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Neutrino masses and the next energy scale

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ABSTRACT

We argue that the dominant contribution to the neutrino masses can be provided by those interactions which incorporate a full quark-lepton correspondence and whose characteristic mass scale furnishes the lightest oasis in the so-called 'energy desert'. In particular we discuss the possibility that the suppression of the neutrino mass spectrum has its origin beyond the single-generation level. This allows us, using the minimal $U(1)$ horizontal symmetry, to correlate the structure of the Majorana mass matrix with the various Dirac mass matrices of the theory.

The strict masslessness of neutrinos, which characterizes the standard Weinberg-Salem (WS) model¹ and persists in the minimal SU(5) unifying theory,² is a combined consequence of the following facts:

- (i) The right-handed neutrinos need not and have never been introduced into the theory.
- (ii) The compact Higgs system is required not to contain a weak isotriplet component.

While the latter is crucial for maintaining the successful $\Delta I = 1/2$ rule, the former violates the correspondence between quarks and leptons according to which ν_R^e is nothing but the color-singlet analog of u_R . Had we introduced right-handed neutrinos into the theory, defined as fermions with zero WS quantum numbers, the theory would have contained a fundamental but unfortunately completely arbitrary mass-scale associated with an SU(3) x SU(2) x U(1) invariant (B - L) - nonconserving Majorana mass term.

Those models which forcefully incorporate right-handed neutrinos include:

- (i) Left-right symmetric models³ - ν_R must be in the theory simply because ν_L is there.
- (ii) Models based on the observation that the lepton number is the fourth color⁴ - ν_R^e must exist as the fourth component of the three colored right-handed u-quarks.
- (iii) Various unifying theories such as⁵ SO(10) or ${}^6E_6 - \nu_R$ is necessarily a member of the fermionic representation.

Most of these models result with essentially the same type⁷ of a neutrino mass matrix given explicitly by

$$M^{\nu} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}. \quad (1)$$

Here, m denotes a Dirac mass matrix with a typical fermionic mass scale, while M is a Majorana mass matrix whose model dependent scale is expected to be very heavy and hence primarily responsible for the extreme lightness $\sim \frac{m^2}{M}$ of the neutrino eigen-masses. The origin and consequently the corresponding scale of M are however quite speculative. The various 'predictions' range from ⁸

$$M \sim gM (W_R^{\pm}) \quad (2)$$

in left-right symmetric models $(W_R^{\pm}$ - the charged gauge bosons associated with $SU(2)_R$), up to

$$M \sim g^2 \frac{m}{M(W_L^{\pm})} M_0 \text{ or even } \nu g M_0 \quad (3)$$

in various $SO(10)$ schemes ⁹ (M_0 - the unification mass). These models share however a common point of view, namely they see the origin of the problem within the so-called single-generation level. They do not make contact with other fundamental problems, such as the well-known generation puzzle, which neither the WS-model nor its naive $SU(5)$ and $SO(10)$ unifying embeddings are capable of dealing with. This is why we would like to adopt here the interesting alternative, ¹⁰ namely that M acquires its scale because of the existence of more than one generation. What we show is the fact that those Higgs fields with large VEV, which must appear in the theory in order to account for the observed suppression of the horizontal ¹¹ gauge interactions

compared with the vertical ones, have in general the proper Yukawa couplings through which the scale of M is fixed. The more so, if the horizontal QN are vertically degenerate and if the generation structure is extended to the Higgs sector as well, these extremely important Yukawa interactions are in fact automatically predicted. Thus, if the horizontal interactions are characterized by the lightest mass scale above $M(W_L^+)$, in a theory where ν_R is a vertical singlet, the suppression of both the neutrino mass spectrum as well as the intergenerational interactions are forcefully attributed to one and the same origin.

We consider an $[SU(2) \times U(1)]_{WS} \times G_H$ local gauge group with the horizontal group factor G_H chosen to be a $U'(1)$ symmetry¹². Apart from being minimal, the choice of $U'(1)$ rather than any other Lie-group has three major physical advantages:

- (i) The original theory, that is prior to the spontaneous symmetry breaking, is free of flavor-changing neutral currents in both the quark as well as the leptonic sectors.
- (ii) In contrast with the flavor-conserving neutral currents, the induced flavor changing neutral currents are necessarily accompanied by non-trivial CP-violating phases. As a consequence, CP can be violated already at the two-generation level where no Kobayashi-Maskawa phase exists.
- (iii) Y' associated with $U'(1)$ may actually represent any diagonal generator of some unspecified horizontal group.

