MASTER

SLAC-TN-81-5
July 1981

A STUDY OF THE EMISSION PERFORMANCE OF HIGH-POWER KLYSTRONS--SLAC XK-5*

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*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

PURPOSE

There are hundreds of high power klystrons operated in the Linac gallery and about fifty to sixty tubes fail every year. The lifetime ranges from a few thousand up to seventy thousand hours except those which fail during an early period. The overall percentage of failures due to emission problems is approximately 25%.[1] It is also noted that a 10% increase in mean lifetime of klystrons will reduce the overall cost per hour as much as a 10% increase in efficiency.[1] Therefore, it is useful to find some method to predict the expected life of an individual tube.

The final goal has not been attained yet, but some useful information was obtained. It is thought that this information might be helpful for those people who will study this subject further.

GENERAL DESCRIPTION

Usually, a klystron is operated under space charge limited condition, its anode current remains constant for a long time during its lifetime, although its cathode is probably decaying gradually. So the operating current is not an indication of the stage of cathode life. It is well-known, the basic parameter of a cathode is its work function, which determines the emission ability. Unfortunately, the operating voltage of SLAC's klystrons is so high that it is impossible to measure the saturation current. Besides, the pulse modulator should work into a matched load; any deviation from the normal condition is not desirable. Therefore, any research work based on high pulse voltage is not practicable. Operating at low voltage, especially in the d.c. condition, is preferable.

Furthermore, the study of a cathode is so complex that there are countless papers concerned with it during this century,[2-11] but still a lot of problems are unclear. Among many kinds of cathodes, oxide cathodes are most complex. They are different from type to type, even from tube to tube and from time to time. Any attempt to find some precise universal theory is recognized as unrealistic.

Nevertheless, we hope to find some relationship between low voltage performance and the saturation current. Then one can estimate indirectly the emission ability. Probably the decay process can be monitored by means of observing its low voltage performance. No doubt it is a statistics problem, for which one has to collect a multitude of data.

Some indications related to the emission ability were certainly discovered from the experiments. A few steps had been taken in order to clarify the problem as described briefly below.

As a first step, the dip-test method was used. A discrepancy between theory and experiments was found. That is, the anode current depends on the cathode emission although the cathode is certainly under space-charge limited condition. But, on the other hand we can make use of this phenomenon to estimate the emission ability. This is exactly what we seek.

As a next step, we used the retarding field method. We found that when the anode voltage is negative, the anode current depends more sensitively on the emission current. Unfortunately, the nonuniformity of tubes is also significant. Besides the measurement of very low currents makes it difficult to obtain reliable data.

To go a step further, it seemed necessary to explain the reason for the discrepancy. Then one can believe that those phenomena are not accidental and they furnish information about cathode emission.

Many possibilities were studied. Among them the most reasonable is the so-called 'edge emission'. Qualitatively, by virtue of it one can explain the deviation of the anode current from the three-halves power law.

Many tubes had been tested at low voltage. It is certain that the three-halves power law is not accurate in that voltage range, although it is a good approximation at normal operating condition. Also it is certain that this deviation depends on the emission as well as the voltage. Because of the complexity and nonuniformity of the tubes, it is difficult to distinguish the extra currents caused by different factors. However, it is worthwhile to monitor the decay process of a working tube by means of checking its low voltage performance, if conditions permit. Of course it is tedious work and involves a lot of statistical data.

A GLANCE AT THE BACKGROUND

Let us recall briefly the conventional and recent theories, and some important work of studying the cathode life.

It is well known that the performance of an electron gun, as well as a diode, follows the three-halves power law, which was derived by Langmuir and Child, providing the cathode is operated under space-charge limited conditions. The perveance is a constant depending only on the shape of the electrodes in a gun but is independent of the cathode emission. When the anode current is raised to saturation, the cathode will be temperature limited, and then the current will remain constant. Recently, R. T. Longo[12,13] developed a new theory and a new formula for the current. It is:

$$j = \frac{j_{SC}\, j_{TL}}{j_{SC} + j_{TL}} \tag{1}$$

where j_{SC} is the space-charge current given by:

$$j_{SC} = KV^{3/2} \tag{2}$$

while j_{TL} is the temperature limited current. This formula is a good approximation when the anode current is close to saturation, and it reduces to the three-halves power law when the anode current is reduced.

As for the case when the anode current is much lower than usual, which is not important in practice, people generally do not care about it and think that it should follow the three-halves power law as well.

As far as cathode life studies are concerned, most of the work is restricted to experimental diodes.[11] But the environment of a cathode in a real tube is quite different; its life is expected to be different too. The analogous work for evaluating the life of a high power microwave transmitting tube was done at Watkins-Johnson Company by D. H. Smith.[14] His test assembly was designed to simulate the environment of a 12.2 GHz 4 kW CW TWT. It is similar to the real tube except for the RF part, which was replaced by a section of drift tube. His purpose was to compare the long life capabilities of different cathode types, rather than to judge an individual tube. However, our purpose is the latter. It seems that the most realistic method to judge an individual tube is the so-called 'dip test', which was originated by

M. G. Bodmer.[15] Since then many scientists used this method to test tubes.[16-18] Particularly, it is valid for oxide cathodes.

As for the measurement of the work function, there are some conventional methods. Unfortunately, all those are merely used in experimental diodes. For a large size convergent gun it is always too complex to make use of them.

THE DIP TEST

The dip test is a common method for testing cathode emission characteristics of actual microwave tubes. Generally the initial voltage and current are the same as in normal operation. After turning off the filament power, the anode current will dip rapidly after a certain amount of time delay, which is a measure of the emission capability of the cathode (see Fig. 2). However, it is not appropriate for a high voltage pulse klystron like the SLAC XK-5, because turning off the filament power in such a high anode voltage tube would risk damaging the cathode. Besides, the pulse modulator is not allowed to work with a varying load as the anode current will drop gradually during the dip process. Therefore, there seems to be no choice but to do it at low voltage.

Due to equipment limitations, operating with a d.c. supply without a focusing system is preferable. Moreover, the cooling problem must be taken into account, otherwise overheating may cause outgassing from the inner surface of the tube body and then possibly damage the cathode. In order to minimize the requirements for facilities, we restrict the applied voltage to less than one kilovolt. The beam power is then only about 100 watts or less, which is certainly safe without cooling.

Since the anode voltage is so low, it is expected that the delay time should be long. One typical experimental dip-test curve is shown in Fig. 1. Since the cathode is so large its temperature drops very slowly, a dip-time of 4 minutes is selected. We note that:

1. The knee point is not obvious. This is probably due to the nonuniformity of the temperature along the cathode. Recent measurement of this nonuniformity indicates about 45 degrees centigrade.

2. The performance violates the theory. According to the theory, the anode current should remain constant until the cathode temperature is so low that its emission is no longer sufficient to satisfy the space-charge limited condition. Then the cathode will be temperature limited. So the idealized dip-test curve should be that as shown in Fig. 2. However the experimental results are far from ideal.

In real tubes, a smooth knee point was obtained.[16-18] It can be explained by Longo's theory, formula (1), and the nonuniformity of the temperature. But at a very low voltage, when the anode current is far less than the saturation current, the dip-test curve is expected to be of the form as shown in Fig. 2(a) with various initial filament power levels. Unfortunately, all the experimental results (one of them is shown in Fig. 1) never support the theory. Furthermore, the initial current corresponds to a perveance higher than normal but depends on the filament power. Some possible factors affecting the current have been considered and excluded, such as instability of the power supply and other disturbances. The different thermal expansion may cause a little change of the perveance, but should not be serious. Therefore, it seems there is no option but to believe that the current does depend on the emission current even though the space-charge limited condition is satisfied perfectly.

Another record of the anode current versus the anode voltage as a function of the filament power is shown in Fig. 3; it also shows the dependence on the emission ability.

However, in spite of the fact that the mechanism is not clear, this dependence is an indication. By making use of it one can evaluate the cathode. We will discuss it further below.

THE RETARDING FIELD TEST

It is found that the lower the anode voltage, the more sensitively the anode current depends on the emission current. Figure 4 shows the performance of two XK-5 klystrons at negative voltage. The anode current is very sensitive to the filament power. Since only those electrons whose initial velocities are large enough to overcome the retarding field can reach the anode, the anode current consists of only a small part of the total emission current. The number of those fast electrons depends on the initial velocity distribution, which is not yet known for oxide cathodes, but it seems reasonable to assume that this rate as well as the distribution are the same for different tubes with same type of cathode and same size of gun. Then the anode current will be a measure of the emission current. As we can see from Fig. 4, the difference of those two tubes is almost one order of magnitude in anode current. For instance, when the filament voltage is 15 V, and the anode voltage is -1.4 V, the anode currents are $7 \cdot 10^{-9}$ amp for M239b and 8.8×10^{-8} amp for M409a.

