ON QCD SUM RULES OF THE LAPLACE TRANSFORM TYPE AND LIGHT QUARK MASSES

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ABSTRACT

We discuss the relation between the usual dispersion relation sum rules and the Laplace transform type sum rules in QCD. Two specific examples corresponding to the ρ-coupling constant sum rule [1b] and the light quark masses sum rules [2] are considered. An interpretation, within QCD, of Leutwyler's formula [11a] for the current algebra quark masses is also given.

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There has been some progress during the last few years in extending the applicability domain of Quantum Chromodynamics (QCD) to obtain predictions on low energy parameters : masses and coupling constants. The approach is based on sum rules which the spectral functions associated to specific two-point functions of current operators must obey as a consequence of general analyticity properties. Depending on how these analyticity properties and the positivity of the spectral functions are exploited there follows a variety of sum rules which have been discussed in the literature. Of particular interest for low energy phenomenology are the sum rules of the type

$$F(M^2) = \frac{1}{\pi}\int_0^\infty dt\ e^{-t/M^2}\ \mathrm{Im}\,\Pi(t) \qquad (1)$$

proposed by Shifman, Vainshtein and Zakharov (SVZ) [1] and collaborators. Here $\frac{1}{\pi}\,\mathrm{Im}\,\widetilde{\Pi}(t)$ denotes a specific spectral function (e.g., the hadronic vacuum polarization measured in the annihilation $e^+e^- \rightarrow$ Hadrons) and $F(M^2)$ is a quantity which in principle can be computed asymptotically in QCD. Equation (1) is a sum rule of the Laplace transform type : $F(M^2)$ is the Laplace transform [1)] of the spectral function $\frac{1}{\pi}\,\mathrm{Im}\,\widetilde{\Pi}(t)$.

The sum rule (1) is to be contrasted with the usual dispersion relation $(Q^2 > 0)$

$$\widetilde{\Pi}(Q^2) = \frac{1}{\pi}\int_0^\infty dt\ \frac{1}{t+Q^2}\ \mathrm{Im}\,\widetilde{\Pi}(t)\ ; \qquad (2)$$

i.e., the Hilbert transform of $\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)$. It is clear that the r.h.s. in (1) is much more selective on the low energy behaviour of the spectral function (small t) than the r.h.s. in (2). Hence the interest to work with the Laplace transform instead of the Hilbert transform if what we aim at is to obtain constraints on low energy parameters from QCD.

The purpose of this letter is to report on some results which clarify the relationship between the asymptotic Hilbert transform $\Pi(Q^2)$, directly calculable in QCD, and the corresponding asymptotic expression for the Laplace transform $F(M^2)$ in general. This is best illustrated with a discussion of two specific examples corresponding to the two-point functions

$$i\int d^4x\, e^{iq\cdot x}\langle 0|T(J^\mu(x)\, J^\nu(0))|0\rangle = -(g^{\mu\nu}q^2 - q^\mu q^\nu)\,\Pi(q^2), \quad (3)$$

where $J^\mu(x)$ denotes the isovector component of the electromagnetic current, and

$$\psi_5(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T(\partial_\mu A^\mu(x)\, \partial_\nu A^\nu(0)^\dagger)|0\rangle\,, \quad (4)$$

where $\partial_\mu A^\mu(x)$ denotes the divergence of the axial current with the quantum numbers of the π^+. As we shall see, there emerges a consistent picture which puts on a firm common framework the SVZ calculation of the ρ - parameters [1b] and our previous work with Becchi and Ynduráin (BNRY) [2] on light quark masses.