Now, consistent with the WS generation structure, the most general QN-assignments of the fundamental fermions are the following:

	T_3	Y	Y'		T_3	Y	Y'
$\begin{pmatrix} u \\ d \end{pmatrix}_L^i$	+1/2	1/3	L_i	$\begin{pmatrix} \nu \\ e \end{pmatrix}_L^i$	+1/2	-1	l_i
d_R^i	-1/2	-2/3	R_i	e_R^i	-1/2	-2	r_i
u_R^i	0	4/3	R'_i	ν_R^i	0	0	r'_i

(4)

with $i=1, \dots, N$ denoting the generation number. To eliminate any superfluous replication, it is natural to assume that

$$L_i \neq L_j, \dots, r'_i \neq r'_j \text{ for } i \neq j. \quad (5)$$

The Higgs system consists first of all of the standard doublets ϕ_α which are distinguished now from each other by means of their different horizontal quantum numbers h_α ($\alpha = 1, \dots, A$). Next it is necessary to control the strength of the horizontal gauge interactions without affecting too much the WS-model. It is therefore crucial to notice that ϕ_β ($0, 0, H_\beta$), with vanishing WS quantum numbers, are the only Higgs fields whose VEV cannot be picked up by the W_L^\pm gauge bosons. Following such a property, these are the extra scalars we are obliged to introduce into the model. Consequently, the characteristic WS mass relation $M(W_L^\pm) = \cos \theta_W M(Z)$ is modified to the form

$$M^2(W_L^\pm) = \cos^2 \theta_W [\cos^2 \chi M^2(Z) + \sin^2 \chi M^2(Z')], \quad (6)$$

where the small mixing angle χ is of order $\left| \frac{\langle \phi \rangle}{\langle \Phi \rangle} \right|^2$ and so is

the mass ratio $\frac{M^2(Z)}{M^2(Z')}$.

Denoting by v_α and V_β ($|v| \ll |V|$) the complex VEV of ϕ_α and Φ_β , respectively, we arrive at the following fermionic mass matrices:

$$m_{ij}^d = C_{ij\alpha} v_\alpha \quad (C_{ij\alpha} \neq 0 \text{ iff } L_i - R_j = h_\alpha), \quad (7a)$$

$$m_{ij}^u = C'_{ij\alpha} v_\alpha^* \quad (C'_{ij\alpha} \neq 0 \text{ iff } L_i - R_j = -h_\alpha), \quad (7b)$$

$$m_{ij}^e = c_{ij\alpha} v_\alpha \quad (c_{ij\alpha} \neq 0 \text{ iff } l_i - r_j = h_\alpha), \quad (7c)$$

and especially

$$m_{ij}^v \cong m_{ij} = c'_{ij\alpha} v_\alpha^* \quad (c'_{ij\alpha} \neq 0 \text{ iff } l_i - r_j = -h_\alpha), \quad (8a)$$

$$M_{ij}^v \cong M_{ij} = c'_{ij\beta} V_\beta \quad (c'_{ij\beta} \neq 0 \text{ iff } r_i + r_j = \pm H_\beta). \quad (8b)$$

Here $C_{ij\alpha}, \dots, c'_{ij\beta}$ are the Yukawa coupling constants, some of which are zero if the linear relations among the relevant Y' -QN are not satisfied. The non-vanishing ones, on the other hand, are expected to be of the same order of magnitude, leaving the fermion mass hierarchy to follow from a suitable hierarchy in the v'_α 's.

The major advantage of incorporating right-handed neutrinos within the minimal horizontal extension of the standard model is best seen in eq. (8b), which is the origin of our first conclusion: The standard WS model suffers from the superfluous replication disease accompanied by a broken quark-lepton correspondence. These two characteristic problems can be cured simultaneously once a horizontal gauge symmetry is introduced. Any such generalization

of the standard model necessitates extra Higgs scalars with vanishing WS-QM to account for the observed suppression of the intergenerational interactions. These fields in turn may in general have Yukawa couplings with right-handed neutrinos. This is the mechanism by which the scale of the Majorana mass matrix becomes

$$M \sim c V \sim \frac{c}{g} M(Z'), \quad (9)$$

where c is a typical Yukawa coupling constant. We remind the reader that a similar correlation, only with $M(W_R^\pm)$ replacing $M(Z')$, has been derived within a single-generation $SU(2)_L \times SU(2)_R \times U(1)$ model by Mohapatra and Senjanovic. If we add a horizontal symmetry to a left-right symmetric model, the smallness of the neutrino masses will be controlled by the heavier mass-scale. If $M(Z')$ furnishes the lightest oasis in the so-called 'energy-desert' ($M(W_L^\pm) < E < M_0$), as advertized in the present note, the horizontal interactions would become the dominant source for the neutrino masses only provided ν_R are vertical singlets. It is thus relevant to mention that the lower bound on the mass of the horizontal gauge bosons has been estimated¹³ to be

$$M(Z') > \frac{1}{g} M(W_L^\pm). \quad (10)$$

This assures that the single Z' exchange would not compete with the conventional WS box diagram¹⁴ in contributing, for example, to the $K^0 - \bar{K}^0$ mass difference.