However, we cannot conclude yet that this difference really indicates the difference of the emission abilities. There are at least two problems that we have to be concerned with.

First, if the initial velocity obeys a Maxwellian distribution law, which is true for a pure metal cathode,[2] the retarding current will decline exponentially with the voltage for a plate diode. The relation is:

$$\log j_a = \log j_e + 0.434 \frac{eV'_a}{KT}$$

$$V'_a = V_a - V_T \qquad (3)$$

where j_e is the emission current density, U_a is the anode voltage and V_T is the contact potential difference. The curve of log j_a ver[illegible] is a straight line. One can deduce the temperature from its slope.

For the oxide cathode, only the fast electrons obey the Maxwellian distribution.[illegible,19] For a diode of coaxial cylindrical electrodes the *retarding current gives a straight line too, providing the applied* negative voltage is sufficiently high, though the slope will be less. For a convergent gun, it is too difficult to get an accurate solution, but in view of Fig. 4, there seems to be a good chance of giving a straight line also if the applied negative voltage is high enough, even though the slope may be different.

Unfortunately, the higher the retarding voltage, the weaker the current. In any attempt to measure the weak current we always encounter difficulty with random noise, especially hum. So we cannot obtain a straight line yet. The nonuniformity of the cathode temperature may make it even more difficult.

[illegible], for a plate diode, theoretically the retarding field current depends on the temperature of the cathode, on the voltage and on the work function of the anode, but is independent of the work function of the cathode. But from experiments of convergent guns, the dependence of the retarding field current is more complex. We found that the variation of the work function of the anode is significant. It is probably due to the fact that the surfaces of the anodes have been contaminated by the active material, so that their work functions are different from tube to tube. It will influence the anode current seriously. As a matter of fact, some new tubes are made of old gun assemblies.

Moreover, it is always necessary to be very careful when measuring a weak current. In addition to noise there exist other possible currents, such as leakage current, ion current, parasitic emission and something due to the irregularity of the cathode itself.

Therefore, the retarding field current tests are not conclusive so far.

THE VIOLATION OF THE THREE-HALVES POWER LAW

A great number of experiments prove that the three-halves power law is not valid at low voltage. The perveance will increase as the anode voltage is decreased. Some typical performance characteristics are shown in Figs. 5 to 12.

Figures 5 and 6 show the performance of two new tubes. Their perveance at low voltage is much higher than that at high voltage. At very low voltages, those data will be inaccurate because the contact potential difference is comparable with the anode voltage. But if it is taken into account, the perveance should be even higher. Figures 7 and 8 show the perveances of different tubes. Figure 9 is typical characteristics which show how the perveance depends on the filament power. Moreover, this phenomenon of the perveance being too high appears not only in standard SLAC XK-5 klystrons, but also in RCA tubes and the SPEAR klystron as shown in Figs. 10 to 12. These tubes failed for low perveance under high voltage operation, but still had a perveance higher than normal at low voltage.

Sometimes the anode current may be a few tens of milliamperes more than expected at the anode voltage of 1KV. Its perveance might be 50% or more than usual. We have to question if there is any reason causing an extra anode current. Some considerations are stated below:

1. Measurement error. This is excluded, as we used precise digital meters all the time.

2. Ion current. This is improbable, since the vacuum is quite good. Besides the ion current cannot be of the same order as the electron current.

3. Leakage current. It cannot be so high. But it may be a factor for negative voltage tests.

4. Parasitic emission. It is quite possible that the focusing electrode emits extra current due to active material evaporated from the cathode. But the temperature of the focusing electrode is only about 570 degrees centigrade, so it should not be significant, and will saturate rapidly. Besides, the contamination is a cumulative process; it should be getting worse and worse during the life of the tube. Instead a new tube has the same performance as an old one.

5. Relativistic effect. The perveance will be slightly lower at high voltage because of the relativistic effect. Correspondingly, the perveance at low voltage will be about 6% higher in contrast with 265 kV. But it will be constant throughout the whole low voltage range.

6. Initial velocity effect. It is true that the three-halves power law is not precise when the initial velocity of the emission is taken into account as some earlier scientists had analyzed. Adams[22] gave an approximate formula for a plate diode as follows:

$$\xi_a = 1.2532\ \eta_a^{3/4} + 1.6685\ \eta_a^{1/4} - \ldots \quad . \qquad (4)$$

where

$$\xi_a = c(\chi - \chi_m)$$

$$\eta_a = \frac{e}{KT}\ (V_a - V_m)$$

are normalized distance and anode voltage respectively, χ_m and V_m are the coordinate and potential of the virtual cathode. The first term corresponds to the three-halves power law. The second term is a correction but it is only about 3% as compared with the first term at an anode voltage of 100 V and a cathode temperature of 1000° Kelvin. So it is negligible in the voltage range we were operating in.

In consideration of all the above arguments, we think that there must be another factor which is more important and which always exists. We hypothesize that it is 'edge emission', which we will discuss next.

EDGE EMISSION

At the edge, the cathode is neither temperature limited nor space-charge limited. It emits a certain amount of current. So the total anode current will be larger than that due to the three-halves power law. The amount of the edge current is:

$$I_{ed} \approx 2\sqrt{\pi}\, W \cdot f(\eta_m) \sqrt{Kj_e}\, V^{3/4} \quad , \tag{5}$$

where W is the width of edge, f is a factor, K is the perveance. It shows the edge emission depends on both emission current density j_e and anode voltage V. The details are discussed in Appendix A.

Appendix B indicates that the total anode current will be:

$$I_a = KV^{3/2} + (1 - \sigma) I_{ed} \tag{6}$$

where σ is a factor less than one. It shows the three-halves power law is valid only if the edge emission is negligible.

CATHODE PERFORMANCE

As the edge current is always present, and cannot be neglected at very low voltage, we can conclude that in that range the violation of the three-halves power law is a normal phenomenon. On the contrary, the coincidence with the three-halves power law at low voltage (for instance 1000 V for XK-5) means the cathode has not been activated perfectly. Figures 5 and 6 show the performances of tubes just after the aging process and after RF testing. It shows that after the aging process the tubes have not been activated perfectly yet, although they follow the three-halves power law. As a matter of fact, the period of high voltage testing is a further activation process. From the figures one notes that the anode current is raised significantly.

SOME SUGGESTIONS

Although almost every tube has some excess current at low voltage, the excess varies widely from tube to tube. Not only the cathode itself may be different, but also there are other factors affecting the results.

It is important to follow some individual tubes from their early age until their retirement for the purpose of monitoring the changing process of emission characteristics. Even though the low voltage test is only an indirect diagnosis, it is after all a simple way and is harmless to tubes.

It is believed that there will be some early indication appearing before the cathode fails.

However, this is no doubt tedious work and it involves taking numerous statistical data, and may take a few years. In the meantime of course, it is worthwhile to record the high voltage performance for comparison, providing the condition is realistic.

Since the oil tank for the tubes in the gallery is constructed in such a way that it is not convenient to test at d.c. conditions, it would be helpful to modify it so that the ground terminal on the high voltage side of the pulse transformer can be disconnected when needed.

ACKNOWLEDGMENTS

This work was performed under the direction of G. T. Konrad. The author wishes to express his thanks to many colleagues who helped a great deal in making the experiments possible. Among them are G. Aske, C. Rivers, F. Coffer, C. Griffin and many others. Especially, T. Schumacher helped a lot with the experiments. The author is particularly indebted to Professor M. Chodorow for his beneficial discussions.

APPENDIX A

EDGE EMISSION

The Role of the Electron Cloud and Its Border

It is generally assumed that near the cathode surface there is a layer of electrons, called the electron cloud, in which the space-charge field results in a lower potential relative to the cathode. The potential minimum is referred to as a virtual cathode or a potential barrier. Only those electrons whose initial velocities are high enough to overcome the barrier can cross the barrier and reach the anode. The higher the barrier, the less the anode current. Therefore, the height of the barrier, or the potential of the virtual cathode, will determine the ratio of the anode current to the emission current.

The virtual cathode is the border of the electron cloud, because the electrons beyond this border are affected by the accelerating field and will never turn back, while the electrons inside this border move randomly in every direction. So this border can be regarded as an emitting surface just like a cathode. The total anode current will be the integral of the emitted current along the whole virtual cathode surface or the border.

