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ABSTRACT

The time necessary to achieve the equilibrium ratio of strange to non strange quarks in heavy ion reactions is estimated in the framework of perturbative QCD. It is found, in the present approximation, to be much larger than the total collision time of even a central U + U collision at E_{LAB} = 2.1 GeV/nucleon bombarding energy.

АННОТАЦИЯ

На основе пертурбативной КХД оценивается время, необходимое для достижения равновесия по числу образовавшихся странных и нестранных кварков в реакциях с тяжелыми ионами. В данном приближении получается, что это время гораздо больше полного времени взаимодействия даже при центральном соударении U + U при энергии 2,1 ГэВ/нуклон.

KIVONAT

A perturbativ QCD keretei között megbecsüljük a ritka és nem-ritka kvarkok közötti egyensuly beállásához szükséges időt nehéz ion reakciókban. Ebben a közelitésben ez jóval nagyobbnak adódik, mint a reakció teljes ideje még a centrális U + U ütközés esetében is 2.1 GeV/nukleon laboratóriumi bombázó energia mellett.

QUARKOCHEMISTRY IN RELATIVISTIC HEAVY ION COLLISIONS

The problem whether a quark gluon plasma phase is produced in the course of a relativistic heavy ion collision (RHIC) remains the most fascinating question of heavy ion research.

Many works [1-4] have dealt with the description of the quark phase itself; in other papers [5-10] the signature, i.e. the experimentally observable consequences of such a phase transition, is investigated. It was shown that the presence of strange quarks influences essentially the state of both the quark and hadronic matter [1,11]. It was pointed out too, that the observation of strange particles may indicate whether they are created in the quark-gluon phase [7,9] or in the hadronic phase [12-15].

In these treatments the chemical equilibrium between strange and non strange quarks was assumed. It remained to be clarified, however, whether during the short lifetime (if any) of the quark-gluon plasma this equilibrium value can be reached.

Our aim in the present work is to estimate the relaxation time, τ, necessary to achieve the equilibrium ratio of strange to non strange quarks. This relaxation time will be compared with the reaction time of the heavy ion collisions. For this purpose we have to introduce dynamic assumptions too, besides the assumptions usually applied in the thermodynamic treatment of the quark phase.

Naturally the complete treatment of the whole collision process by QCD is beyond the possibilities of present day theories. Therefore we shall assume that a given initial state is created by processes not discussed here. Then we shall follow up the consequences of this initial condition. Presumably one will have to investigate the consequences of different initial conditions to find the most realistic one. One also has to decide what elementary events should be regarded as the most important for the processes investigated and what events should be left in the backround.

In our present consideration the total effect of all the background processes is assumed to be much quicker than the selected one, so these background events ensure, for example, the thermal equilibrium of quark matter.

For the evolution of heavy ion collision we suggest the following scenario:

i/ In the near central collision of relativistic, large heavy ions a piece of hot and dense matter, the firecloud, is formed, which undergoes a phase transition to form a quark-gluon plasma.

ii/ This plasma initially contains "up" and "down" quarks only (denoted by q) and has high temperature and density. Later on in this plasma other kinds of quarks will be created, namely anti-up, anti-down, strange and anti-strange quarks $(\bar{q}, s, \bar{s})$. Although strange quarks may have already been produced in the previous, hadronic, phase of the collision, this can change the initial condition only but not the relaxation time, τ.

iii/ The non strange anti-quarks $(\bar{q})$ are produced by much quicker processes than the strange ones (s and $\bar{s}$). Actually the equilibrium between non strange quarks and anti-quarks is regarded as being achieved at the beginning of the strange and anti-strange quark production. (This assumption is motivated by the observation that the threshold energy is much smaller for the $q\bar{q}$ than for the $s\bar{s}$ pair creation.)

iv/ Our considerations are based on the applicability of the perturbative QCD for the description of elementary processes going on in the quark-gluon plasma. The strange and anti-strange quarks are hypothesized as being created by the following process: $q\bar{q} \longrightarrow s\bar{s}$ (see Fig. 1a).

v/ On cooling down, this plasmaball will somehow break up to the experimentally observed hadrons after a certain time.

