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# COULOMB DISPLACEMENT ENERGY CALCULATIONS FOR THE A = 28 ; T = 1 TRIAD

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Abstract

Coulomb energy displacements have been calculated for the A=28;T=1 triad using shell model wave functions. Averaging the small deviations between calculated and available experimental values, one has calculated six additional $^{28}$Si;T=1 levels of which four can be associated with experimental levels with previously unknown T-values.

## 1) Introduction

From a study of the $^{29}Si(^3He,t)$ reaction [1] we have obtained spectroscopic informations on levels in $^{29}P$ up to $E_x \sim 4$ MeV. From a comparison with the levels in $^{28}Al$ [ref.2] a clear connection appears between excited states in these nuclei up to $\sim 3.3$ MeV, except for the second $0^+$ in $^{28}Al$ and second $5^+$ in $^{28}P$. This connection locates the $T_z=1$ and $T_z=-1$ partners of the first sixteen positive parity triplets of the T=1 isobaric triad A=28. In the corresponding part of $^{28}Si$ level scheme, there are only eleven known levels to have T=1 [ref.2] two of them, the levels at 10597.3 and 10724.7 keV both with $J^\pi=1^+$, are considered to be split analogue states due to isospin mixing.

For the lower six levels Coulomb displacement energy (CDE) calculations have already been made [ref.3,4] and it would be interesting to extend these calculations to all ten states. This is a way to check the nuclear model wave functions generated to describe the A=28;T=1 states because the CDE for triplet of states are sensitive to wave functions. If the agreement between experimental and calculated CDE is satisfying for the known triplets, these calculations could give predictions on the positions of the T=1 missing states in the $^{28}Si$ spectrum.

## 2) Coulomb displacement energies calculations

### 2.1) METHOD

The difference between the binding energies of two isobaric analogue states arises from the mass difference between proton and neutron and from the electrostatic and electromagnetic Coulomb energy difference $\Delta E_c$ between them. $\Delta E_c$ can be extracted experimentally from the excitation energies and masses by:

$$\Delta E_c^{exp} = E_x(Z+1,N) - E_x(Z,N+1) - Q(p,n)$$

De Meijer et al [3]) have developped an effective model to calculate the CDE that can be used when the wave function is a mixture of shell model configurations. A complete description of this model is given ref.[3]); here we will recall only briefly its main features. Since all configurations are orthonormal to each other, the CDE for the wave function is the weighted sum of the CDE for the configurations. The weightfactor is the intensity of the configuration in the wave function. For each configuration, the Coulomb energy shift can be calculated as a weighted sum of single particle CDE. Each single particle CDE is calculated by factorizing the configuration in nucleus A into a "core" in the nucleus A-1 and a neutron. Subsequently the neutron is changed into a proton without changing the other quantum numbers. The configuration in the nucleus A-1 is named "core" because the $T^-$ operator only works on the neutron. In order to know the binding energy of the neutron to that core, it is necessary to calculate the model wave functions for the nucleus A and for the core nucleus A-1. The electrostatic part of single particle CDE results from the interaction between the last added proton and the charge distribution of all other protons. The electromagnetic part arise from difference between the spin-orbit coupling energies for proton and neutron. The charge distribution in the model is spherical.

The radial wave function of the single neutron is calculated with a Wood-Saxon nuclear potential as:

$$V(r) = - V_o \left[ f(r) - v \, (\hbar/2\mu c) \, \frac{1}{r} \frac{d}{dr} f(r) \, \underset{\sim}{L}.\underset{\sim}{S} \right]$$

where $f(r) = \left[ 1 + \exp (r-R)/\alpha \right]^{-1}$ and $R = r_o A^{1/3}$, $\mu$ is the reduced mass of the core plus nucleon and v is the spin-orbit parameter. The well depth $V_o$

is adjusted to reproduce the neutron separation energy with respect to the core. The best values for the two parameters, radius $r_o$ and diffuseness $a$ have been determined in ref.[3]) as: $r_o$= 1.28 fm, $a$ = 0.63 fm for A = 13-28 and $r_o$= 1.26 fm, $a$ = 0.63 fm for A = 29-40.