Let us now consider the edge of the cathode, beyond which there is no emission (see Fig. 13(a)). Because the motion inside the cloud is directed in every direction with the same probability, there must be some electrons moving transversely and across the original edge into the region above the nonemitting surface. This is the 'edge emission region' as shown in Fig. 13(a). Its density will decrease gradually. The electron cloud region is then expanded beyond the cathode. As we can see from Fig. 13, above the center part of the cathode, the distance of the potential minimum relative to the cathode is uniform and the height of the barrier is uniform too. But at the edge the distance as well as the height gradually decrease to zero. This means the boundary of the electron cloud will end on the nonemitting surface. The lower barrier implies that there are more electrons escaping from a unit area of the border.

Under general conditions, the anode current is so high that this extra current can be neglected. However, when the anode voltage is [illegible] that the barrier in the center part is so high that the current emitting from it is very weak, the extra emission may play a significant role, because the height of the edge barrier always ranges to zero. There are always some electrons escaping from this border no matter how high the barrier in the center part is.

The Edge Emission Current

In order to estimate the edge emission, we make the following assumptions for the sake of simplicity:

1. The electron initial velocity follows Maxwell's distribution law. This is not exactly true for oxide cathodes; however, the distribution probability decreases approximately exponentially with the velocity, so a Maxwellian distribution will be an approximation.

2. The emission and the field on the surface of the cathode are uniform, and the virtual cathode is very close to the cathode. Thus we can use a one dimensional approximation except at the edge.

3. The border of the electron cloud on the edge is a straight line as shown in Fig. 13(c).

4. Inside the electron cloud, the equipotential lines are parallel to the cathode instead of perpendicular to the edge as it should be (see Fig. 13(a,c)).

Suppose the cathode potential is zero, so the potential of the virtual cathode V_m, and V inside the electron cloud are always negative, viz. $V < 0$, $V_m < 0$.

By virtue of the assumptions 1 and 2, one can obtain the following relation:[22]

$$\frac{j_a}{j_e} = e^{\frac{eV_m}{KT}} = e^{-\eta_m} \tag{A.1}$$

and the potential distribution function is:

$$\xi(\eta) = \int_0^{\eta} \frac{d\eta}{\sqrt{e^{\eta} - 1 + e^{\eta}\,\phi(\sqrt{\eta}) - 2\sqrt{\eta/\pi}}} \quad . \tag{A.2}$$

where j_e is the emission current density, j_a is the anode current density, ξ and η are the normalized coordinate and potential relative to the virtual cathode, on which $\xi = \eta = 0$ (see Fig. 14). η_m is the normalized height of the barrier. ϕ is the error function. They are:

$$\xi = C(z - z_m)$$

$$\eta = \frac{e}{KT}(V - V_m)$$

$$\eta_m = -\frac{eV_m}{KT}$$

$$\phi(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} \cdot dt \tag{A.3}$$

$$C^2 = \frac{e}{\varepsilon_0}(2\pi m)^{1/2}\ (KT)^{-3/2}\ j_e\ e^{-\eta_m}$$

$$C = 9.2\times10^5\ T^{-3/4}\ j_a^{1/2}$$

where z is the distance from the cathode, z_m is the position of the virtual cathode. The above quantities are dimensionless except C, which has units cm^{-1}. The units of j_a are amp/cm^2, and T, degree Kelvin.

The function (A.2) is shown in Fig. 15. z_m vs anode current density [illegible] as a function of the ratio of the emission current to anode current is shown in Fig. 16.

In addition, the current density crossing the equipotential line with potential V, is:

$$j = j_e\, e^{\frac{eV}{KT}} = j_e\, e^{\eta-\eta_m} \tag{A.4}$$

Suppose the width of the edge border is W (Fig. 13(c)), then we have:

$$\sin\alpha = \frac{z_m}{W} = \frac{\xi_m}{CW} \qquad (\xi_m = Cz_m) \tag{A.5}$$

and

$$dW = \frac{dz}{\sin\alpha} \quad .$$

The emission current from an elemental area of the border is only determined by the potential thereof, therefore:

$$\begin{aligned} dj_{ed} &= j_e\, e^{\frac{eV}{KT}}\, dW \\ &= j_e\, e^{-\eta_m}\, e^{\eta}\, \frac{dz}{\sin\alpha} \end{aligned} \tag{A.6}$$

From assumptions 3 and 4, the total edge current should be:

$$j_{ed} = \int_0^{z_m} j_e \frac{e^{-\eta_m}}{\sin\alpha} e^{\eta}\, dz = \frac{j_e}{C \sin\alpha}\, e^{-\eta_m} \int_0^{\xi_m} e^{\eta}\, d\xi \qquad \text{(A.7)}$$

Substituting C and sinα one can obtain:

$$j_{ed} = \sqrt{j_e j_a}\, W f(\eta_m)$$

$$f(\eta_m) = e^{-\eta_m/2} \int_0^{\eta_m} \frac{e^{\eta}\, d\eta}{\sqrt{e^{\eta} - 1 + e^{\eta}\phi(\sqrt{\eta}) - 2\sqrt{\eta/\pi}}} \Bigg/ \int_0^{\eta_m} \frac{d\eta}{\sqrt{e^{\eta} - 1 + e^{\eta}\phi(\sqrt{\eta}) - 2\sqrt{\eta/\pi}}} \qquad \text{(A.8)}$$

The numerical solution of this function is shown in Fig. 17. It shows that $f(\eta_m)$ changes not much when η_m varies over a wide range.

The anode current em[illegible]ed from the main part of the cathode is:

$$\ \ = \pi R^2 j_a \ , \qquad \text{(A.9)}$$

where R is the radius of the cathode. The total edge current should be a line integral along the edge. It is:

$$I_{ed} = 2\pi R\, j_{ed} = \sqrt{j_e j_a}\, 2\pi R \cdot W f(\eta_m) \qquad \text{(A.10)}$$

The ratio of these two is:

$$\frac{I_{ed}}{I_{ao}} = \sqrt{\frac{j_e}{j_a}}\, \frac{2W}{R} f(\eta_m) \qquad \text{(A.11)}$$

Since the width of the edge is much less than the radius, generally I_{ed} is negligible. However, when j_a is far less than j_e, the edge current may be substantial, even higher than the normal current I_{ao}.

Providing the anode current from the main part of the cathode follows the three-halves power law, then we have:

$$I_{ed} = 2\sqrt{\pi}\ W\ f(\eta_m)\ \sqrt{Kj_e}\ V^{3/4} \quad . \tag{A.12}$$

This expression means the edge current depends on both the emission current and the anode voltage, but follows the three-fourths power of the anode voltage rather than the three-halves power.

As an example for estimating the quantity of the edge current we suppose $V = 1000$ V, $j_e = 10$ a/cm, $K = 2\times10^{-6}$ a/$V^{3/2}$, $I_{ao} = 63$ ma, $j_a = 1.2\times10^{-3}$ a/cm^2. From Fig. 16 one obtains $z_m = 1.4\times10^{-2}$ cm. $f(\eta_m) = 0.55$. Suppose $W = 2z_m$ then we obtain: $I_{ed} = 43$ ma.

This is the same order of magnitude as the extra current we obtained by measurement on tubes. Since the emission current as well as the edge width W vary over wide ranges, there is no doubt that the extra current is different from tube to tube.

The practical situation is more complex than what we have considered here, particularly the state of the electron cloud is not well known. It becomes more complex when a cathode is large so that the temperature as well as the work function are not uniform. The width of the edge is by no means a constant. In a real tube the edge of the cathode is not a boundary between two sections of the same plane as in the idealized case (Fig. 13); e.g., Fig. 18 shows the cathode assembly of a SLAC XK-5 klystron. The border of the electron cloud is ambiguous. The violation in minor spacings of an assembled multiple element cathode structure will lead to variations in measured data which might be due to edge effects.

Conclusion

All the above deductions are approximate and only a qualitative analysis rather than a precise quantitative analysis. It does not include other possibilities which also contribute a certain amount of extra current, such as parasitic emission. However, we can conclude that the edge current plays an important role at low voltage, and it results in the violation of the three-halves power law. Furthermore, the extra current will serve as an additional means for estimating the emission capability of the cathode.

APPENDIX B

THE THREE-HALVES POWER LAW

Introduction

The three-halves power law originated by Langmuir and Child is well known as an intrinsic characteristic of a diode or an electron gun operated under space-charge limited conditions. Recently R. T. Longo[13] presented a new formula relating the current density-voltage-temperature, which is coincident with the experimental data in the regime of practical operation that is not far from saturated condition. Longo's formula shows that the perveance is lower than normal. According to the discussion above if the anode current is much lower than saturation, the three-halves power law is not accurate; but conversely, the perveance will be higher than normal. It then seems necessary to inquire to what extent the three-halves power law is valid. This appendix is intended to explain the three-halves power law from a different point of view.