We are interested in the short-distance behaviour of the two-point functions (3) and (4). This can be analyzed via the Wilson's operator product expansion method. In practice we shall only retain the unit operator (i.e., the usual asymptotically free perturbative contributions), and the operators which contribute to the leading non-perturbative $1/Q^2$ power. In order to avoid the dependence in the external renormalization of the two-point function (subtraction terms in the Hilbert transform) it is convenient to work with derivatives of the functions $\Pi(q^2)$ and $\psi_5(q^2)$; i.e., moments of the Hilbert transform. One derivative is required for $\Pi(q^2)$ and two for $\psi_5(q^2)$. In general, for an arbitrary number N of derivatives [2)], we have ($Q^2 = -q^2 > 0$) :

$$\chi^{(N)}(Q^2) \equiv \frac{1}{(N-1)!}(Q^2)^N(-1)^N\frac{\partial^N}{(\partial Q^2)^N}\,\Pi(q^2) =$$

$$\int_0^\infty dt\,\frac{N}{t+Q^2}\left(\frac{Q^2}{t+Q^2}\right)^N\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)\,;\quad N\geq 1 \quad (5)$$

and

$$\psi_5^{(N)}(Q^2) \equiv \frac{1}{(N-2)!}\,(Q^2)^{N-2}\,(-1)^N\,\frac{\partial^N}{(\partial Q^2)^N}\,\psi_5(q^2) =$$

$$\int_0^{\infty} dt\,\frac{(N-1)\,N}{(t+Q^2)^3}\left(\frac{Q^2}{t+Q^2}\right)^{N-2}\frac{1}{\pi}\,\mathrm{Im}\,\psi_5(t)\,;\quad N \geqslant 2\,. \tag{6}$$

The limit advocated by SVZ is : $N \rightarrow \infty$ and $Q^2 \rightarrow \infty$ with $Q^2/N \equiv M^2$ fixed [3]. It can be readily seen that this limit, when applied to the r.h.s. of equations (5) and (6), leads to the Laplace transform of the spectral functions $\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)$ and $\frac{1}{\pi}\,\mathrm{Im}\,\psi_5(t)$ respectively. With

$$\hat{L} \equiv \lim_{\substack{Q^2 \to \infty \\ N \to \infty}\,/\,Q^2/N = M^2} \frac{1}{(N-1)!}\,(Q^2)^N\,(-1)^N\,\frac{\partial^N}{(\partial Q^2)^N}\;,$$

it then follows from equations (5) and (6) that

$$\hat{L}\;\Pi(Q^2) = \frac{1}{M^2}\int_0^{\infty} dt\; e^{-t/M^2}\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)\;; \tag{7}$$

and

$$\hat{L}\;\psi_5^{(2)}(Q^2) = \frac{1}{M^6}\int_0^{\infty} dt\; e^{-t/M^2}\,\frac{1}{\pi}\,\mathrm{Im}\,\psi_5(t)\;. \tag{8}$$

The problem now is to evaluate in QCD the limits on the l.h.s. of these equations. This we discuss in the next two paragraphs.

Once we choose a renormalization scheme, say the $\overline{\mathrm{MS}}$-scheme for convenience, the functions $\chi^{(1)}(Q^2)$ and $\psi_5^{(2)}(Q^2)$, besides their dependence on Q^2, also depend on the renormalized quark masses $m_i(\nu)$, i = up, down, ... ; on $\alpha_s = g^2(\nu)/4\pi$, where $g(\nu)$ is the renormalized QCD coupling constant ; and on ν, the arbitrary mass scale introduced via renormalization. The functions $\chi^{(1)}(Q^2)$, $\psi_5^{(2)}(Q^2)$ as well as their successive derivatives $\chi^{(N)}(Q^2)$, $N > 1$; and $\psi_5^{(N)}(Q^2)$, $N > 2$ obey simple renormalization group equations [4]. Using the fact that the renormalization group operator is an homogeneous function in Q^2 and in $m_i(\nu)$, it then follows that the SVZ-limits $\hat{L}\,\Pi(Q^2)$ and $\hat{L}\,\psi_5^{(2)}(Q^2)$ also obey simple renormalization group equations $\left(Q^2\frac{\partial}{\partial Q^2} = \frac{Q^2}{M}\frac{\partial}{\partial Q^2/_M} \to M^2\frac{\partial}{\partial M^2}\right)$:

