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MEANING OF THE NEGATIVE IMPEDANCE

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ABSTRACT

It is shown that the negative real part of an input impedance does not mean instability of the related circuit. A negative real part of the input impedance means only that the concerned circuit is active.

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I. OPERATIONAL IMPEDANCE AND STABILITY

If the unit impulse of a current is fed between two nodes of a linear time invariant network, a voltage V (the Green function) is developed between the two nodes. The circuit is stable if, and only if, this voltage goes to zero when the time goes to infinity.*

The L-transform of the voltage V is, by definition, the operational impedance z(s) that appears between the considered nodes. Consequently, the circuit is stable if, and only if, the poles of the impedance are located on the left-hand side of the complex s plane.

II. THE STEADY STATE IMPEDANCE

The operational impedance z(s) becomes the steady state impedance if we put $s = jw$, as is well known. The complex function z(w) can be divided into the real and imaginary parts: $z(w) = r(w) + jz(w)$. This operation involves both the poles and the zeros of the operational impedance, and it appears obvious that the signs of the function r(w) cannot give any information about the stability of the related network even if it is always negative. Moreover, depending upon the sign of its real part the whole impedance is, normally but improperly, referred to as positive or negative.

III. A SIMPLE MATHEMATICAL EXAMPLE

Suppose that an operational impedance has the form:

$$z(s) = A\frac{s + z}{s + p} ,$$

where A, z and p are real quantities and p is positive. In this case, the stability is ensured but r(w) may be positive or negative depending upon the sign and the values of A and z as can be immediately checked with the before mentioned substitution.

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* A pure capacitor is considered stable even if the voltage remains constant at its terminals.

IV. A PHYSICAL EXAMPLE

One of the most important, and simple examples is given by the cathode follower amplifier as indicated in Figure 1. Y_o and Y_k are general operational admittances; G_m and R are the transconductance and plate resistance of the tube. We are concerned

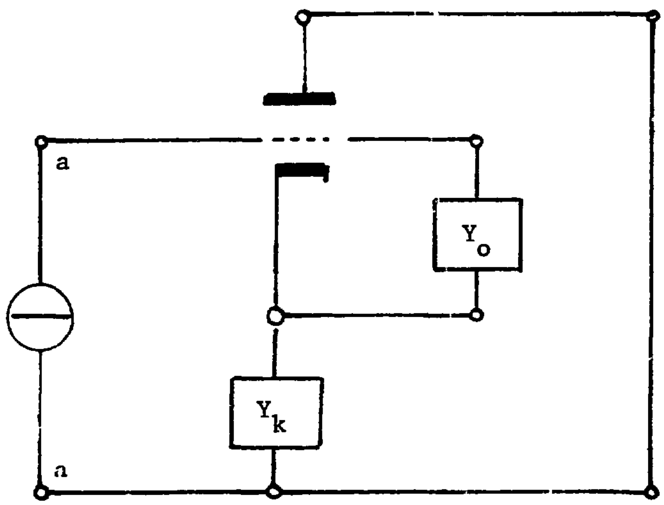

Figure 1

with the impedance appearing between the terminals a-a. A very simple calculation shows that we have

$$z_a(s) = \frac{1/R + G_m + Y_o + Y_k}{Y_o(1/R + Y_k)} \quad . \tag{1}$$

If $Y_o = sC_o$ and $Y_k = sC$, we obtain

$$z_a(s) = \frac{1/R + G_m + s(C + C_o)}{sC_o(1/R + sC)} \quad . \tag{2}$$

We have two poles, $s_1 = 0$; $s_2 = - 1/RC$, and it is clear the circuit is stable. Upon substituting s = jw, we find r(w):

$$r(w) = \frac{1/R - G_m C/C_o}{(1/R)^2 + (wC)^2} \quad . \tag{3}$$

It is evident that r(w) can assume negative values. Negative values for the real part of the input impedance also means negative values for the real part of the input admittance. It we assume that the input terminals are connected with a very large resistor (an open circuit in our case), the situation does not change very much. Therefore, we are considering a circuit that is stable while exhibiting a negative input conductance.

The presented scenario would change drastically if we connect the input terminals with an inductor L. The operational impedance now appearing between the terminals a-a is easily calculated inverting the general expression 1 and adding the term 1/sL. After some algebra, we obtain:

$$z_a(s) = sL \frac{1 + G_m R + sR(C + C_o)}{s^3 LCC_o R + s^2 LC_o + sR(C + C_o) + RG_m + 1} .$$

The Hurwitz criterion applied to this third-order equation indicates that poles with a positive real part are present if

$$LRC_o(C_o - G_m RC) < 0 \quad ,$$

and the above condition can be easily met. We see that while the added inductor cannot modify the real part of the input admittance, it can bring the circuit into instability.

V. MEANING OF "NEGATIVE IMPEDANCE"

A passive network is always absolutely stable; this means that any operational impedance between any two nodes must have poles and zeros on the left-hand side of the complex plane. An active network which exhibits impedances having poles located only in the left-hand semi-plane and zeros located also in the right-hand semi-plane is stable, but the steady impedances may show a negative real part. In conclusion; impedances with a negative real part only mean that we are dealing with an active network. (This situation might be referred to as "conditionally stable" because the addition of a passive element between two nodes may bring the system into oscillations.)

VI. REMARKS

All the previous considerations are valid for the "admittances" and the Green function is a current.