The Field Solution

As is well known, Poisson's equation can be solved by a Green's function as follows:

$$V(M) = \int_S V(M_A) \frac{\partial G(M,M_A)}{\partial n} dS - \frac{1}{\epsilon_0} \int_D \rho(M_0) G(M,M_0)\, dv \qquad (B.1)$$

where dS and dv denote the integral elemental surface and elemental volume, respectively. Now we consider the problem of an electron gun. The physical meaning of the above formula is as follows. The first integral on the right-hand side is a surface integral taken along the whole boundary, M_A is the boundary point, at which the potential is $V(M_A)$. $G(M,M_A)$ is the Green's function, and n denotes the normal direction. Since the potential at the cathode is zero, the integral vanishes and so we are concerned only with the anode. Thus the first term is determined by and is proportional to the anode voltage V_A, but is independent of the space charge. We can express it as follows:

$$\int_S V(M_A) \frac{\partial G(M,M_A)}{\partial n} dS = V_A \int_S \frac{\partial G(M,M_A)}{\partial n} dS = V_A \alpha(M) \qquad (B.2)$$

When the shape of the electrodes is known, the function $\alpha(M)$ is uniquely determined.

The second integral of formula (B.1) is a volume integral taken along the whole space-charge area. M_0 is the source location, and M is the field location, $\rho(M_0)$ is the charge density, and can be expressed as follows:

$$\rho(M_0) = j(M_0)/v(M_0) \qquad (B.3)$$

where $v(M_0)$ is the scalar velocity at point M_0, $j(M_0)$ is the current density. If there exists crossover between trajectories, it will be multi-valued function, or

$$j(M_0) = \sum_k j_k(M_0) \qquad (B.4)$$

For a static problem, $v(M_0)$ is determined uniquely by the potential at the point where it is located, regardless of crossover, provided initial velocity is negligible. Hence:

$$\rho(M_0) = \frac{\sum_k j_k(M_0)}{\sqrt{2\eta V(M_0)}} \quad . \qquad (B.5)$$

η is the electronic charge-to-mass ratio.

Therefore, formula (B.1) can be rewritten as below:

$$V(M) = V_A \alpha(M) - \frac{1}{\varepsilon_0} \int_D G(M,M_0) \frac{\sum_k j_k(M_0)}{\sqrt{2\eta V(M_0)}} \, dv \quad , \qquad (B.6)$$

where α and G are definitive functions determined only by the shape of the electrodes.

When j and V are unknown functions, the above expression is an integral equation. It is equivalent to Poisson's differential equation.

Now suppose the anode voltage is V_A, and $j(M)$, $V(M)$ is a set of solutions of the above equation. We want to find a new solution $V'(M)$,

$j'(M)$ when the anode voltage is raised n-fold to nV_A. Apparently, if we let the potential everywhere rise n-fold to $nV(M)$, but the current rises $n^{3/2}$-fold to $n^{3/2}$ $j(M)$ everywhere, viz:

$$\begin{aligned} V'_A &= nV_A \\ V'(M) &= nV(M) \\ j'_k(M) &= n^{3/2}\, j_k(M) \end{aligned} \tag{B.7}$$

they will satisfy the equation. In fact, substituting them into (B.6), we have:

$$nV(M) = nV_A a(M) - \frac{1}{\varepsilon_0} \int_D G(M,M_0) \frac{\sum_k j_k(M_0) n^{3/2}}{\sqrt{2\eta\, V(M_0)\, n}}\, dv$$

This is exactly the same as Eq. (B.6). So $V'(M)$, $j'(M)$ is a new set of solutions, a self-consistent solution. And from (B.7) the ratio of the currents is:

$$\frac{j'(M)}{j(M)} = \left(\frac{V'_A}{V_A}\right)^{3/2} . \tag{B.8}$$

Obviously, the total anode current is the surface integral of j along the anode surface. Since the ratio on the left-hand side of (B.8) is constant everywhere, the ratio of the anode currents will be the same constant. Therefore we have:

$$\frac{I'_A}{I_A} = \left(\frac{V'_A}{V_A}\right)^{3/2} \tag{B.9}$$

This means that we obtain the three-halves power law, which is valid no matter what the shapes of the electrodes are, and no matter if crossover takes place or not. Strictly speaking, we have not proved this statement. It is correct only if the solution is also satisfied for the equation of motion and then the trajectories in both cases have similar shapes. This is true as shown in Ref. 20. Furthermore, formula (B.9) is also valid in the case where there exists magnetic field in the gun region, providing a scaling principle is applied. That includes (B.7) with the addition of the following expression:

$$B' = n^{1/2} B \qquad (B.10)$$

Briefly speaking, if the above conditions are satisfied, the radius of curvature R of the trajectories will hold everywhere as one can deduce from the following equation:[20]

$$\frac{mv^2}{R} = e(E_\perp + |v \times B|_\perp) \qquad (B.11)$$

Moreover, we should point out that the three-halves power law will be inaccurate under relativistic conditions, for which the velocity is no longer proportional to the square root of the potential. The denominator of formula (B.5) is no longer an expression of the velocity, nor is that in Eq. (B.6). In this analysis we will not concern ourselves with relativity.

However, we want to emphasize that the above solution is not unique, because the boundary condition is not certain yet. For example, when the cathode is saturated, the solution of the current will be different. But the scaling principle is still valid; that is: if the cathode temperature is adjusted so that the condition (B.7) is satisfied, then all the trajectories will remain unchanged. In practice, under normal conditions, the electric field vanishes on the surface of the virtual cathode, which is very close to the real cathode, and the initial velocity is very low as compared with the potential everywhere except the neighborhood of the cathode; then the boundary condition is defined. This corresponds to space-charge limited condition, and the three-halves power law will be valid in spite of the cathode emission.

For a common electron gun, one generally does not care about a slight variation of the perveance due to various factors such as the d.c. magnetic field. On the other hand, in a magnetron injection gun it is found that the perveance strongly depends on the magnetic field.[21] As far as this analysis is concerned, we want to emphasize the perveance is not constant.

Flow Mode

As is well known, in a microwave cavity there exist one or more modes. Each mode corresponds to a certain field distribution. Similarly, in an electron gun, if there is no outer magnetic field and the boundary condition on the cathode is ideal, the shape of the beam flow is uniquely defined. So there is only one 'flow mode'. When the anode voltage is raised, the current strength follows the three-halves power law, but the shape of the flow remains unchanged, and the perveance is constant. When an outer magnetic field is present, this flow mode might be changed to a new mode and the perveance may also be changed. However, if the magnetic field follows the scaling principle with raised anode voltage, the flow mode is unchanged and so is the perveance.

Therefore, each flow mode corresponds to a certain flow shape and a certain perveance, but the current magnitude has no effect. The relationship between the perveance and the flow mode is something like that of an eigenvalue and the eigenfunction. Of course the former is not a linear operator problem.

We define two normalized parameters as follows:

$$j_1(M) = \frac{j(M)}{I_A} \quad , \quad j_{1k}(M) = \frac{j_k(M)}{I_A} \quad , \quad V_1(M) = \frac{V(M)}{V_A} \qquad (B.12)$$

Obviously, as soon as the flow mode is known, these parameters are independent of the anode voltage or total anode current. In fact we have:

$$\int_S j_1(M)\, dS = 1 \quad , \qquad (B.13)$$

where S is the anode surface or any cross section of the beam.

The Electric Field

Being similar to the potential, the electric field strength can be also expressed in two terms. In fact, from (B.6) we obtain:

$$\vec{E}(M) = -\nabla V(M) = -V_A \nabla \alpha(M) + \frac{1}{\varepsilon_0} \int_D \nabla G(M,M_0) \frac{\sum_k j_k(M_0)}{\sqrt{2\eta V(M_0)}}\, dv \qquad (B.14)$$

This formula is satisfied in the whole gun space except the electron cloud region, which is between the virtual cathode and the cathode.

The gradient of $G(M,M_0)$ in the integrand is the derivative with respect to M, while the integral is over M_0. The first term on the right-hand side represents a component of the electric field induced by and proportional to the anode voltage. The function $\nabla\alpha(M)$ only depends on the shape of the electrodes, but is independent of space charge. The second term is the component of the field induced by the space charge. Using the normalized parameters we have:

$$\vec{E}(M) = -V_A \nabla\alpha(M) + \frac{I_A}{\sqrt{V_A}} \frac{1}{\epsilon_0} \int_D \nabla G(M,M_0) \frac{\sum_k j_{1k}(M_0)}{\sqrt{2\eta V_1(M_0)}} dv \qquad (B.15)$$

The above integral as well as $\nabla\alpha(M)$ are functions of location only, providing the flow mode is known. So the above expression can be re-written as follows:

$$\vec{E}(M) = V_A \vec{e}_V(M) + \frac{I_A}{\sqrt{V_A}} \vec{e}_I(M) \qquad (B.16)$$

The physical meaning of e_V and e_I is that they are the electric field induced by unit anode voltage and current, respectively.

