.3 \cdot 10^{-3}$, a value which is of the order of common CP-violation but which is clearly negligible for cross-sections and decay rates of B- and L-violating processes since their order of magnitude is determined by the expression $\bar{f}^2(1 + |a|^2)$.

The phenomenological Lagrangian of Eq.(3) immediately gives rise to two interesting observations:

i) <u>Parity of leptons</u>:

The smallness of the parameter a in certain transitions (listed in Table I) means, that there exists an (almost) parity-conserving transition between baryons and leptons. We are far away from the maximal parity violation of conventional weak interactions. It is therefore meaningful to assign an intrinsic parity quantum number to the leptons, after one has chosen the parity of the proton to be + 1. Obviously we find, that the leptons e^- and μ^- behave as antiprotons, therefore their parity - 1.

Let us look more to the cases yielding $a \approx 0$. If we adopt the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)$, the bare value of a_e and a_μ is exactly zero, and the renormalization factors from $q^2 = m^2_{W_R} \sim (400\ \mathrm{GeV})^2$ down to low energies yield $a_{e,\mu} \approx 10^{-3}$; therefore parity violation is of the order of possible CP-violation effects.

The situation is more complicated in the gauge group SU(5). Here, the transition $\Sigma^+ \to \mu^+$ has bare value a_μ equal zero; this value becomes renormalized to ~ -0.029. Parity violation is therefore a 3% effect; treating it as a small perturbation, we are allowed to define the parity of the muon. On the other hand, the transition of proton $\to e^+$ starts with an unrenormalized value $a_e = -1/3$; therefore we cannot make any conclusions. However, there exists a transition between positron and a $C = 2$, $S = 0$ ccd-state (N^{*+}_{cc}), which yields the bare value $a_e = 0$, and we find similar arguments as for the muon. Assuming the N_{cc}^{**+} to have the same parity as the proton (via strong interactions), an (approximate) definition of the parity of the electron can be given even in SU(5).

If the unification group is SO(10), we have to make specific assumptions on the breaking mechanisms to find how many gauge bosons are relevant for baryon decay. Following the line given in Wilczek and Zee [4], we find a pure left-handed transition with $\alpha = 1$, therefore no conclusion about parity can be done.

It should be noted that in all our values of α, we have neglected the Cabibbo angle. However, $\sin \Theta$ enters quadratically in the expressions for α and will shift the values of the parity violating amplitude by $\sim 5\%$, still a small perturbation. Finally, this conjecture can be extended to τ-leptons by looking to transitions between τ and (hypothetical) baryons containing a b-quark.

ii) Diffractive production of baryons by antileptons

We now look for the cross-section of a positron (momentum q_ℓ) to scatter on some target (e.g. a proton) yielding only hadrons; i.e. the process $e^+ p \to X_H$. The corresponding amplitude is given by

$$A_\ell(q_\ell, p_i \ \ldots) = O(q_\ell, p_i \ \ldots) \ u_\ell^c(q_\ell) \tag{9}$$

where we have suppressed spinor indices on the quantity O and used a shorthand p_i for the other momenta involved. If we make an off-mass-shell extrapolation in the variable q^2, we find a pole in the amplitude for $q^2 = m_p^2$, the mass of the proton:

$$\lim_{q_\ell^2 \to m_p^2} O(q_\ell, p_i \ \ldots)(\not{q}_\ell - m_p) = \bar{O}(q_\ell, p_i \ \ldots) \ \vec{f}(1 + \alpha\gamma_5) \ . \tag{10}$$

Here we have defined the strong interaction amplitude for a proton to scatter on the same target, yielding hadrons; i.e. $pp \to X_H$

$$\bar{A}_B(q_p, p_i \ \ldots) = \bar{O}(q_p, p_i \ \ldots) \ u_B(q_p) \ . \tag{11}$$

We therefore find that antileptons produce baryons diffractively by an off mass-shell lepton-baryon transition. Neglecting lepton mass effects we have

$$\sigma_{\ell\to X}(s) \approx \frac{\bar{f}^2(1+|a|^2)}{m_p^2}\,\sigma_{p\to X}(s) \quad . \tag{12}$$

Since the value $\bar{f}/m_p$ turns out to be about 10^{-33}, the cross-sections have the order of magnitude of 10^{-65} mb and therefore seem to be of no importance for creation of baryons in our present universe.

III. Calculation of f

In order to calculate the effective coupling f we make use of the Bethe-Salpeter (BS) formalism developed by Bethe, Salpeter, Gell-Mann, Low, Wick and Mandelstam [20-23]. Within this formalism a baryon B with mass M, momentum P and spin projection s is described as a bound state of three quarks by a three-field BS-amplitude, which is defined by

$$\Phi(x_1,x_2,x_3)_{\alpha\beta\gamma;abc;ABC} = \langle 0|T(\psi_{\alpha aA}(x_1)\psi_{\beta bB}(x_2)\psi_{\gamma cC}(x_3))|B(P,s)\rangle \ . \qquad (13)$$