$$\left(-M^2\frac{\partial}{\partial M^2} + \beta(\alpha_s)\,\alpha_s\frac{\partial}{\partial \alpha_s} - \sum_i \left(1+\gamma(\alpha_s)\right) x_i\frac{\partial}{\partial x_i}\right)\begin{pmatrix}\hat{L}\,\Pi(Q^2)\\ \hat{L}\,\psi_5^{(2)}(Q^2)\end{pmatrix} = 0, \qquad (9a,b)$$

where $x_i \equiv m_i(\nu)/\nu$; and $\beta(\alpha_s)$ and $\gamma(\alpha_s)$ are the usual functions associated to the coupling constant renormalization and the mass renormalization (which in the $\overline{MS}$-scheme is the same for all quark flavours). From this result and equations (7) and (8) we then conclude that, in the SVZ-limit, the natural choice for the scale variable ν^2 in the renormalized coupling constant and the renormalized masses is $\nu^2 = M^2$ [5]. We have carried out this procedure explicitly for the two two-point functions in question [6]. The details of the derivation will be published elsewhere. The final results are given in equations (11) and (12) below.

An alternative procedure is to scale the asymptotic QCD result for $\Pi(Q^2)$ and $\psi_5^{(2)}(Q^2)$ at $\nu^2 = Q^2$, as it is usually done, and then compute the limits $\hat{L}\,\Pi(Q^2)$ and $\hat{L}\,\psi_5^{(2)}(Q^2)$. From the practical point of view what one then needs to do is to evaluate limits of the type

$$\hat{L}\;\frac{1}{x^{\alpha+1}}\;\frac{1}{(\log x)^{\beta+1}}\;, \quad \text{where } x \equiv Q^2/_{\Lambda^2}\,.$$

This can be done in a very suitable way, once it is recognized that $\frac{1}{x^{\alpha+1}}\frac{1}{(\log x)^{\beta+1}}$ is the Laplace transform of the function $\mu(t,\beta,\alpha)$ defined by Erdélyi et al., [5]. We find after some algebra, that

$$\hat{L}\;\frac{1}{x^{\alpha+1}}\;\frac{1}{(\log x)^{\beta+1}} = \frac{1}{y}\,\mu\left(\frac{1}{y},\beta,\alpha\right), \quad \text{where } y \equiv \frac{M^2}{\Lambda^2}\,;$$

and the asymptotic expansion, for large y, gives then [7]

$$\hat{L}\;\frac{1}{x^{\alpha+1}}\;\frac{1}{(\log x)^{\beta+1}} = \frac{1}{\Gamma(\alpha+1)}\;\frac{1}{y^{\alpha+1}}\;\frac{1}{(\log y)^{\beta+1}}\left\{1 - (\beta+1)\,\psi(\alpha+1)\,\frac{1}{\log y} + O\left(\frac{1}{\log^2 y}\right)\right\}, \qquad (10)$$

where $\psi(z) \equiv \frac{1}{\Gamma(z)} \frac{d\Gamma(z)}{dz}$.

Using either of the two procedures described above we finally obtain the following results :

$$\frac{1}{M^2}\int_0^\infty dt\, e^{-t/M^2} \frac{1}{\pi} \mathrm{Im}\, \Pi(t) = \frac{1}{4\pi^2}\Big\{ 1 + \frac{\bar{\alpha}_s(M^2)}{\pi} + \left(\frac{\bar{\alpha}_s(M^2)}{\pi}\right)^2 \left[R_2 - \frac{\beta_1}{2}\gamma_E - \frac{\beta_2}{\beta_1} \log\log \frac{M^2}{\Lambda^2}\right] + 4\pi^2 \frac{1}{M^4}\left[m_u \langle \bar{\psi}_u \psi_u \rangle + m_d \langle \bar{\psi}_d \psi_d \rangle \right] + \frac{\pi}{3}\frac{1}{M^4}\langle \alpha_s G^2 \rangle + O\left(\frac{\bar{m}_i^2}{M^2}\right) + O\left(\frac{\bar{\alpha}_s}{\pi}\right)^3 + O\left(\frac{1}{M^4}\bar{\alpha}_s\right) + O\left(\frac{1}{M^6}\right) \Big\} ; \tag{11}$$