2.1) SHELL MODEL WAVE FUNCTIONS

The shell model wave functions have been calculated with a MSDI residual interaction and with a configuration space comprising the 1d5/2, 2s1/2 and 1d3/2 subshells, with at least eight particles in 1d5/2 shell. For the positive parity A=28;T=1 states, the calculations are identical to the ones of de Voigt et al [5]) with a minor extra restriction on the configuration space: for computational reasons we have suppressed the $(1d5/2)^6(2s1/2)^2(1d3/2)^2$ configurations in order to reduce the matrices sizes. We expect the effect of this restriction to be negligible in the present Coulomb calculations: because (i) the centroid of these configurations is very likely situated at high excitation energies and so they mix only weakly with the first levels of each J;T (ii) in the CDE calculations we have only take into account the configurations with at least to 5% intensity. The wave functions for the A=27;T=1/2 and 3/2 positive parity states have been calculated under the conditions as ref.[5,6]): no configurations are excluded

3) Results

3.1) THE CALCULATION OF THE KNOWN T=1 LEVELS IN A=28

As a first step, we have calculated the CDE for the first ten A=28;T=1 triplets. In order to associate the experimental levels, we have started from the T=1 propositions of ref.[2]) for the first seven triplets; for the $^{28}P;T_z$=-1 partner, our results of ref.[1]) confirm these suggestions.

The next three experimental triplets are identified from suggestions of ref.[2]) for $^{28}$Al and $^{28}$Si and from our previous results for $^{28}$P. Table 1 and figure 1 present the results of CDE calculations made for $r_o$= 1.26 fm. Table 1 lists also the $\Delta E_c$ values calculated previously by de Meijer et al[3]) and Miehe et al [4]) for the first six triplets. The Q(p,n) and the neutron separation energies involved in the calculations have been extracted from ref.[7]). Both calculations have used $r_o$= 1.28 fm and as can be seen the results of de Meijer et al are systematically too low. In fact our calculations with $r_o$= 1.28 fm reproduce these results within 30 keV. By taking $r_o$= 1.26 fm the systematical deviation can be removed without noticeably changing the relative level spacings. This change in $r_o$ might reflect the effects due to the limited number of 1d5/2 holes in the wave functions. As can be seen the overall agreement with the data is fairly good; especially the changes in level spacing are well reproduced.

3.2) CALCULATION OF THE UNKNOWN T=1 LEVELS IN $^{28}$Si

As has been pointed out in subsection 2.1, experimental excitation energies are available for ten triplets and for the $T_z$=1 and $T_z$=-1 components of six additional triplets. In this subsection, we try to locate the six levels missing in the low lying part of $^{28}$Si;T=1 spectrum.

The calculations are performed with $r_o$= 1.26 fm. In order to deduce $^{28}$Si;T=1 excitation energies from the $^{28}$P ones, the calculations are made in a same way as before except that $^{28}$Si energies are calculated from the $^{28}$P excitation energies by subtracting the calculated CDE. There are two sets of calculated excitation energies available for $^{28}$Si;T=1: $E_x^{cal}$(Al→Si) from $^{28}$Al and $E_x^{cal}$(P→Si) from $^{28}$P. We obtain the so called predicted values by averaging the two values and correcting for the average deviations in

each of the two sets:

$$E_x^{pred}(^{28}Si;T=1) = 1/2\left[E_x^{cal}(Al\rightarrow Si) + E_x^{cal}(P\rightarrow Si) + \bar{x}(Al) + \bar{x}(Si)\right]$$

where $\bar{x}(X) = (1/N)\left[E_x^{exp}(Si) - E_x^{cal}(X\rightarrow Si)\right]$, calculated for the N known experimental excitation energies of $^{28}Si;T=1$. In this way we have corrected for the average observed deficiencies due to the shell model wave functions and/or the $r_o$ parameter. The resultant standard deviation regarded as uncertainty on the predicted values, is about 20 keV. Table 2 presents the results.