To make our discussion less qualitative, we need to specify the model, that is to fix the various horizontal QN. Once

is done, the explicit structure of all the fermionic mass matrices is known up to Yukawa coupling constants which are assumed to be of the same order of magnitude. It is also obvious that the existence of several relations among the horizontal QN must have a direct translation into correlations among mass matrices in various fermionic sectors. To demonstrate this point we consider three different types of models:

- (i) Simple-minded models where the horizontal QN are determined empirically.
- (ii) Models which treat the lepton number (or alternatively the B-L combination) as the fourth color.
- (iii) Models which result from flavor grand unifying theories.

As a typical model of the first class we discuss an $SU(2) \times U(1) \times U'(1)$ model¹³ where the generation structure is extended to the Higgs sector and the form of the fermion mass matrices are derived by making some physical assumptions associated with the VEV hierarchy. Among the notable features of this model one can find the following: (i) The naturalness of $m_u \approx m_d$ necessitates a third generation of fermions and uniquely determines the structure of the two quark mass matrices, (ii) The mass scale of the third generation fermions is fixed by the light ones, (iii) The Cabibbo universality is recovered with $\theta_c \approx (m_d^2 m_b^{-3} m_c^{-3})^{1/2}$, and (iv) The main CP-violating parameter associated with the $K^0 - \bar{K}^0$ system is derived to be $\epsilon \sim 10^{-3}$ as observed. In such a model, where experimental information helps us choose the horizontal QN of the quarks, the only constraints imposed on the leptons are due to the anomaly-free requirement. The corresponding equations are:

$$\begin{aligned}
\sum l_i &= -3 \sum L_i, \\
\sum (l_i - r_i) &= -\frac{1}{3} \sum (5L_i - R_i - 4R'_i), \\
\sum (l_i^2 - r_i^2) &= \sum (L_i^2 + R_i^2 - 2R_i'^2), \\
\sum (2l_i^3 - r_i^3 - r_i'^3) &= -3 \sum (2L_i^3 - R_i^3 - R_i'^3).
\end{aligned} \tag{11}$$

Thus, although we may not have enough information about the structure of the leptonic mass matrices, the above constraints along with the various conditions in (7a) - (8b) are capable of excluding several candidates for m^e , m and M .

Models⁴ which favor the idea of the lepton number being the fourth color are compatible with the choice

$$l_i = L_i, r_i = R_i, r_i' = R_i'. \tag{12}$$

A quick glance at eqs. (7a)-(8b) along with the above assignments show the existence of two striking correlations among the fermionic mass matrices. first of all, still assuming that the Yukawa coupling constants are very similar to each other, it is possible to derive the typical SU(5) - originated correlation

$$m_{ij}^e \sim m_{ij}^d, \tag{13}$$

from entirely different arguments. The other interesting correlation

which reads

$$m_{ij}^{\nu} \sim m_{ij}^u, \quad (14)$$

is more important from the viewpoint of the present note. It tells us that for the Dirac part of the neutrino mass matrix we can use information taken from the up-quark sector. However, as far as the Majorana part of the mass matrix is concerned, we may say almost nothing about its structure. This is simply because, apart from the general anomaly-free constraint involving r'_i which now reads

$$\sum_i r'_i \cdot 3 = \sum_i (2l_i^3 - r_i^3), \quad (15)$$

r'_i is not necessarily related to l_i and r_i . This is not the case in our next example.

Finally we arrive at models which may be regarded as the low-energy manifestations of grand unifying theories. Following our primary motivation that the horizontal interactions are in fact responsible for the smallness of the neutrino masses, the unifying theories of relevance are those which are capable of dealing with the flavor problem¹⁵. Among these there is a certain class of theories, namely flavor-chiral semi-simple gauge theories¹⁶, which are subject to a global so-called Vertical-Horizontal discrete symmetry. Such theories have been proposed with the primary motivation that the generation structure exists at all energy regimes and that the single-generation group factor is not a consequence of the SSB. A theory of this kind is based on the local gauge group $SO(10)_V \times SO(10)_H$. It has the characteristic property that all members of a given generation have the same horizontal QN. Thus, if we assume, just for the technical reason of making contact with the present discussion, that $SO(10)_H$ is broken

at medium energies to $U'(1)$, we obtain

$$L_i = -R_i = -R'_i = \ell_i = -r_i = -r'_i \equiv x_i, \quad (16)$$

with x_i being their common value. Note that the opposite sign in the QN of the right versus the left-handed fermions is due to the flavor chirality of the original theory. Using the above QN-assignments, the fermionic mass matrix elements are restricted as follows

$$\begin{aligned} m_{ij}^e &\sim m_{ij}^d \neq 0 \text{ iff } x_i + x_j = h_\alpha, \\ m_{ij} &\sim m_{ij}^u \neq 0 \text{ iff } x_i + x_j = -h_\alpha, \\ M_{ij} &\neq 0 \text{ iff } x_i + x_j = \pm H_\beta. \end{aligned} \quad (17)$$