Since E will vanish everywhere on the surface of the virtual cathode, where we define $M = M_c$. $\vec{e}_V$ has only a vertical component, and so has $\vec{e}_I$. Therefore we have:

$$\frac{I_A}{V_A^{3/2}} = -\frac{e_{V\perp}(M_c)}{e_{I\perp}(M_c)} \quad . \qquad (B.17)$$

The right-hand side is independent of the anode voltage and current, so it is a constant for a certain flow mode. This constant is just the perveance. Thus we obtain the three-halves power law once again. Generally, tubes are operated under space-charge limited, then the electric field vanishes on the surface of the cathode. The physical meaning of its vanishing is that the electric field e_V will always be compensated by the space-charge field. It implies a principle of uniformity of emission as explained below.

The integral $e_I(M)$ referred to in Eq. (B.15) is with respect to the whole space, but near the surface of the cathode, on which the potential is zero, the main contribution is due to the space charge nearby. This argument is due to the fact that a space charge near the metal surface can only induce a vertical electric field nearby, its electric field at a far distance will be compensated by its mirror charge, while the field on the surface induced by the space charge far from the metal surface is rather uniform and will diminish rapidly. Besides the space-charge density itself reduces with distance according to Langmuir's relation. So approximately, $e_I(M_c)$ is proportional to the local emission current density (emitting from the virtual cathode, viz. anode current density). The same is true about $e_V(M_c)$ because it is proportional to $e_I(M_c)$ Correspondingly, if one requires the emission current density along the cathode to be uniform, which is usually what a gun designer desires, one should arrange the electrodes so that the electric field induced by the anode voltage is also uniform along the cathode.

The Perturbation by the Edge Emission

Suppose there exists an extra beam current I_{ed}, which is emitted from the edge of the cathode, and adheres to the outer 'shell' of the original beam. Of course, it will induce an extra space-charge field, and in turn affect the original beam emission.

As a first order approximation, we think of this extra current only as a disturbance, namely, the flow mode is not distorted seriously, but the anode current may be slightly changed from I_A to I'_A. Then the electric field can be expressed as follows:

$$E(M) = V_A\, e_V(M) + \left[I'_A\, e_I(M) + I_{ed}\, \sigma(M)\, e_I(M)\right]/\sqrt{V_A} \qquad \text{(B.18)}$$

Since E must vanish on the cathode surface, thus:

$$V_A\, e_V(M_c) + \left[I'_A\, e_I(M_c) + I_{ed}\, \sigma\, e_I(M_c)\right]/\sqrt{V_A} = 0 \qquad \text{(B.19)}$$

σ is a measure of the space-charge field due to the edge current. It might be a complex function. However, it will always be less than one, because the electric field on the cathode surface induced by a 'shell' layer of the beam (like a hollow beam) will always be less than that induced by a solid beam. From (B.17) and (B.19) one obtains:

$$I'_A + I_{ed}\,\sigma = -\frac{e_V(M_c)}{e_I(M_c)}\, V^{3/2} = I_A$$

where I_A is the anode current without perturbation, and here σ is a measure of the average effect. The total anode current will be:

$$I_{tot} = I'_A + I_{ed} = I_A + (1 - \sigma)\, I_{ed}$$

$$= K\, V^{3/2} + (1 - \sigma)\, I_{ed}$$

This means the three-halves power law is distorted by the perturbation, and the edge current is partly compensated by a reduction of the original beam current.

Conclusion

By solving the wave equation with boundary conditions one can obtain a series of eigenfunctions and eigenvalues. Similarly, solving Poisson's equation of an electron gun, combining with the equation of motion and boundary conditions one can obtain a series of 'flow modes' and corresponding perveances. Each flow mode obeys the three-halves power law. Any distortion of the boundary or change of the magnetic field will also distort the flow mode. For instance, at the edge of the cathode the common boundary condition is violated. The edge emission will influence the mode and so results in a violation of the three-halves power law.

REFERENCES

1. J. Z. Lebacqz, "RF sources developments," Linear Accelerator Conference, Upton, NY, September 1979.
2. G. Herrmann and P. S. Wagener, "The oxide coated cathode," vol. 2, Chapman Ltd., 1951.
3. I. S. Nergaard, "Studies of the oxide cathode," RCA, rev. 13, December 1952, p. 464.
4. ?. S. Rittner et al., "Studies on the mechanism of operation of the L cathode," JAP28, no. 2, February 1957, p. 156.
5. R. W. Springer and T. W. Hass, "Auger electron spectroscopy study of cathode surfaces during activation and poisoning. I. The barium-on-oxygen-on-tungsten dispenser cathodes," JAP45, no. 12, December 1974, p. 5260.

6. R. Forman, "Surface studies of barium and barium oxide on tungsten and its application to understanding the mechanism of operation of an impregnated tungsten cathode," JAP47, no. 12, December 1976.

7. R. Forman, "Use of auger spectroscopy in the evaluation of thermionic cathode," IEEE Trans. on Electron Device, vol. ED-24, no. 1, January 1977, p. 56.

8. E. S. Rittner, "On the mechanism of operation of the type B impregnated cathode," JAP48, no. 10, October 1977, p. 4344.

9. D. Jones, "Surface and emission characterization of the impregnated dispenser cathode," Application of the Surface Science 2, 1979, p. 232.

10. R. Forman, "A proposed physical model for the impregnated tungsten cathode based on auger surface studies of the Ba-O-W system," Application of Surface Science 2, 1979, p. 258.

11. P. Palluel and A. M. Shroff, "Experimental study of impregnated-cathode behavior, emission and life," JAP51, no. 5, May 1980, p. 2894.

12. R. T. Longo, "Long life, high current density cathodes," IEDM, 1978, p. 152.

13. R. T. Longo, "A study of thermionic emitters in the regime of practical operation," IEDM, 1980, p. 467.

14. D. H. Smith, "Design, construction and long life endurance testing of cathodes assemblies for use in microwave high power transmitting tubes," Watkins-Johnson Co., NASA CR-135371.

15. M. G. Bordmer, "Dip testing, a new method for measuring cathode activity," IRE Trans. on Electron Device, vol. ED-5, January 1958, p. 43.

16. D. W. Maurer, "Cathode activity measurement: a modification of the dip test," BSTJ, vol. 46, no. 10, December 1967, p. 2363.

17. C. P. Foster, "Dip testing, effect on life of tube," IRE Trans. on Electron Device, vol. ED-12, May 1965, p. 296.

18. R. Strass, "Traveling wave tubes for communication satellites," PIEEE, vol. 65, March 1977, p. 387.

19. C. S. Hung, "Thermionic emission from oxide cathodes: retarding and accelerating field," JAP21, January 1950, p. 37.

20. M. Chodorow and C. Susskind, "Fundamentals of Microwave Electronics," McGraw-Hill Book Co., New York, 1964, chap. 2.

21. G. S. Kino and N. J. Taylor, "The design and performance of a magnetron-injection gun," IRE Trans. ED-9, 1, January 1962.

22. I. Langmuir, "The effect of space charge and initial velocities on the potential distribution and thermionic current between parallel plane electrodes," Phys. Rev. 21, 1923, p. 419.