From the comparison between calculated and experimental $^{29}Si$ excitation energies, which are possible T=1 candidates, four more levels are found to agree with the calculations: (i) the experimental level at 10883 keV ($1^- - 4^+$) could be the second $2^+;1$ calculated at 10878 ± 20 keV; (ii) the 11778.8 keV level agrees in energy with the fourth $2^+;1$ calculated at 11756 ± 20 keV; (iii) the experimental $3^+$ 12240.8 keV level could be the third $3^+;1$ calculated at 12254 ± 20 keV; (iv) the second $4^+;1$ level predicted at 11952 ± 20 keV could agree with the experimental level lying at 11933 keV. On the other hand, there are no obvious candidate for the first $4^+$ and first $5^+$ levels.

## 4) Conclusions and summary

For shell model wave functions with an MSDI interaction it was found that the calculated Coulomb displacement energies and deduced excitation energies are in fairly good agreement with available experimental values. Averaging the small deviations, one compensates effectively the deficiencies in the wave functions due to the limited configuration space and the choice of the parameters in the CDE calculations. With this improvement six additional T=1 levels in $^{28}Si$ have been calculated of which four can be associated with experimental levels with previously unknown T values.

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TABLE 1

Excitation energies and Coulomb displacement energies for known T=1 levels in A=28 nuclei

| Shell model | | $^{28}Al^{exp}$ Ref.[2] | | $^{28}Si^{exp}$ Ref.[2] | | $\Delta E_c^{exp}$ (keV) | $\Delta E_c^{cal}$ (keV) Ref.[3] | $\Delta E_c^{cal}$ (keV) Ref.[4] | $\Delta E_c^{cal}$ (keV) | $E_x^{cal}$ $^{28}Si$ (keV) |
|---|---|---|---|---|---|---|---|---|---|---|
| $J^\pi$ | n | $E_x$(keV) | $J^\pi$ | $E_x$(keV) | $J^\pi;T$ | | $r_o$=1.28 | 1.28 | 1.26 | 1.26 fm |
| $0^+$ | 1 | 972.2 | $0^+$ | 10272.3 | $(0,1)^+;1$ | 5439.4 | 5273 | 5494<br>5473 a) | 5371 | 10204 |
| $1^+$ | 1 | 1372.8 | $1^+$ | 10597.3<br>10724.7 | $1^+;0+1$<br>$1^+;0+1$ | 5363.8<br>5491.2 | 5331 | 5485 | 5463 | 10697 |
| | 2 | 1620.1 | $1^+$ | 10901.0 | $1^+;1$ | 5420.2 | 5394 | 5489 | 5496 | 10977 |
| | 3 | 2201.6 | $1^+$ | 11446.2 | $1^+;1$ | 5383.9 | | | 5429 | 11492 |
| | 4 | 3105. | $(1,3)^+$ | 12331.7 | $1^+;1$ | 5367. | | | 5369 | 12335 |
| $2^+$ | 1 | 30.6 | $2^+$ | 938.5 | $2^+;1$ | 5489.2 | 5317 | 5480 | 5383 | 9275 |
| | 3 | 2138.5 | $2^+$ | 11434.3 | $(2,3)^+;1$ | 5435.1 | | | 5412 | 11411 |
| $3^+$ | 1 | 0. | $3^+$ | 9315.9 | $3^+;1$ | 5455.2 | 5328 | 5478 | 5379 | 9240 |
| | 2 | 1014.0 | $3^+$ | 10376. | $3^+;1$ | 5501.3 | 5375 | 5455 | 5523 | 10398 |
| | 4 | 3296. | $3^+$ | 12542.1 | $(2^+,3);1$ | 5385.4 | | | 5282 | 12439 |