and, for the π^+- channel,

$$\frac{1}{M^4}\int_0^\infty dt\, e^{-t/M^2} \frac{1}{\pi} \mathrm{Im}\, \psi_5(t) = \frac{3}{8\pi^2} \frac{(\hat{m}_u + \hat{m}_d)^2}{\left(\log \frac{M}{\Lambda}\right)^{2\gamma_1/\beta_1}} \Big\{ 1 - \frac{\bar{m}_u^2(M^2) + \bar{m}_d^2(M^2) + [\bar{m}_u(M^2) - \bar{m}_d(M^2)]^2}{M^2} + \frac{\bar{\alpha}_s(M^2)}{\pi}\left[\frac{11}{3} + 2\gamma_E + \frac{2}{-\beta_1}\left(\gamma_2 - \gamma_1 \frac{\beta_2}{\beta_1}\right) + \frac{2\gamma_1\beta_2}{\beta_1^2} \log\log\frac{M^2}{\Lambda^2} \right] - \frac{8}{3}\pi^2 \frac{1}{M^4}\left[\left(m_d - \frac{m_u}{2}\right)\langle \bar{\psi}_u \psi_u \rangle + \left(m_u - \frac{m_d}{2}\right)\langle \bar{\psi}_d \psi_d \rangle \right] + \frac{\pi}{3}\frac{1}{M^4}\langle \alpha_s G^2 \rangle + O\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 + O\left(\frac{1}{M^4}\bar{\alpha}_s\right) + O\left(\frac{1}{M^6}\right) \Big\} . \tag{12}$$

Here, $\bar{\alpha}_s(M^2)/\pi = 1/-\beta_1 \log \frac{M}{\Lambda}$; $\hat{m}_i$ are the invariant quark masses in the $\overline{\mathrm{MS}}$-scheme at the two-loop level ; γ_E is the Euler constant (γ_E = 0.5772...);

R_2 is the 3-loop calculation [6] of the ratio R : $R_2 = 1.986 - 0.115\, n_f$, with n_f the number of flavours ; $\beta_1 = -\frac{11}{2} + \frac{1}{3} n_f$; $\gamma_1 = 2$; $\beta_2 = -\frac{51}{4} + \frac{19}{12} n_f$, [7] ; and $\gamma_2 = \frac{101}{12} - \frac{5}{18} n_f$, [8]. For the K^+-channel there is a corresponding expression like equation (12) with the replacement $d \rightarrow s$. The leading non-perturbative contributions are parametrized by the vacuum expectation values $\langle \bar{\psi}\psi \rangle$ and $\langle \alpha_s \vec{G}_{\mu\nu} . \vec{G}^{\mu\nu} \rangle$. These contributions can be fixed from the PCAC relation : $(m_u + m_d) \langle \bar{\psi}_u \psi_u + \bar{\psi}_d \psi_d \rangle \simeq -2 f_\pi^2 m_\pi^2$; and the recent estimate [9], via charmonium sum rules, $\langle \alpha_s G^2 \rangle = (0.044 \begin{smallmatrix} +\ 0.014 \\ -\ 0.006 \end{smallmatrix})\ \mathrm{GeV}^4$.