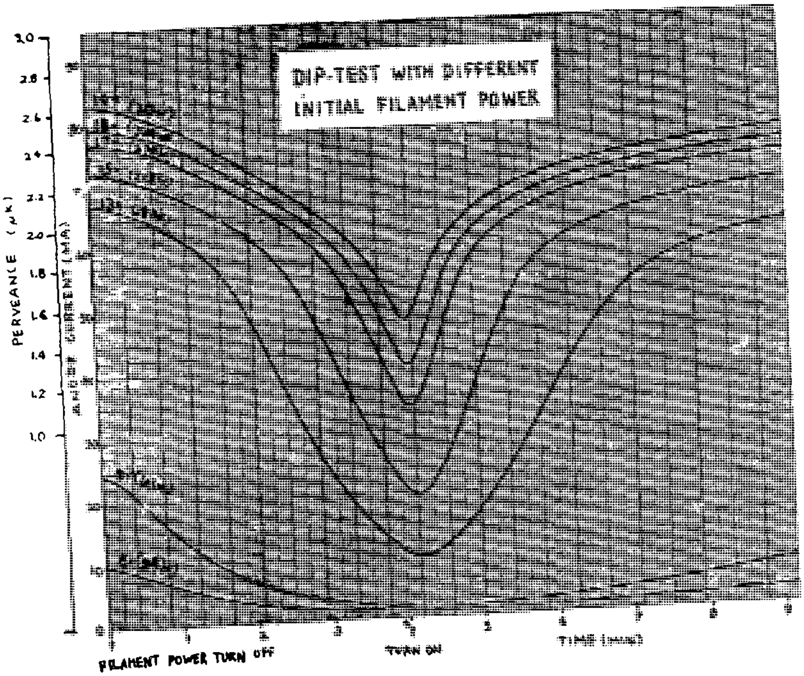
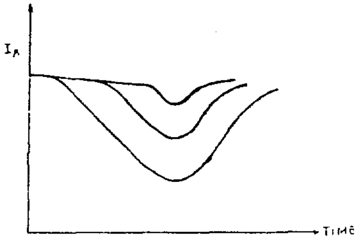
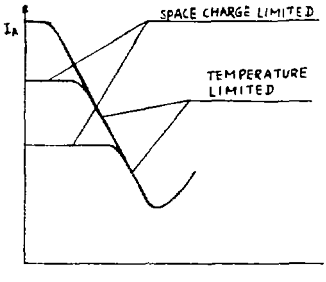


Fig. 1. The experimental dip-test curve (M239).



(a) Dip-test curves for different activity (or different initial temperature) V_A = const.



(b) Ideal dip-test curves for different anode voltage.

Fig. 2. The ideal dip-test characteristics.

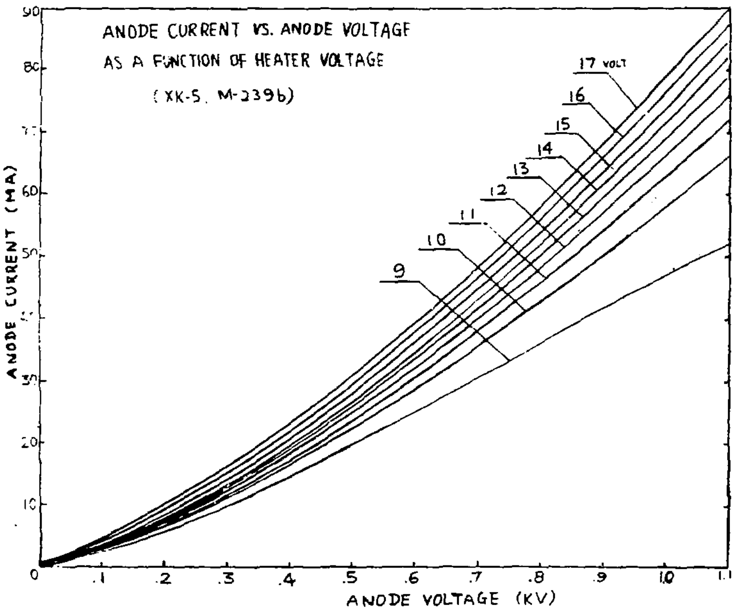


Fig. 3. $I_A - V_A$ characteristics.

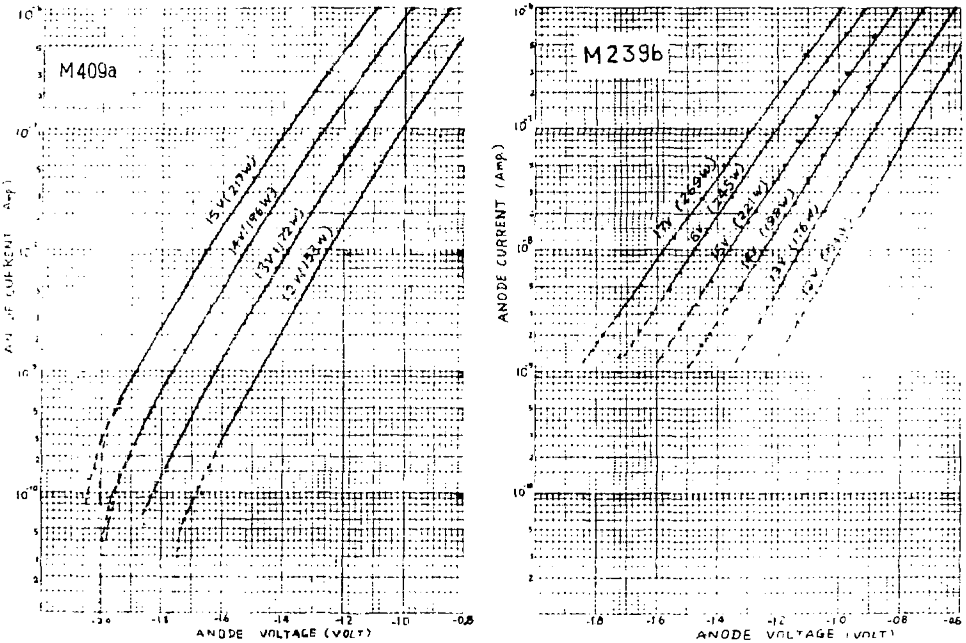


Fig. 4. The typical retarding field current characteristics.

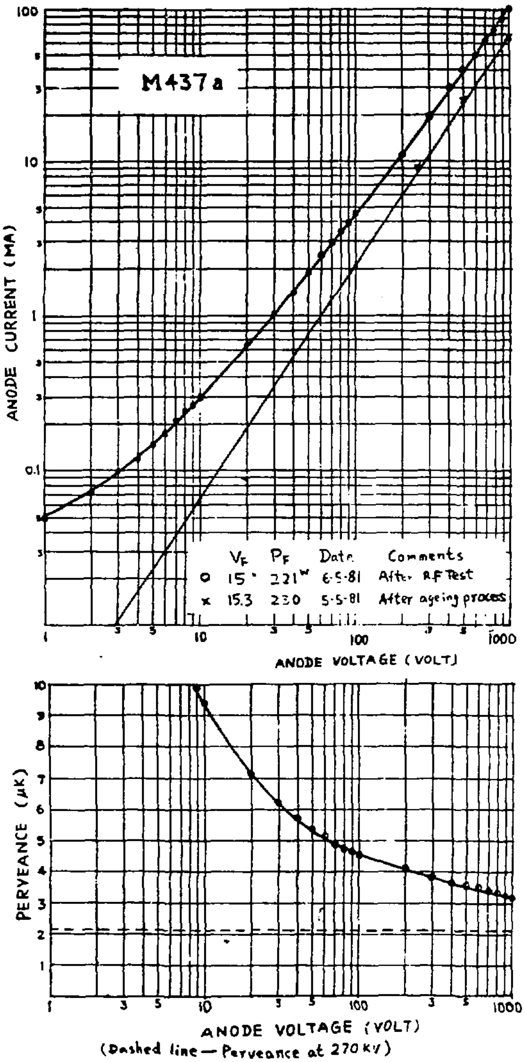


Fig. 5. The characteristics for different activity levels.

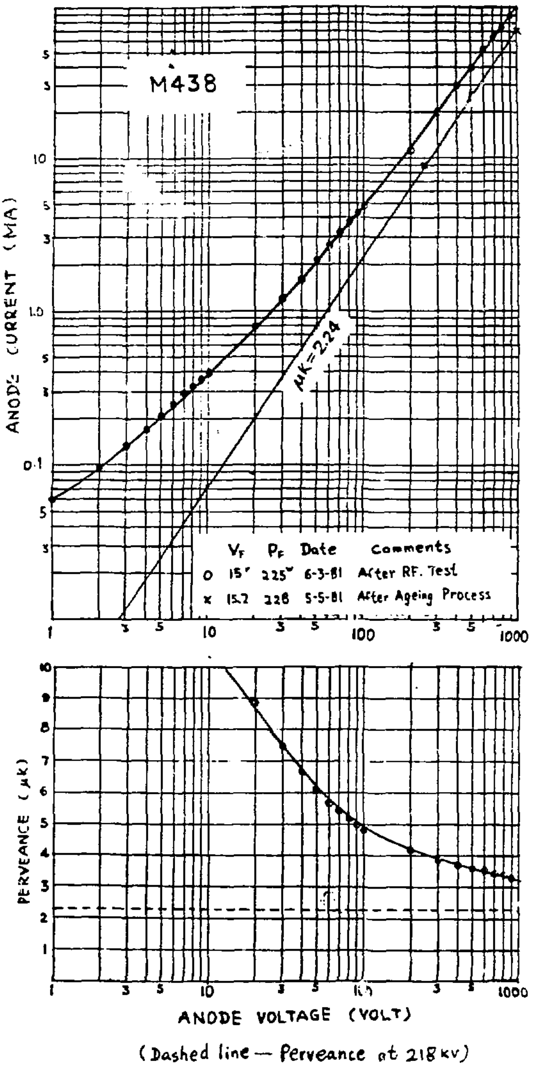


Fig. 6. The characteristics for different activity levels.