Here $|B(P,s)\rangle$ denotes the baryon state; α,β,γ are the spin indices, a,b,c the colour indices, and A,B,C the flavour indices of the renormalized quark fields $\psi(x)$. Separation of the center-of-mass motion leads to

$$\Phi(x_1,x_2,x_3) = e^{-iPX}\ \Phi(\tfrac{1}{3}s+\tfrac{1}{2}r, \tfrac{1}{3}s-\tfrac{1}{2}r, -\tfrac{2}{3}s) \ , \qquad (14)$$

where the center-of-mass coordinate X and the relative coordinates r and s are given by

$$\begin{aligned} X &= \frac{1}{3}(x_1+x_2+x_3) \ , \\ r &= x_1 - x_2 \ , \\ s &= \frac{1}{2}(x_1+x_2-2x_3) \ . \end{aligned} \qquad (15)$$

Note that X belongs to the symmetric representation S, whereas s and r belong to the mixed representations M^+ and M^-, respectively, of the permutation group of three elements S_3. The Fourier transform of $\Phi(x_1,x_2,x_3)$ is defined by

$$\chi(k_1,k_2,k_3;P,s)\ \delta^{(4)}(P-k_1-k_2-k_3) =$$
$$= \frac{1}{(2\pi)^6}\int d^4x_1 d^4x_2 d^4x_3 \exp\,[i\sum_{i=1}^{3} k_i x_i]\ \Phi(x_1,x_2,x_3) \ . \qquad (16)$$

With the help of Eq.(14) the BS-amplitude in momentum space χ may be rewritten to

$$\chi(p,q;P,s)_{\alpha\beta\gamma;abc;ABC} = \tag{17}$$

$$= \frac{1}{(2\pi)^2} \int d^4r d^4s \; e^{i(pr+qs)} \langle O|T(\psi_{\alpha aA}(\tfrac{1}{3}s+\tfrac{1}{2}r)\psi_{\beta bB}(\tfrac{1}{3}s-\tfrac{1}{2}r)\psi_{\gamma cC}(-\tfrac{2}{3}s))|B(P,s)\rangle .$$

The center-of-mass momentum P and the relative momenta p and q are defined in terms of the quark momenta k_i ($i = 1,2,3$) in the following way:

$$\begin{aligned} P &= k_1 + k_2 + k_3 \quad , \\ p &= \frac{1}{2}(k_1 - k_2) \quad , \\ q &= \frac{1}{3}(k_1 + k_2 - 2k_3) \quad . \end{aligned} \tag{18}$$

The graphical representation of the BS-amplitude χ is given in Fig. 1.

The BS-amplitude $\chi(p,q;P,s)$ is the solution of the homogeneous BS-equation for bound states [20]

$$\begin{aligned} &\prod_{i=1}^{3} S_F^{-1}(k_i)\; \chi(p,q;P,s) = \\ &= \sum_{i=1}^{3} S_F^{-1}(k_i) \int d^4p_i' \; K_i(p_i,p_i';K_i)\; \chi(p_i',q_i;P,s) + \\ &+ \int d^4p' d^4q' \; K(p,q;p',q';P)\; \chi(p',q';P,s) \; . \end{aligned} \tag{19}$$

The physical significance of this equation becomes probably more transparent in its graphical representation, Fig. 2. $S_F(k)$ denotes the quark Feynman-propagator which for heavy quarks is assumed to be of the form of the free Feynman propagator [24]

$$S_F(k) = \frac{1}{\not{k} - m} , \tag{20}$$

but with m being interpreted as an effective quark mass the numerical value of which has to be determined by comparison of the theoretical predictions resulting within the framework of the BS formalism with experimental data, e.g. for the pion decay constant f_π [25-27]. The relative momenta p_i and q_i are obtained from Eq.(18) by cyclic permutation of the quarks:

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p_3 \\ q_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{3}{4} \\ 1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{3}{4} \\ -1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} p_2 \\ q_2 \end{pmatrix} . \tag{21}$$

The momenta K_i are defined as sums of the quark momenta k_i:

$$K_1 = k_2 + k_3 , \qquad K_2 = k_1 + k_3 , \qquad K_3 = k_1 + k_2 . \tag{22}$$

The dynamics of the theory is contained in the interaction kernels $\mathcal{K}_i$ and $\mathcal{K}$, where the $\mathcal{K}_i$ $(i = 1,2,3)$ denote the two-particle kernels and $\mathcal{K}$ the three-particle kernel. These kernels may be considered as phenomenological, relativistically generalized potentials describing the two-quark and three-quark interactions which lead to the three-quark bound state B.

At this point one has to decide how one wants to approximate the forces between quarks. That means one has to choose a reasonable form of the interaction kernels. We adopt the model of Kielanowski [15], because its predictions are in good agreement with experiment. It reproduces correctly the baryon spectrum [15] and it gives reasonable results for the strong decay widths of the $\frac{3^+}{2}$-baryon resonances and for the electromagnetic and weak form factors of the $\frac{1^+}{2}$-baryons [28]. Furthermore, one even can obtain a not too bad value for the ratio of axial vector coupling constant to vector coupling constant G_A/G_V for nucleons [15].