It is instructive, for the purpose of simplicity, to discuss equations (11) and (12) at the approximation where all the corrections to asymptotic freedom except for the leading non-perturbative contribution $\langle \alpha_s G^2 \rangle$ are neglected. From the contribution of the ρ to the l.h.s. in equation (11), and from the contribution of the π to the l.h.s. in equation (12), one can then respectively derive the SVZ inequality [8]

$$\frac{\gamma_\rho^2}{4\pi} \gtrsim \frac{\pi}{2e} \left(1 - \frac{\pi}{3} \frac{\langle \alpha_s G^2 \rangle}{M_\rho^4} \right) \simeq 0.5 \quad ; \tag{13}$$

and the BNRY inequality [9]

$$\hat{m}_u + \hat{m}_d \gtrsim \sqrt{\frac{2\pi}{3}} \; \frac{8 f_\pi m_\pi^2}{3 \langle \alpha_s G^2 \rangle^{1/2}} \simeq 33\ \mathrm{MeV} . \tag{14}$$

The significance of these interesting results can be now examined in a more rigorous way if the full information contained in equations (11) and (12) is taken into account.

One can use equation (11) to make a comparison of QCD with the low energy data $(\sqrt{t} < 2\ \mathrm{GeV})$ on $e^+e^- \rightarrow$ Hadrons in $I = 1$. This is precisely what the authors of ref. [10], EKV, have done. In doing that EKV have also included a non-perturbative contribution of order M^{-6} but not the $(\bar{\alpha}_s(M^2)/\pi)^2$ corrections we show in equation (11). We have checked that this correction does not change significantly their conclusions. From their analysis, EKV find that

$$70 \text{ MeV} \lesssim \Lambda \lesssim 210 \text{ MeV} \quad .$$

Lower bounds on $\hat{m}_u + \hat{m}_d$ follow from equation (12) from the fact that the π-pole contribution to the l.h.s. : $2 f_\pi^2 \left(\frac{m_\pi}{M}\right)^4 e^{-m_\pi^2/M^2}$ has to be larger than the calculated r.h.s. ($\frac{1}{\pi} \mathrm{Im}\, \psi_5(t) \geq 0$ for all t). The lower bounds thus obtained are shown in Fig. 1 as a function of M^2 for the accepted range : 70 MeV $< \Lambda <$ 210 MeV. The bounds are rather sensitive to the value of Λ . They show that for $\Lambda \gtrsim 150$ MeV there are important corrections to the rough approximation made to derive equation (14). For each value of Λ there is an optimum for $\hat{m}_u + \hat{m}_d$ with an error from the uncalculated corrections. Taking as an estimate of this error a value equal to the square of the calculated corrections to the asymptotic freedom term we find :

$$\hat{m}_u + \hat{m}_d \gtrsim 30 \pm 7 \text{ MeV} , \quad \Lambda = 70 \text{ MeV}$$

$$\hat{m}_u + \hat{m}_d \gtrsim 20 \pm 5 \text{ MeV} , \quad \Lambda = 140 \text{ MeV}$$

$$\hat{m}_u + \hat{m}_d \gtrsim 13 \pm 3 \text{ MeV} , \quad \Lambda = 210 \text{ MeV} \quad .$$

The corresponding lower bounds for the combination $\hat{m}_u + \hat{m}_s$ are shown in Figure 2. Here, the mass correction term $O\left(\frac{m_s^2}{M^2}\right)$ is important and has been taken into account by an iterative procedure. Because of the vicinity of the continuum threshold, we expect the bounds to be less good in the case of $\hat{m}_u + \hat{m}_s$ than in the case of $\hat{m}_u + \hat{m}_d$.