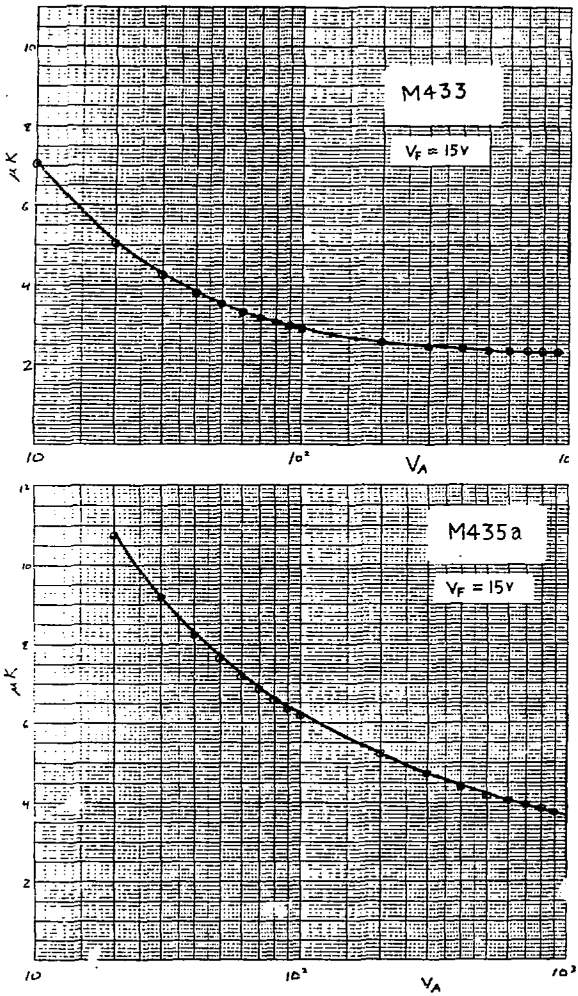


Fig. 7. The perveance characteristics at low voltage.

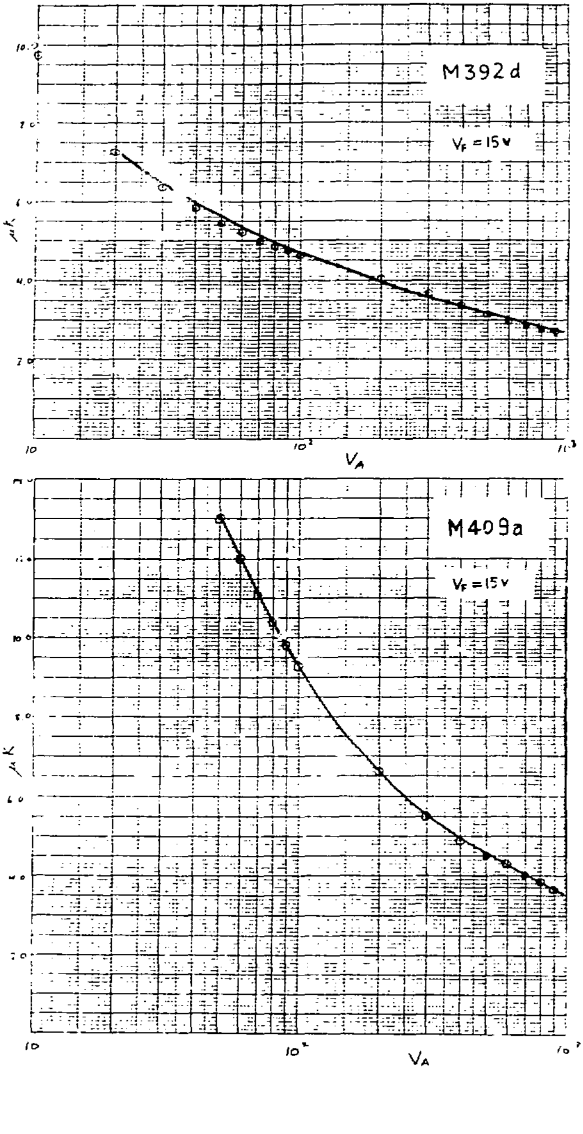


Fig. 8. The perveance characteristics at low voltage.

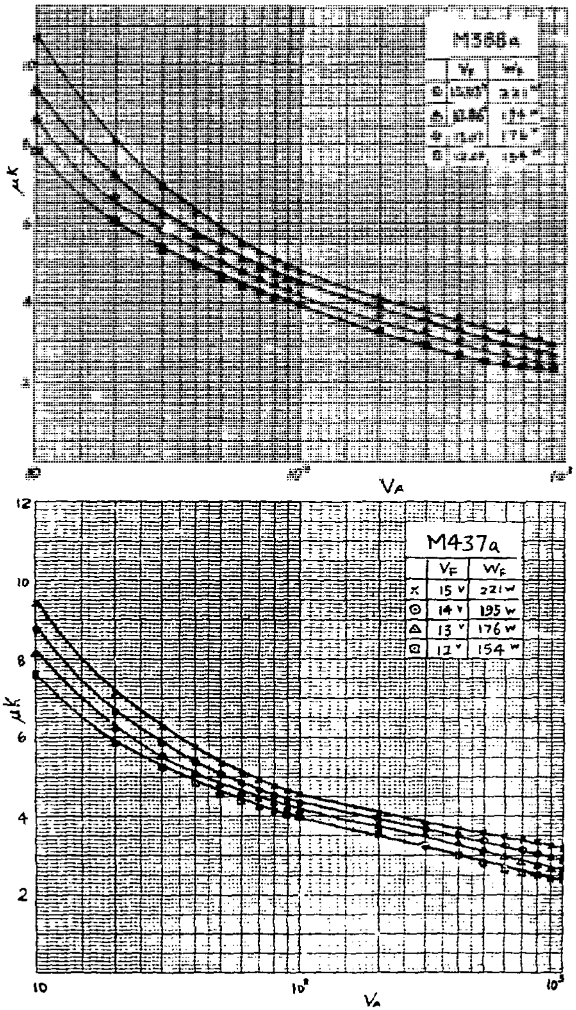


Fig. 9. The typical perveance characteristics as a function of filament power.

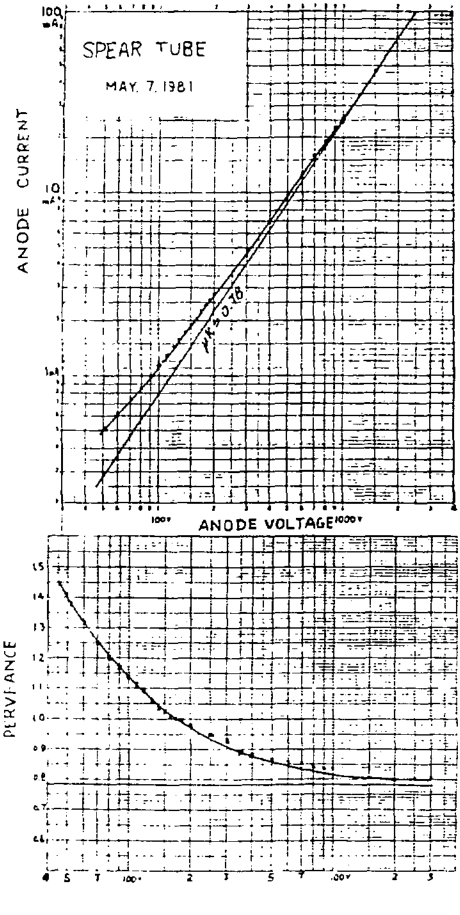


Fig. 10. SPEAR tube (this tube was undergoing re-aging after it was rejected for lack of emission).