The model of Kielanowski is characterized by the following features: It assumes strong binding, i.e. the quark mass is large compared with the mass of the bound state:

$$\frac{M}{3m} << 1 . \tag{23}$$

The two-particle kernels K_i and the three-particle kernel K are assumed to be of the "convolution type",

$$K_i(p_i,p_i';K_i) = K_i(p_i - p_i') \; ,$$
$$K(p,q;p',q';P) = K(p - p',q - q') \; , \tag{24}$$

and to factorize into a spin part U and a space part V each of which has to be a Lorentz scalar:

$$K_i(p_i - p_i') = V_i(p_i - p_i')\, U_i \; ,$$
$$K(p - p',q - q') = V(p - p',q - q')\, U \; . \tag{25}$$

A consequence of the factorization of the interaction kernels is a similar factorized structure of the BS-amplitudes. The space parts of the kernels are approximated by kernels of the harmonic oscillator type:

$$V_i(p_i - p_i') = (\alpha - \beta\Box_{p_i})\, \delta^{(4)}(p_i - p_i') \quad ,$$
$$V(p - p',q - q') = m[\alpha_o - \beta_o(\tfrac{3}{2}\Box_p + 2\Box_q)]\, \delta^{(4)}(p - p')\, \delta^{(4)}(q - q') \; . \tag{26}$$

Here the parameters β and β_o are related to the level spacing of the baryons. The potential depth parameters α and α_o are expected to be of the order of m^2,

$$\alpha = O(m^2) \; , \qquad \alpha_o = O(m^2) \; , \tag{27}$$

so that they may compensate the large quark mass m in order to achieve the desired small bound state mass M. The structure of the spin parts U_i of the two-particle kernels is taken from the meson case, where it has been used with great success [24,26,27]:

$$U_1 = 1 \times \gamma_5 \times \gamma_5 \; , \qquad U_2 = \gamma_5 \times 1 \times \gamma_5 \; , \qquad U_3 = \gamma_5 \times \gamma_5 \times 1 \; . \tag{28}$$

The spin part U of the three-particle kernel is a straightforward generalization of the corresponding two-particle kernels U_i:

$$U = \frac{1}{2}(U_1U_2 + U_2U_1 + U_1U_3 + U_3U_1 + U_2U_3 + U_3U_2) = U_1 + U_2 + U_3 \ . \tag{29}$$

It is clear that this generalization is not unique. The only justification of all of this phenomenological ansatz can be done by successfully predicting experimentally available quantities, such as hadronic couplings, etc.

Under these assumptions the BS-equation, Eq.(19), can be solved [15] after Wick-rotation of its space part [22]. The validity of Eq.(23) allows one to expand the solution χ into a power series in $1/m$:

$$\chi = \sum_{n=0}^{\infty} \left(\frac{1}{m}\right)^n \chi_n \ . \tag{30}$$

The parameters α, α_0, β and β_0 are then specified to take the values

$$\alpha = \frac{1}{3} m^3 \ , \qquad \alpha_0 = 0 \ , \qquad \frac{\beta}{\beta_0} = -1 \ . \tag{31}$$

The mass spectrum one obtains from the eigenvalues of the radial equation of the orbital part of the BS-equation is given by

$$\begin{gathered} M^2 = M_0^2 + 36\sqrt{\beta}\,[n_1 + n_2 + 2(r_1 + r_2)] \ , \\ \begin{matrix} n_i = 0,1,2,\dots \\ r_i = 0,1,2,\dots \end{matrix} \quad \text{for } i = 1,2 \ . \end{gathered} \tag{32}$$

M_0 denotes the mass of the ground state solution, the n_i and r_i correspond to the quantum numbers of the angular momentum and radial excitations, respectively. Note that the resulting spectrum is linear in mass squared. Therefore the baryon resonances described by this model lie on linear Regge trajectories. The comparison of Eq.(32) with the experimental mass spectrum yields for the so far undetermined parameter β

$$\sqrt{\beta} \simeq 0{,}029 \ \text{GeV}^2 \ . \tag{33}$$

The BS-amplitude χ may be written in the form

$$\chi(p,q;P,s) = N[1+\frac{1}{2m}\sum_{i=1}^{3} K_i + \frac{1}{4m^2}(K_1K_2+K_1K_3+K_2K_3)+O(\frac{1}{m^3})]\chi_o(p,q;P,s) \;, \tag{34}$$

where N denotes a normalization constant and K_1, K_2, K_3 are short for $K_1 \times 1 \times 1$, $1 \times K_2 \times 1$, $1 \times 1 \times K_3$, respectively. For the octet baryons χ_o is given by

$$\chi_o(p,q;P,s) = \frac{1}{\sqrt{2}}(|\tfrac{1}{2}\,0;\tfrac{1}{2}\,s;+M_u^+> \chi_{SU(3)}^{(M^+)} + |\tfrac{1}{2}\,0;\tfrac{1}{2}\,s;+M_u^-> \chi_{SU(3)}^{(M^-)})\cdot$$
$$\cdot\ \chi_C \exp\,[-\frac{p_E^2}{6\sqrt{\beta}} - \frac{q_E^2}{8\sqrt{\beta}}] \;. \tag{35}$$