The constraint associated with the Majorana matrix elements becomes very similar to those associated with all the other matrices. To decide whether or not the ij matrix element in any given fermionic sector takes a zero value, one has first of all, as far as the fermions are concerned, to calculate the only relevant combination which is $x_i + x_j$. Only then, it becomes necessary to check if those quantities coincide with $\pm h_\alpha$ and $\pm H_\beta$. If, as an interesting example, we assume that $h_\alpha = H_\alpha$, i.e. the same horizontal structure for all the Higgs families, M_{ij} becomes a combination of two parts, one proportional to m_{ij}^e and the other to m_{ij} . This way, the structure of the Majorana mass matrix is determined by the other fermion mass matrices. The only question is the following: Do we really have a good reason to believe that the horizontal QN obey the restriction

$$h_\alpha = H_\alpha. \quad (18)$$

With regard to this, it is crucial to note that in a theory where all the fermions associated with a given generation have the same horizontal QN, it is only natural to expect that the same persists in the Higgs sector as well. Such a situation is favored by the technicolor scheme where the Higgs particles actually represent bound states of fermions. Thus, if in an $SU(3) \times SU(2) \times U(1) \times U'(1)$ theory,

(i) The generation structure persists in the Higgs sector, and

(ii) The horizontal QN are vertically degenerate,

the majorana mass matrix can be formally (i.e. up to Yukawa coupling constants of the same order of magnitude) written as

$$M_{ij} \sim \left| \frac{V}{v} \right| (m_{ij}^e + m_{ij}^{\nu}). \quad (19)$$

To conclude we reemphasize that the qualitative nature of our discussion is mainly due to the fact that we have chosen to make our point within an effective low-energy model which is as simple as possible. After all, it becomes relevant to ask about the specific details of the neutrino mass matrix only after getting some general idea concerning the puzzle where do the tiny neutrino eigen-masses come from. Our main observation is that the dominant contribution to the neutrino masses may originate from those interactions which do incorporate a full quark-lepton correspondence and whose mass-scale furnishes the lightest oasis in the 'energy-desert'. This opens the door for a mutual suppression of both the neutrino eigen-masses as well as the horizontal gauge interactions. Moreover, using a $U'(1)$ horizontal symmetry, we have established general correlations

among the various fermionic mass matrices, especially including both the Dirac as well as the Majorana neutrino mass matrices.

References

1. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in "Elementary Particle Theory", edited by N. Svartholm (Wiley, N.Y. 1969), p. 367.
2. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
3. J.C. Pati and A. Salam, Phys. Rev. Lett. 31, 661 (1973); R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566, 2558 (1975).
4. J.C. Pati, Phys. Rev. D10, 275 (1974).
5. H. Georgi, in "Particles and Fields" (1975), (AIP Press, N.Y.), p. 575; H. Fritzsch and P. Minkowski, Ann. Phys. N.Y. 93, 193 (1975).
6. F. Gürsey, P. Ramond and P. Sikivie, Phys Lett. 60B, 177 (1976); Y. Achiman and B. Stech, Phys. Lett. 77B, 389 (1978).
7. M. Gell-Mann, P. Ramond and R. Slansky, "Supergravity" - P. Van Nieuwenhuizen and D.Z. Freedman (eds.), (North-Holland), 1979.
8. R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
9. E. Witten, HUTP-79/A076; M. Gell-Mann, P. Ramond and R. Slansky, ref. 7.
10. T. Yanagida, TU/80/208.
11. F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421 (1979); A. Davidson, M. Koca and K.C. Wali, Phys. Rev. Lett. 43, 92 (1979); C.L. Ong, Phys. Rev. D19, 2738 (1979); T. Maehara and T. Yanagida, Prog. theor. Phys. 60, 822 (1978).

12. For the first attempt to use a $U'(1)$ horizontal symmetry see A. Davidson, M. Koca and K.C. Wali, Phys. Rev. Lett. 43, 92(1979).
13. A. Davidson and K.C. Wali, Phys. Rev. 21D, 787 (1980).
14. M. Gaillard and B. Lee, Phys. Rev. D10, 897 (1974).
15. The first attempt is due to H. Georgi, Nucl. Phys. B156, 126 (1979).
16. A. Davidson, Phys. Lett 90B, 87 (1980); A. Davidson, Phys. Lett. 93B, 183 (1980); K.C. Wali, Talk given at the first workshop on Grand Unification (New-Hampshire), 1980.
17. A. Davidson, K.C.Wali and P.D. Manheim (SU-4217-169), (to be published in Phys. Rev. Lett.)