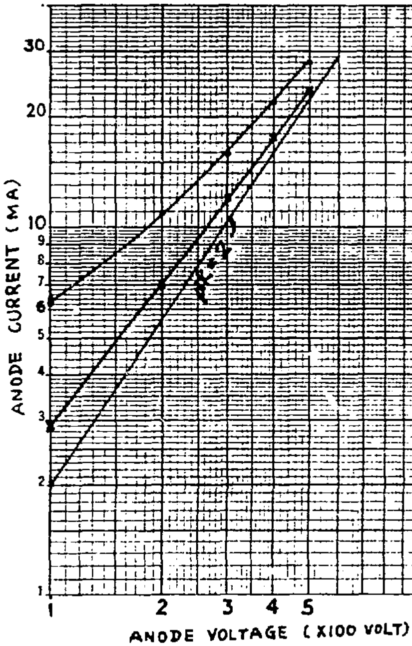


Fig. 11. Two failed RCA tubes: o — U-39 Apr.10.81
x — XT-22 Apr.30.81.

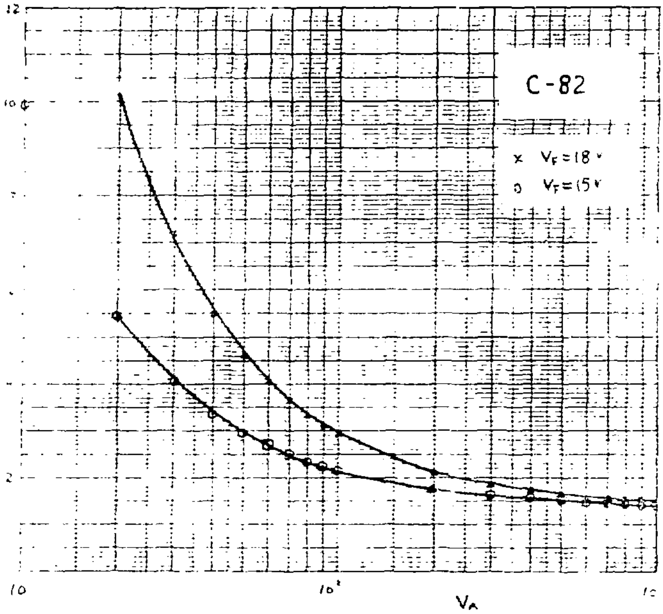
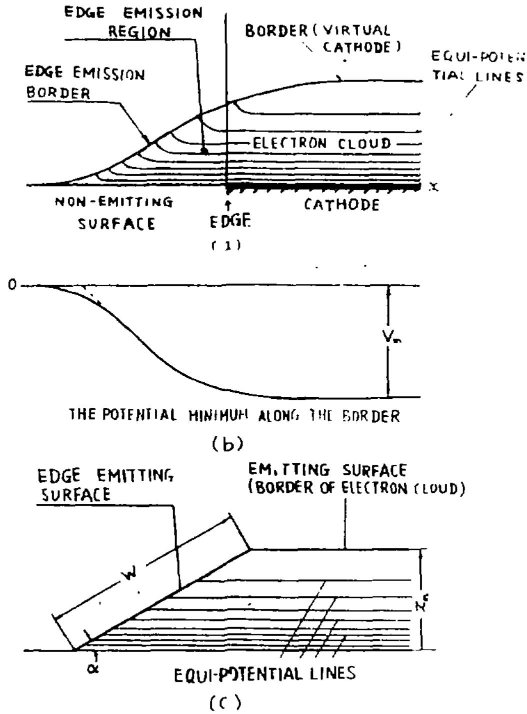


Fig. 12. A failed RCA tube (this failed RCA tube has abnormal performance).



EDGE EMISSION REGION
BORDER (VIRTUAL CATHODE)
EDGE EMISSION BORDER
ELECTRON CLOUD
NON-EMITTING SURFACE
EDGE
CATHODE
x
(a)
0
V_m
THE POTENTIAL MINIMUM ALONG THE BORDER
(b)
EDGE EMITTING SURFACE
EMITTING SURFACE (BORDER OF ELECTRON CLOUD)
W
α
EQUI-POTENTIAL LINES
(c)

Fig. 13.

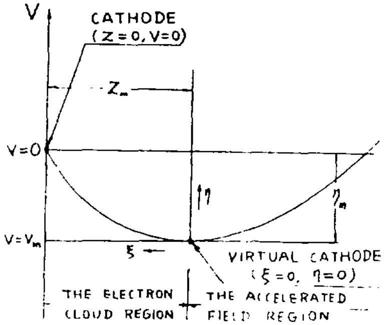


Fig. 14. The potential distribution in front of the cathode.

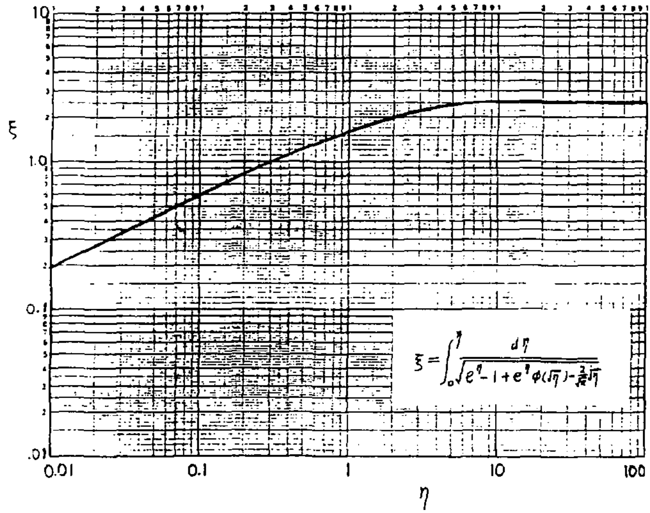

10
1.0
0.1
.01
ξ
0.01
0.1
1
10
100
η
$\xi = \int_0^{\eta} \frac{d\eta}{\sqrt{e^{\eta} - 1 + e^{\eta}\phi(\sqrt{\eta}) - \frac{2}{\sqrt{\pi}}\sqrt{\eta}}}$

Fig. 15.

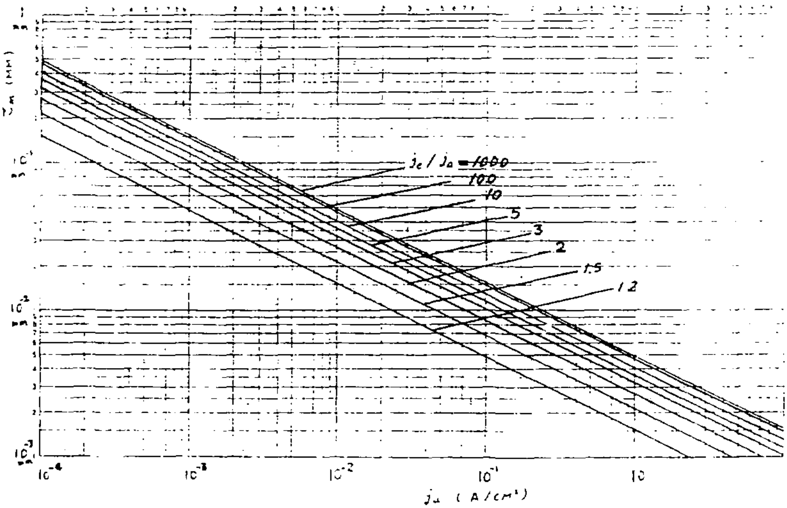


Fig. 16.

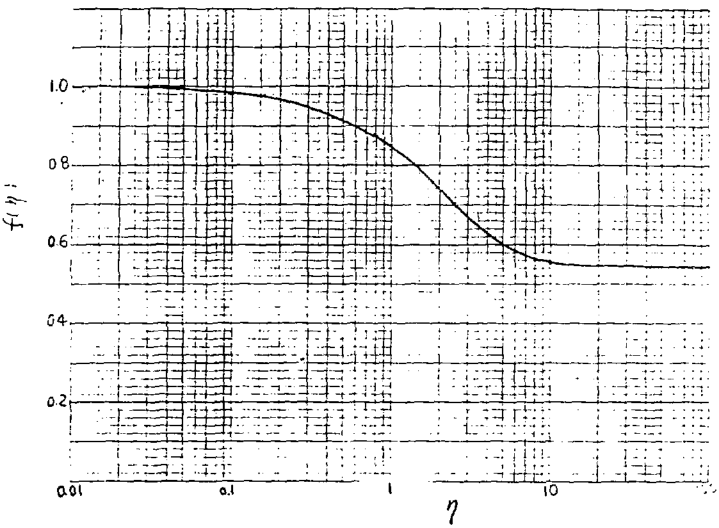


Fig. 17.

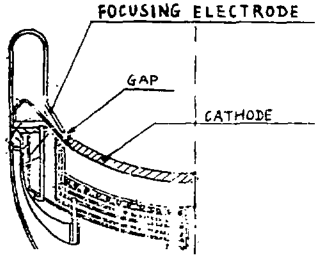


Fig. 18. The real structure of the cathode and its edge.