The notation for the spin wave functions $|\frac{1}{2}\,0;\frac{1}{2}\,s;+M_u^+>$ and $|\frac{1}{2}\,0;\frac{1}{2}\,s;+M_u^->$ is the following [29]: the first two numbers denote the type of the irreducible representation of the group O(4) according to which the states are classified; the next two numbers denote the spin of the bound state and its projection; the sign gives the parity of the spin state and the last character the symmetry under permutation of the quarks: M_u^+ and M_u^- denote states of mixed symmetry with left- and right-handed spinors unmixed, symmetric and antisymmetric with respect to permutation of the first two quarks. The spin wave functions may be expressed in terms of quark-diquark amplitudes [14,29]:

$$|\tfrac{1}{2}\,0;\tfrac{1}{2}\,s;+M_u^+>_{\alpha\beta\gamma} = \frac{iP^\lambda}{2\sqrt{6}M}[(\sigma_{\lambda\mu}C)_{\alpha\beta}(\gamma_5\gamma^\mu u(s))_\gamma + (\sigma_{\lambda\mu}\gamma_5 C)_{\alpha\beta}(\gamma^\mu u(s))_\gamma]$$
$$|\tfrac{1}{2}\,0;\tfrac{1}{2}\,s;+M_u^->_{\alpha\beta\gamma} = \frac{1}{2\sqrt{2}}\,[(C)_{\alpha\beta}(\gamma_5 u(s))_\gamma + (\gamma_5 C)_{\alpha\beta}(u(s))_\gamma] \;. \tag{36}$$

Here C stands for the charge conjugation matrix. The mixed-symmetric SU(3)-flavour wave functions denoted by $\chi_{SU(3)}^{(M^+)}$ and $\chi_{SU(3)}^{(M^-)}$ are represented e.g. for the proton by

$$\chi_{SU(3)}^{(M^+)}(\text{proton})_{ABC} = -\frac{1}{\sqrt{6}}(d_A u_B u_C + u_A d_B u_C - 2u_A u_B d_C)$$
$$\chi_{SU(3)}^{(M^-)}(\text{proton})_{ABC} = -\frac{1}{\sqrt{2}}(d_A u_B u_C - u_A d_B u_C) \;. \tag{37}$$

χ_C symbolizes the colour wave function of the bound state, which for baryons is totally antisymmetric:

$$\chi_{C\ abc} = \frac{1}{\sqrt{6}} \varepsilon_{abc} \quad . \tag{38}$$

χ_C guarantees the whole BS-amplitude to be totally antisymmetric as it must be in order to describe a baryon correctly, whereas the spin-flavour-space part alone is obviously symmetric under exchange of any pair of quarks. The subscript E in the exponential term stands for Euclidean; p_E and q_E are the relative quark momenta after Wick rotation: $p_E^2 = -p^2$, $q_E^2 = -q^2$.

For the applications of Eq.(34) the value of the normalization constant N is of great interest. The normalization condition for BS-amplitudes is provided by the inhomogeneous BS-equation [23]. If the baryon states are normalized according to

$$\langle B(P',s')|B(P,s)\rangle = \delta^{(3)}(\vec{P} - \vec{P}')\ \delta_{ss'} \tag{39}$$

insertion of Eq.(34) into the normalization relation yields for N

$$N = \frac{1}{(2\pi)^{3/2}\sqrt{2P_0}}\frac{1}{\beta\sqrt{m}} \tag{40}$$

for the ground state solutions ($n_1 = n_2 = r_1 = r_2 = 0$ in Eq.(32)).

The first and simultaneously main step in the calculation of f is to write down the matrix element for the virtual (off-shell) transition $B \leftrightarrow \ell^c$ expressed in terms of a BS-amplitude describing the hadronic part of the process. This BS-amplitude serves to decompose the baryon into quarks, the baryon- and lepton-number violating interactions of which are already given within the framework of grand unified theories. The usage of the BS-formalism is therefore nothing else but a convenient way to treat the so far unsolved problem of the confinement of colour nonsinglets for our special case. Neglecting Cabibbo-suppressed interactions there are seven types of off-shell transitions of this kind:

$$
\begin{array}{ll}
p \leftrightarrow e_R^+ & \Sigma^+ \leftrightarrow \mu_R^+ \\
p \leftrightarrow e_L^+ & \Sigma^+ \leftrightarrow \mu_L^+ \\
n \leftrightarrow \nu_{e\,R}^{\,c} & \Sigma^0 \leftrightarrow \nu_{\mu\,R}^{\,c} \\
 & \Lambda \leftrightarrow \nu_{\mu\,R}^{\,c} \; .
\end{array} \tag{41}
$$

Considering, for example, the strangeness-conserving nucleon decay into an anti-neutrino

$$N \rightarrow \nu_e^{\,c}\, X_{ns} \tag{42}$$

(N denotes the nucleon, X_{ns} a nonstrange hadronic state) we have to deal with the effective Lagrangian

$$L_{eff}(x) = \frac{G}{\sqrt{2}}\, \varepsilon_{ijk} [\overline{\psi_\nu^{\,c}}(x)\gamma_\mu(1-\gamma_5)\psi_{d_i}(x)][\overline{\psi_{u_k}^{\,c}}(x)\gamma^\mu(1+\gamma_5)\psi_{d_j}(x)] \; . \tag{43}$$