It would be nice to be able to compare these results with previous estimates of the so called current algebra masses [11] . Sometime ago, Leutwyler [11a] , using SU(6) symmetry to relate the matrix elements : $\langle 0|\bar{d}\, i\gamma_5\, u\,|\pi\rangle$ and $\langle 0|\bar{d}\gamma_\mu u\,|\rho\rangle$, obtained the formula

$$\frac{1}{2}(m_u + m_d) = \frac{2}{3} f_\pi \frac{m_\pi^2}{M_\rho^2} \gamma_\rho \simeq 5.3 \text{ MeV} \quad . \tag{15}$$

Inspired by the symmetry assumption in the derivation of this formula, we suggest comparing the two sum rules equations (11) and (12) at the <u>same</u> M^2-value. Fixing $M^2 = M_\rho^2$, a choice which reproduces well the

ρ-coupling constant, and is consistent with the bounds we have derived for $\hat{m}_u + \hat{m}_d$ within the range $70\ \mathrm{MeV} \lesssim \Lambda \lesssim 210\ \mathrm{MeV}$, we obtain the relation

$$\frac{1}{2}\left[\bar{m}_u(M_\rho^2) + \bar{m}_d(M_\rho^2)\right] \simeq \sqrt{\frac{2e}{3}}\, f_\pi \frac{m_\pi^2}{M_\rho^2}\gamma_\rho \left\{ 1 - \frac{\gamma_\pi^2}{M_\rho^4} f_\pi^2 m_\pi^2 + O\left(\frac{\bar{\alpha}_s(M_\rho^2)}{\pi}\right) + O\left(\frac{1}{M_\rho^6}\right) \right\} . \tag{16}$$

We consider this result to be an improved QCD-version, at the one-loop approximation, of Leutwyler's formula, equation (15). Numerically, the r.h.s. in equation (16) gives

$$\bar{m}_u(M_\rho^2) + \bar{m}_d(M_\rho^2) \simeq 21.4\ \mathrm{MeV} \tag{17}$$

which, for $70\ \mathrm{MeV} \lesssim \Lambda \lesssim 210\ \mathrm{MeV}$, corresponds to

$$24\ \mathrm{MeV} \lesssim \hat{m}_u + \hat{m}_d \lesssim 31\ \mathrm{MeV} . \tag{18}$$

From these results, it seems fair to conclude that the combination of current algebra quark masses $m_u + m_d$, if interpreted as the running QCD masses at M_ρ (or as the scale invariant masses $\hat{m}_u + \hat{m}_d$), have very likely been underestimated by a factor of two (or of three).

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- FOOTNOTES -

1) We consider the terminology of Laplace transform more suitable than that of Borel transform used by SVZ. We have found that several properties of Laplace transforms help usefully in deriving some of the results below.

2) The interest to work with moments of the Hilbert transform in connection with the comparison between $\sigma(e^+e^- \to \text{hadrons})$ and QCD was first recognized by Ynduráin, ref.[3].

3) The same limit appears in ref.[2] in the optimization of moments of the Hilbert transform of $\frac{1}{\pi}\,\mathrm{Im}\,\psi_5(t)$.

4) See e.g. refs. [4] and [2].

5) The authors are grateful to Jan Dash for an enlightening discussion on this point.

6) Useful formulae to do that can be found in the appendix of BNRY, ref.[2].

7) Our result for the leading term in equation (10) agrees with the one quoted by SVZ (their equation (5.22) in ref.[1a]). We find, however, that the corrections are of order $\frac{1}{\log y}$ and not $O(\frac{1}{\log x})$ as stated in their equation (5.22). The relevance of these $1/\log y$ corrections already appears at the two-loop level. This is why we give explicitly the coefficient of the next to leading term.

8) Notice that the coupling constant g_ρ^2 of SVZ is 4 times our γ_ρ^2 . SVZ choose $M^2 = M_\rho^2$ and in fact they write their result as an "estimate" and not as an inequality.

9) BNRY choose M^2 so as to optimize the lower bound expression for $\hat{m}_u + \hat{m}_d$. This corresponds to $M^2 = \left(\frac{\pi\langle\alpha_s G^2\rangle}{2}\right)^{1/2}$.

- FIGURE CAPTIONS -

Figure 1. Lower bounds for the sum of invariant quark-masses $\hat{m}_u + \hat{m}_d$ versus M^2 (see equation (12)) for various choices of Λ .