TABLE 1 continued

| Shell model | | $^{29}Si^{exp}$ | | $^{29}P^{exp}$ | | $\Delta E_c^{exp}$ (keV) | $\Delta E_c^{cal}$ (keV) | $\Delta E_c^{cal}$ (keV) | $\Delta E_c^{cal}$ (keV) | $E_x^{cal}$ $^{29}P$ (keV) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Ref.[2]) | | Ref.[1,2]) | | | Ref.[7]) | Ref.[4]) | | |
| $J^\pi$ | n | $E_x$(keV) | $J^\pi;T$ | $E_x$(keV) | $J^\pi$ | | $r_o$=1.28 | 1.28 | 1.26 | 1.26 fm |
| $0^+$ | 1 | 10272.3 | $(0,1)^+;1$ | 877 | $0^+$ | 5719. | 5697 | 5923<br>5898[a]) | 5791 | 949 |
| $1^+$ | 1 | 10597.3<br>10724.7 | $1^+;0+1$<br>$1^+;0+1$ | 1313 | $1^+$ | 5830.<br>5702. | 5784 | 5913 | 5885[b]) | 1432 b) |
| | 2 | 10901.0 | $1^+;1$ | 1569 | $1^+$ | 5782. | 5812 | 5916 | 5920 | 1709 |
| | 3 | 11446.2 | $1^+;1$ | 2143 | $1^+$ | 5811 | | | 5852 | 2184 |
| | 4 | 12331.7 | $1^+;1$ | 2973 | $1^+$ | 5731 | | | 5791 | 3009 |
| $2^+$ | 1 | 9380.5 | $2^+;1$ | 106 | $2^+$ | 5840. | 5731 | 5902 | 5802 | 68 |
| | 3 | 11434. | $(2,3)^+;1$ | 2104 | $2^+$ | 5784. | | | 5833 | 2153 |
| $3^+$ | 1 | 9315.9 | $3^+;1$ | 0 | $3^+$ | 5798. | 5742 | 5902 | 5799 | 1 |
| | 2 | 10376. | $3^+;1$ | 1133 | $3^+$ | 5871. | 5798 | 5873 | 5946 | 1208 |
| | 4 | 12542. | $(2^+,3);1$ | 3164 | $3^+(4)^+$ | 5736 | | | 5701 | 3129 |

n is the ordering number of the eigenvector of the shell model wave function.

[a]) The shell model calculations used in ref.[4]) give two $0^+;1$ levels at almost the same energies. [b]) The calculations have been made from the mean value of the excitation energies for the two $1^+;0+1$ levels.

TABLE 2

Summary of the calculated, experimental and proposed T=1 states following from the adjusted CDE calculations