Defining for convenience a pure quark operator O_q by

$$O_q(x) = \varepsilon_{ijk}[\gamma_\mu(1-\gamma_5)\psi_{d_i}(x)][\psi_{u_k}^T(x)C^\dagger\gamma^\mu(1+\gamma_5)\psi_{d_j}(x)] \tag{44}$$

we find for the desired S-matrix element

$$
\begin{aligned}
&\langle\bar{\nu}\,(k,\sigma)|S|N(P,s)\rangle = \\
&= (2\pi)^4\,\delta^{(4)}(P-k)\frac{G}{\sqrt{2}}\,\langle\bar{\nu}\,(k,\sigma)|\overline{\psi_\nu^{\,c}}(0)|0\rangle\;\langle 0|O_q(0)|N(P,s)\rangle
\end{aligned} \tag{45}
$$

where $\langle 0|O_q(0)|N(P,s)\rangle$ has necessarily to involve the BS-amplitude for the nucleon N. Then, by inspection of Eq.(17), it is easy to convince oneself that this matrix element is given by the expression

$$
\begin{aligned}
\langle 0|O_q(0)|N(P,s)\rangle = &\sum_{\alpha,\beta,\gamma}\;\sum_{a,b,c}\;\sum_{A,B,C} \varepsilon_{abc}(d_A u_B d_C)[\gamma_\mu(1-\gamma_5)]_{\delta\gamma}[C^\dagger\gamma^\mu(1+\gamma_5)]_{\beta\alpha}\cdot \\
&\cdot \frac{i}{(2\pi)^6}\int d^4p\; d^4q\; \chi(p,q;P,s)_{\alpha\beta\gamma;abc;ABC} \; .
\end{aligned} \tag{46}
$$

The integration over the relative quark momenta p and q is the remnant of the integration over the internal quark momenta k_1, k_2, and k_3 when taking into account energy-momentum conservation:

$$\int d^4k_1 d^4k_2 d^4k_3 \; \chi(k_1,k_2,k_3;P',s) \; \delta^{(4)}(P-k_1-k_2-k_3) = $$
$$= \int d^4P' d^4p d^4q \; \chi(p,q;P',s) \; \delta^{(4)}(P-P') = \int d^4p d^4q \; \chi(p,q;P,s) \quad . \tag{47}$$

It can be seen from Fig. 3, which shows Eq.(45) in a symbolical manner, that the matrix element of Eq.(46) has to be a two-loop integral as stated by Eq.(47).

Inserting the BS-amplitude found by Kielanowski, Eq.(34), into Eq.(46) and comparing the matrix element obtained in this way with the phenomenological matrix element resulting from Eq.(2), one can extract the effective coupling f. Our result is identical for all seven cases of Eq.(41):

$$f = \frac{G}{\sqrt{2}} \frac{2\sqrt{6}}{\pi^2} \frac{\beta}{\sqrt{mM}} (\sqrt{6}\, f^+ - \sqrt{2}\, f^-)[6 \frac{M}{m} + \frac{M^2}{m^2} + 72 \frac{\sqrt{\beta}}{m^2}] + O(\frac{1}{m^3}) \tag{48}$$

where f^+ and f^- denote the scalar products of the flavour part of the quark operator O_q with the flavour wave functions of the BS-amplitude $\chi^{(M^+)}_{SU(3)}$ and $\chi^{(M^-)}_{SU(3)}$ which take e.g. for the proton the values

$$f^+(\text{proton}) = \sum_{A,B,C} u_A u_B d_C \; \chi^{(M^+)}_{SU(3)\;ABC}(\text{proton}) = \frac{2}{\sqrt{6}} \quad ,$$
$$f^-(\text{proton}) = \sum_{A,B,C} u_A u_B d_C \; \chi^{(M^-)}_{SU(3)\;ABC}(\text{proton}) = 0 \quad . \tag{49}$$

The relative minus sign between the terms involving f^+ and f^- is due to the definition of O_q, Eq.(44), in terms of left- and right-handed quarks. A definition of O_q in terms of only one kind of quarks, either left- or right-handed, which can be given by application of the transformation

$$\bar{\psi}_1 \gamma_\mu (1 \pm \gamma_5) \psi_2 = - \overline{\psi_2^c} \, \gamma_\mu (1 \mp \gamma_5) \psi_1^c \quad , \tag{50}$$

would manifest itself in a relative plus sign between f^+ and f^-. It is remarkable that in f the lowest order of the power series in 1/m vanishes. This is due to the fact that the effective Lagrangians we are dealing with, like Eq.(43), are formed by the exchange of vector bosons. As a consequence of that f depends decisively on the quark mass m.

When one tries to find an acceptable numerical value for f one has to be aware of the fact that there are a number of uncertainties entering in this attempt. The most uncertain aspect is the size of the effective quark mass m. Although there exist different estimates of m [24-26] we follow the choice of Ref. 27 and Ref. 28 and choose $m = 1$ GeV+). Of less importance, however, is the arbitrariness in the choice of the model for the BS-kernel caused by our lack of knowlwdge about the interactions taking place between quarks. It has been shown [14] that one is able to obtain in other reasonable models quite similar results for the BS-amplitude, differing only slightly in the value of the internal momentum cut-off $\beta^{1/4}$.