Figure 2. Lower bounds for the sum of invariant quark-masses $\hat{m}_u + \hat{m}_s$ versus M^2 (replace $m_d \to m_s$ in equation (12)) for various choices of Λ .

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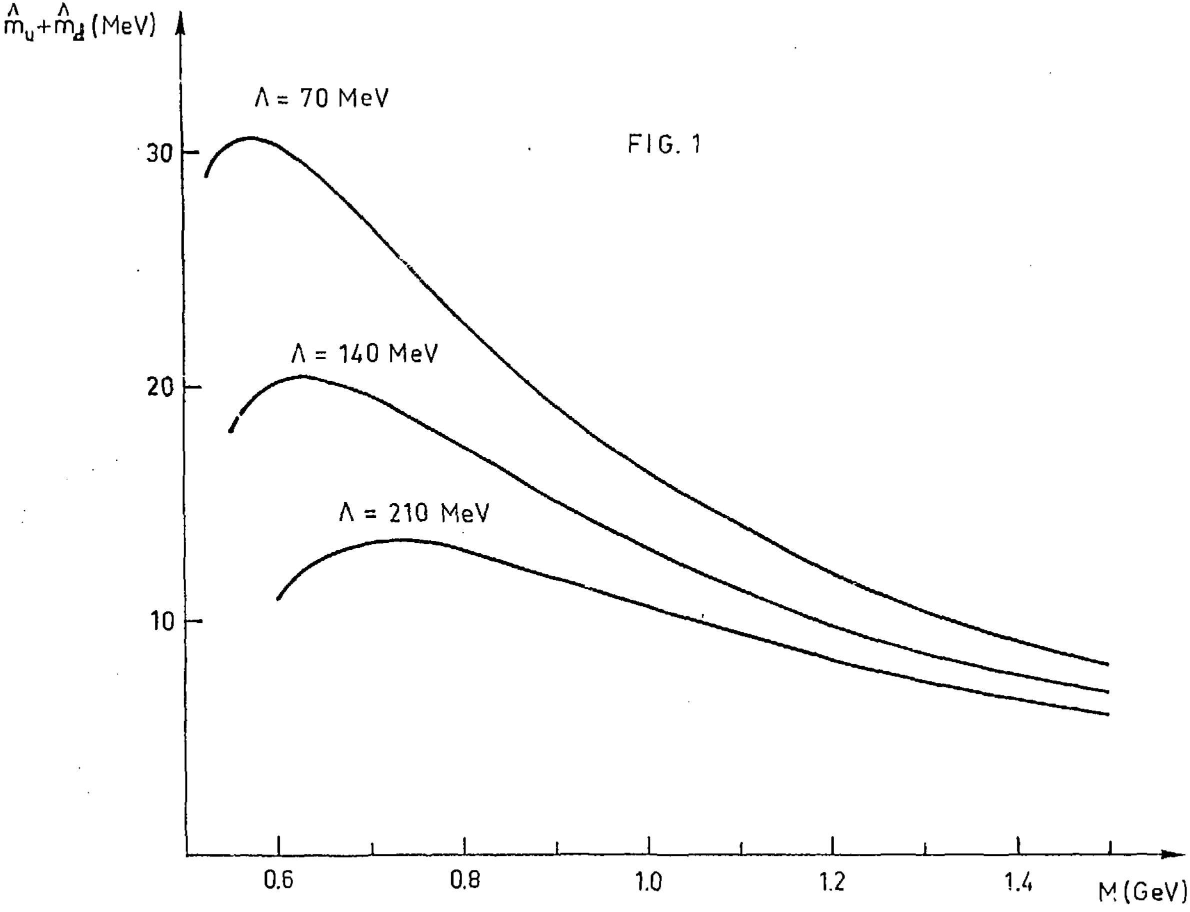