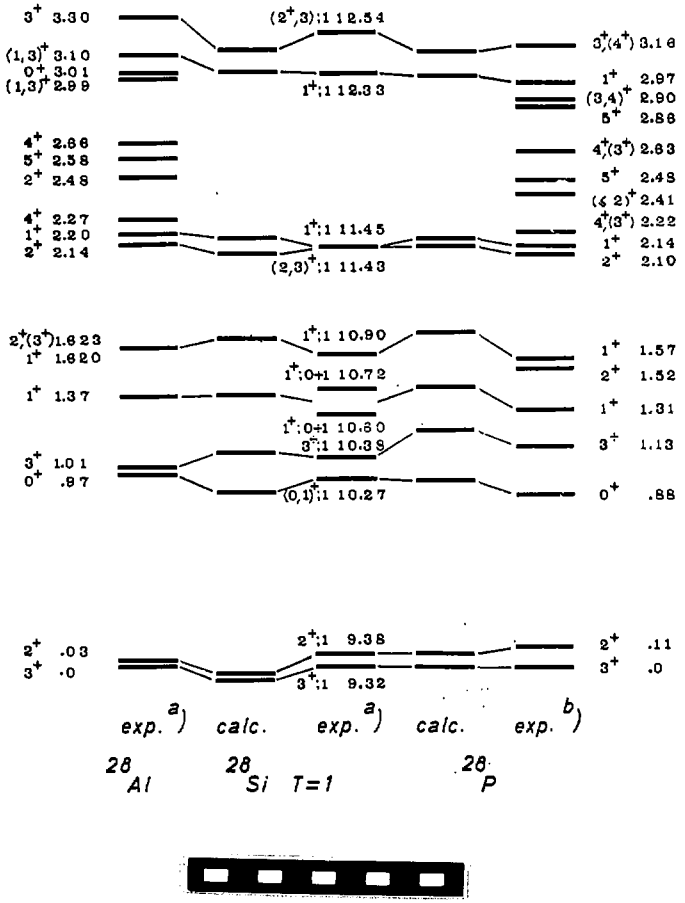
| $J^\pi$ | $^{28}$Al Ref.[2]) $T_z = 1$ $E_x^{exp}$(keV) | $J^\pi$ | $^{28}$Si Ref.[2]) $T_z = 0$ $E_x^{pred}$ (keV) | | $E_x^{exp}$ (keV) | $J^\pi;T$ | $^{28}$P Refs.[1,2]) $T_z = -1$ $E_x^{exp}$(keV) | $J^\pi$ |
|---|---|---|---|---|---|---|---|---|
| $0^+$ | 972.2 | $0^+$ | 10232 | | 10273.3 | $(0,1)^+;1$ | 877 | $0^+$ |
| $1^+$ | 1372.8 | $1^+$ | 10648 | | 10597.3<br>10724.7 | $1^+;0+1$<br>$1^+;0+1$ | 1313 | $1^+$ |
| $1^+$ | 1620.1 | $1^+$ | 10900 | | 10901.0 | $1^+;1$ | 1569 | $1^+$ |
| $1^+$ | 2201.6 | $1^+$ | 11480 | | 11446.2 | $1^+;1$ | 2143 | $1^+$ |
| $1^+$ | 3105. | $(1,3)^+$ | 12347 | | 12331.7 | $1^+;1$ | 2973 | $1^+$ |
| $2^+$ | 30.6 | $2^+$ | 9380 | | 9380.5 | $2^+;1$ | 106 | $2^+$ |
| $2^+$ | 1622.7 | $2^+(3^+)$ | 10878 | * | 10883. | $(1^-,4^+)$ | 1516 | $2^+$ |
| $2^+$ | 2138.5 | $2^+$ | 11429 | | 11434.3 | $(2,3)^+;1$ | 2104 | $2^+$ |
| $2^+$ | 2485.0 | $2^+$ | 11756 | * | 11778.8 | | 2406 | $(\geq 2)^+$ |
| $3^+$ | 0. | $3^+$ | 9310 | | 9315.9 | $3^+;1$ | 0 | $3^+$ |
| $3^+$ | 1014.0 | $3^+$ | 10381 | | 10376. | $3^+;1$ | 1133 | $3^+$ |
| $3^+$ | 2987. | $(1,3)^+$ | 12254 | * | 12240.8 | $3^+$ | 2896 | $(3,4)^+$ |
| $3^+$ | 3296 | $3^+$ | 12542 | | 12542.1 | $(2^+,3);1$ | 3164 | $3^+(4^+)$ |
| $4^+$ | 2272. | $4^+$ | 11554 | | | | 2216 | $4^+(3^+)$ |
| $4^+$ | 2656. | $4^+$ | 11952 | * | 11933. | | 2628 | $4^+(3^+)$ |
| $5^+$ | 2582. | $5^+$ | 11844 | | | | 2483 | $5^+$ |

* : proposed T=1 level.

Caption of figure 1

A comparison between the A=28;T=1 levels schemes.

$r_o$ = 1.26 fm. [a]) taken from ref.[2]); [b]) taken from refs.[1,2]). All energies are in MeV.


3+ 3.30
(1,3)+ 3.10
0+ 3.01
(1,3)+ 2.99
4+ 2.86
5+ 2.58
2+ 2.48
4+ 2.27
1+ 2.20
2+ 2.14
2+(3+)1.623
1+ 1.620
1+ 1.37
3+ 1.01
0+ .97
2+ .03
3+ .0
(2+,3);1 12.54
1+;1 12.33
1+;1 11.45
(2,3)+;1 11.43
1+;1 10.90
1+;0+1 10.72
1+;0+1 10.60
3+;1 10.38
(0,1)+;1 10.27
2+;1 9.38
3+;1 9.32
3+(4+) 3.16
1+ 2.97
(3,4)+ 2.90
5+ 2.86
4+(3+) 2.63
5+ 2.48
(≤2)+ 2.41
4+(3+) 2.22
1+ 2.14
2+ 2.10
1+ 1.57
2+ 1.52
1+ 1.31
3+ 1.13
0+ .88
2+ .11
3+ .0
exp. a)
calc.
exp. a)
calc.
exp. b)
28 Al
28 Si T=1
28 P