Another already well-known problem is the magnitude of the effective four-fermion coupling G [9,30]. Taking the values of Goldman and Ross [9] for the grand unification group SU(5), $\alpha = 0.0244$ and $m_X = 4.2 \cdot 10^{14}$ GeV, one finds

$$\frac{G}{\sqrt{2}} = 2.17 \cdot 10^{-31}\ \mathrm{GeV}^{-2} \ . \tag{51}$$

With this value for the coupling strength G we obtain in the case of the transition $p \leftrightarrow e^+$ (i.e. $M = 0.938$ GeV)

$$f(\mathrm{proton}) = 1.61 \cdot 10^{-33}\ \mathrm{GeV} \ . \tag{52}$$

Similar results are obtained for the other transitions of Eq.(41). In Table II we listed the values of the effective coupling strength, $f_{eff} = \bar{f}\sqrt{1+|\alpha|^2}$, for different processes and unification groups.

+) This value is, of course, a lower limit to the initial assumption of heavy quarks, Eq.(23). An increase of m would lead to a better agreement with this assumption but on the other hand it would reduce f proportional to $m^{3/2}$.

IV. Application: The Proton Decay

Apart from the applications of the effective Lagrangian of Eq.(3) already mentioned in Sec. II one may use the baryon-lepton coupling constant $\bar{f}$ in order to calculate the decay rates for the baryon- and lepton-number violating decay of the nucleons. There are already some calculations of that sort present in the literature. They can be classified into two types according to the way they treat the problem of binding quarks to hadrons. One method uses the static SU(6) spin-flavour wavefunctions of quarks inside of hadrons for weighting the possible tree-graph processes described by Eq.(1) [5-10]. The other method uses a bag model for estimating the initial diquark overlap in the nucleon bag and the quark-antiquark overlap in the various exclusive final state meson bags [11-13]. All of these works contain a direct treatment of the three-quark system called baryon. We propose here a new kind of handling the puzzle of confinement for the special case of proton decay: A use of the effective baryon-lepton coupling constant is equivalent of taking the baryon pole in the three-quark system.

We calculate the exclusive two-body decay rates of proton and neutron according to Fig. 4: The nucleon N decays with a probability determined by the phenomenological meson-baryon-baryon coupling constant g [31] into a pseudoscalar meson P or vector meson V and an off-shell baryon B which converts into the antilepton ℓ^c with a strength given by the baryon-lepton coupling constant $\bar{f}$. We take into account only pole contributions of the relevant baryons with lowest mass, i.e. of the $J^P = \frac{1^+}{2}$ baryons p, n, Σ^+, Σ^0 and Λ, because higher baryon states are represented by angular excitations in the harmonic oscillator potential and it turns out that the contributions to the amplitude of the correctly normalized corresponding solutions of the BS-equation are suppressed at least by a factor

$$\frac{\beta^{1/4}}{m}\,\frac{M_B}{M_B^{**}} = 0.26 \quad ,$$

where M_B/M_B^{**} denotes the mass ratio of ground state to excited state solution of the BS-equation. We therefore think that confining ourselves to the

$J^P = \frac{1}{2}^+$ baryons of the 56 representation of static SU(6) and ignoring the in principle possible poles of the $J^P = \frac{1}{2}^-$ baryons N(1535), N(1700), Δ(1650) and so on should give the major contributions to the decay widths. Denoting by M_N, M_B, m_P, and m_V the masses of the nucleon, the off-shell baryon, the pseudoscalar, and the vector meson and by g_P, g_V, g_T the pseudoscalar, vector, and tensor baryon-baryon-meson coupling constants and neglecting the lepton mass the nucleon decay rates are given for production of a pseudoscalar meson by

$$\Gamma(N \to P\ell^c) = \frac{1}{16\pi} \frac{g_P^2}{M_B^2} \bar{f}^2 (1 + |a|^2) \frac{(M_N^2 - m_P^2)^2}{M_N^3} , \tag{53}$$

and for production of a vectormeson by

$$\Gamma(N \to V\ell^c) = \frac{1}{16\pi} \frac{g_V^2}{M_B^2} \bar{f}^2 (1 + |a|^2) \frac{(M_N^2 - m_V^2)^2}{M_N^3} \cdot \left\{3(1 + \frac{\kappa}{2})^2 + (1 - \frac{m_V^2}{M_N^2} \frac{\kappa^2}{4}) \frac{(M_N^2 - m_V^2)}{m_V^2}\right\} , \tag{54}$$

where κ means the ratio tensor to vector coupling constant

$$\kappa = \frac{g_T}{g_V} . \tag{55}$$

Using the values of a given in Table I and the empirical data for the phenomenological meson-baryon-baryon coupling constants reported by Nagels et al. [31] we obtain for the various mesons M the two-body branching ratios

$$\frac{\Gamma(N \to M\ell^c)}{\Gamma(N \to \text{two-body})}$$

listed in Table III separately for proton and neutron. If the final state contains a muon, its mass has been taken into account.

Comparing our results for the two-body branching fractions with those of Refs. 7, 8, 10 to 13 we find a good qualitative agreement with Machacek [7], Gavela et al. [8], case (a) of Din et al. [11] and the recoil model of Kane and Karl [10] except for a larger enhancement of the decay modes involving a pion which is due to the different orders of magnitude of the phenomenological coupling constants [31]. For the discrepancy of our results with those of Donoghue [12] and Golowich [13] see footnote 1 of Ref. [8]. In addition we found the kaon modes somewhat more suppressed than cited in Ref. [8].