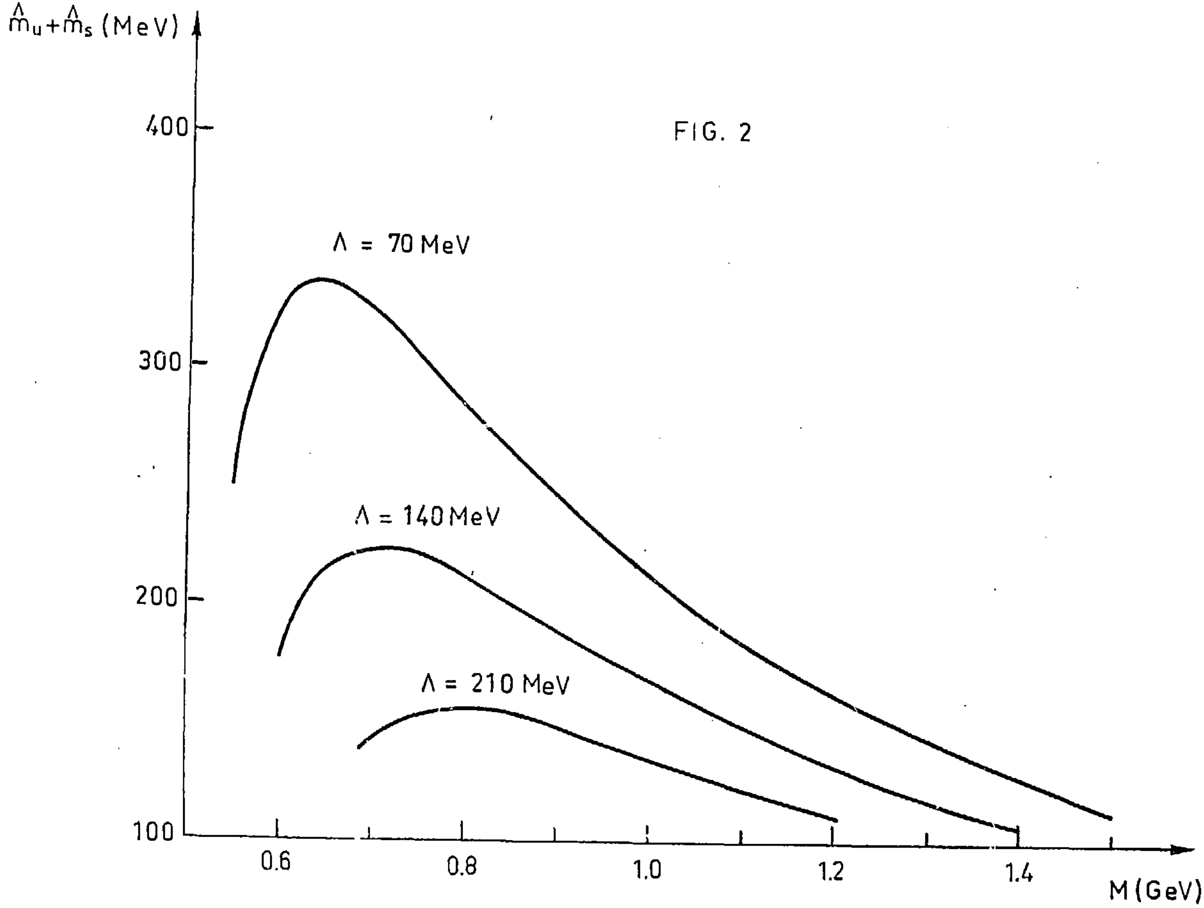


$\hat{m}_u+\hat{m}_d$ (MeV)
Λ = 70 MeV
Λ = 140 MeV
Λ = 210 MeV
30
20
10
0.6
0.8
1.0
1.2
1.4
M (GeV)

FIG. 1



$\hat{m}_u+\hat{m}_s$ (MeV)
400
300
200
100
Λ = 70 MeV
Λ = 140 MeV
Λ = 210 MeV
0.6
0.8
1.0
1.2
1.4
M (GeV)

FIG. 2