For the estimation of the lifetimes of proton and bound neutron we include the three-body decays of the nucleons. As already pointed out by Wise et al. [32] the three-body non-resonant background yields a non-negligible contribution to the total decay rate. Using simple Born terms for the meson-baryon amplitude we found the semi-inclusive branching ratios

$$\frac{\Gamma(N \to \ell^c + \text{hadrons})}{\Gamma(N \to \text{all})}$$

of Table IV. Our computations show a particular large contribution of the decay modes $p \to e^+\pi^+\pi^-$ and $n \to \nu_e^c\pi^+\pi^-$ due to the absence of the interference term between s- and u-channel. Our results are not very different from those found by Jarlskog and Yndurâin [6], Machacek [7] and Goldman and Ross [9].

Adding up all calculated decay widths we obtain the ratios τ_p/τ_n of the proton lifetime versus that of the bound neutron given in Table V. In contrast to most of the already published values for this ratio [33], in our work for the case of SU(5) the proton lifetime appears to be larger than that of the neutron. This fact is brought about by the dominance of the pion decay modes in connection with the isospin relation [34]

$$\Gamma(n \to e^+\pi^-) = 2\Gamma(p \to e^+\pi^0) \ . \tag{56}$$

The nucleon lifetimes resulting within the SU(5) model are mainly determined by the value of the baryon-lepton coupling strength f, Eq.(52),

$$\begin{aligned} \tau_p &= 2.6\cdot 10^{31}\ \text{yr}\ , \\ \tau_n &= 2.3\cdot 10^{31}\ \text{yr}\ . \end{aligned} \tag{57}$$

These estimates are of course affected with all the uncertainties considered in Refs. 9 and 30.

We have shown that the description of the new baryon-number violating force in terms of an effective Lagrangian yields a convenient way of treating phenomenologically the corresponding processes, like decay rates, branching ratios and cross-sections. All one has to know are the coupling constant $\bar{f}$ and the ratio a of parity violation, as well as the usual strong interaction meson-baryon coupling constants. In addition, the smallness of the parameter a in certain transitions allows for a definition of parity of leptons with respect to nucleons.

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	SU(5)	SO(10)	$SU(2)_L \otimes SU(2)_R \otimes U(1)$
r_e	2	0	1
r_μ	1	0	1
a_e	-0.359	1	0
a_μ	-0.029	1	0

Table I. Unrenormalized values of r and renormalized values of a for three different unification groups.

	f_{eff}		
transition	SU(5)	SO(10)	$SU(2)_L \otimes SU(2)_R \otimes U(1)$
$p \leftrightarrow e^+$	14.87	6.35	8.98
$n \leftrightarrow \nu_e^{\,c}$	6.35	6.35	6.35
$\Sigma^+ \leftrightarrow \mu^+$	9.25	6.35	8.98
$\Sigma^0 \leftrightarrow \nu_\mu^{\,c}$	4.50	4.50	4.50
$\Lambda \leftrightarrow \nu_\mu^{\,c}$	2.59	2.59	2.59

Table II. Values of $f_{eff} = \bar{f}\sqrt{1 + |a|^2}$ in units of $10^{-3}\,\frac{G_{GUM}}{\sqrt{2}}\,m_p^3\,\Lambda_R$ ($= 6.3\cdot 10^{-34}$ GeV for SU(5)). Here m_p denotes the proton mass.

Branching ratio [%]

Decay mode	SU(5)	SO(10)	$SU(2)_L \otimes SU(2)_R \otimes U(1)$
$p \to e^+\pi^0$	44.4	20.6	29.4
$p \to e^+\eta$	1.2	0.6	0.8
$p \to e^+\rho^0$	8.1	3.8	5.4
$p \to e^+\omega$	16.4	7.6	10.9
$p \to \nu_e{}^c\pi^+$	16.1	41.0	29.2
$p \to \nu_e{}^c\rho^+$	2.9	7.5	5.4
$p \to \mu^+K^0$	7.5	10.4	12.8
$p \to \nu_\mu{}^cK^+$	3.3	8.5	6.1
$p \to \nu_\mu{}^cK^{*+}$	0.1	0.2	0.1
$n \to e^+\pi^-$	73.3	45.8	59.5
$n \to e^+\rho^-$	13.4	8.4	10.9
$n \to \nu_e{}^c\pi^0$	6.7	23.0	14.9
$n \to \nu_e{}^c\eta$	0.2	0.6	0.4
$n \to \nu_e{}^c\rho^0$	1.2	4.2	2.7
$n \to \nu_e{}^c\omega$	2.5	8.5	5.5
$n \to \nu_\mu{}^cK^0$	2.7	9.5	6.0
$n \to \nu_\mu{}^cK^{*0}$	0.0	0.2	0.1

Table III. Two-body decay branching ratios $\frac{\Gamma(N \to \ell^c M)}{\Gamma(N \to \text{2-body})}$.

Decay mode	Branching ratio [%] SU(5)	SO(10)	$SU(2)_L \otimes SU(2)_R \otimes U(1)$
$p \to e^+ X_{ns}$	73.0	35.7	50.0
$p \to \nu_e^c X_{ns}$	17.1	46.0	32.2
$p \to \mu^+ X_s$	6.8	9.9	12.0
$p \to \nu_\mu^c X_s$	3.1	8.4	5.8
$n \to e^+ X_{ns}$	85.1	51.0	67.6
$n \to \nu_e^c X_{ns}$	12.0	39.6	26.1
$n \to \mu^+ X_s$	0.2	0.2	0.3
$n \to \nu_\mu^c X_s$	2.8	9.1	6.0

Table IV. Inclusive nucleon decay branching ratios $\frac{\Gamma(N \to \ell^c X)}{\Gamma_{tot}(N)}$. X_{ns} and X_s denote non-strange and strange hadronic states.

Unification group	τ_p/τ_n
SU(5)	1.11
SO(10)	0.90
$SU(2)_L \otimes SU(2)_R \otimes U(1)$	0.96

Table V. Comparison of the ratio τ_p/τ_n for the different grand unification groups.

Figure Captions

Fig. 1 Baryon BS-amplitude χ for a bound state B of three quarks q_i ($i = 1,2,3$) with momenta k_i, spin indices α, β, γ, colour indices a, b, c, and flavour indices A, B, C.

Fig. 2 The homogeneous three-particle BS-equation for bound states, multiplied by $\prod_{i=1}^{3} S_F(k_i)$.

Fig. 3 The transition $B \leftrightarrow \ell^c$ decomposed into B- and L-violating effective four-fermion interaction with strength G and BS-amplitude χ.

Fig. 4 The decay of a nucleon N into an antilepton ℓ^c and a pseudoscalar meson P or a vector meson V.

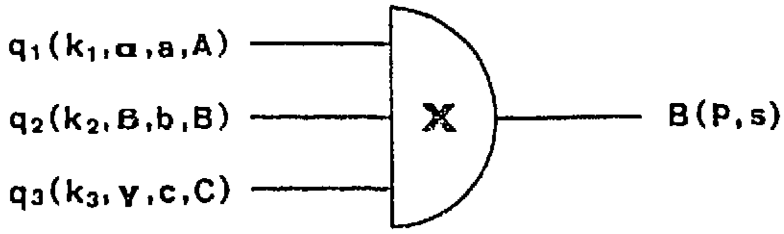

Fig. 1

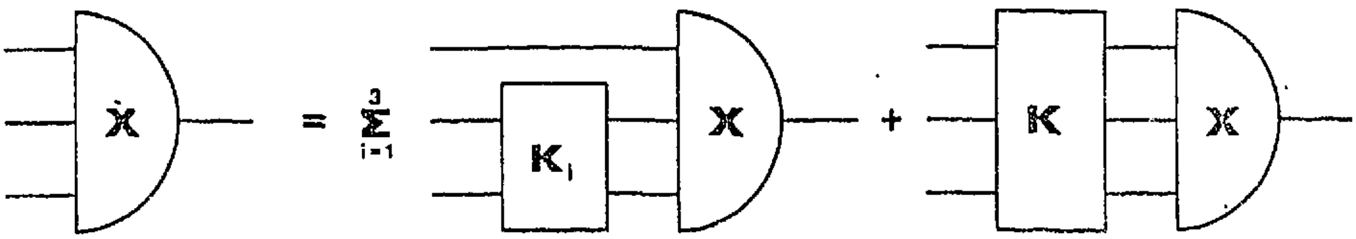

Fig. 2

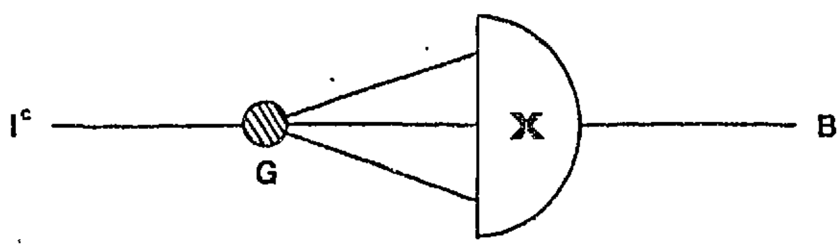

Fig. 3

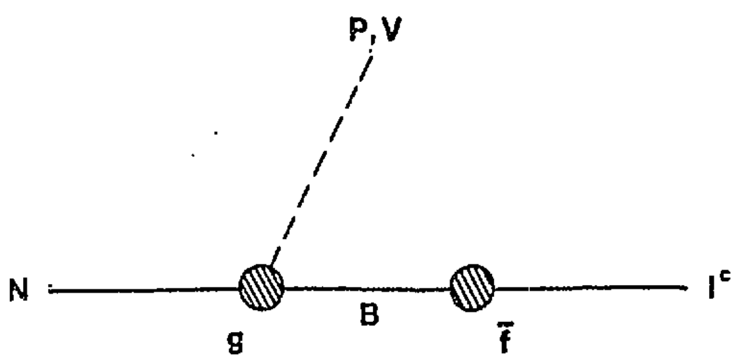

Fig. 4