According to this scenario the time development of the strange-non strange quark composition of the system is described by the following set of differential equations, similary as in Ref. [16]:

$$
\begin{aligned}
\dot{n}_q &= \nu(n_q, n_{\bar{q}}, \dots) \approx 0 , \\
\dot{n}_{\bar{q}} &= \nu(n_q, n_{\bar{q}}, \dots) \approx 0 , \\
\dot{n}_s &= \lambda \cdot (n_q n_{\bar{q}} - r \cdot n_s n_{\bar{s}}) , \\
\dot{n}_{\bar{s}} &= \lambda \cdot (n_q n_{\bar{q}} - r \cdot n_s n_{\bar{s}}) ,
\end{aligned}
\qquad (4)
$$

where the kinetic coefficient, λ, is determined as the rate factor averaged over the momentum space of colliding q and $\bar{q}$ particles (eq.(2)) and ν represents the total effect of $q\bar{q}$ pair production and annihilation. Because those processes are supposed as being quicker than $s\bar{s}$ pair production, the number density of non strange quarks and anti-quarks (n_q, $n_{\bar q}$) is assumed to keep its equilibrium value. In eqs.(1), r denotes the equilibrium ratio between non strange and strange quark anti-quark pairs.

Due to its high temperature (100–300 MeV) the quark-gluon plasma will be treated as a multicomponent relativistic ideal Boltzmann gas. If we denote the cross section of process $q\bar q \to s\bar s$ shown in Fig.1a by σ then the rate factor, λ, and the equilibrium ratio, r, are given as:

$$\lambda = \langle \sigma \cdot v_{rel} \rangle = \frac{\iint \sigma \cdot v_{rel} f_1 f_2 \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3}}{\iint f_1 f_2 \frac{d^3p_1}{(2\pi)^3} \cdot \frac{d^3p_2}{(2\pi)^3}} = \frac{\beta^4}{16} \cdot \int_{4m_s^2}^{\infty} s^2 \cdot \sigma(s) K_1(\beta\sqrt{s}) \frac{ds}{2\sqrt{s}} \qquad (2)$$

and

$$r = \left(\frac{n_q \cdot n_{\bar q}}{n_s \cdot n_{\bar s}} \right)_{eq} = \left(\frac{Q(\beta, m_q)}{Q(\beta, m_s)} \right)^2 = \left(\frac{d_q / d_s}{\frac{1}{2}(\beta m_s)^2 K_2(\beta m_s)} \right)^2 . \qquad (3)$$

Here $s=(p_1+p_2)^2$ is the invariant C.M. energy squared, β is the inverse temperature, m_s and m_q are the rest masses of strange and non strange quarks, respectively (they are assumed to be: $m_s \approx 300$ MeV, $m_q \ll m_s$, the latter is neglected here (see Refs [9,17])), d_s and d_q are the spin isospin and color degeneracy factor of quarks (6 and 12) and finally $K_n(x)$ denotes the n-order Bessel-function of the imaginary argument.

The function $Q(\beta,m)$ denotes the canonical partition function of a Boltzmann-distributed set of particles each having rest mass m at $T=1/\beta$ temperature. The general form of it for the relativistic Boltzmann gas is:

$$Q(\beta, m) = \frac{d}{\pi^2 \beta^3} \cdot \frac{1}{2} (\beta m)^2 K_2(\beta m) .$$

(The value of the function $f(x) = 0.5x^2K_2(x) \leq 1.0$ for all $x \geq 0.0$.)

The equilibrium value of the number density of non strange quarks and anti-quarks - which ensure the stationary conditions for the slower $s\bar s$ creation process - can be determined from Boltzmann statistics:

$$n_q = Q(\beta,0)\, e^{\beta\mu} \quad , \quad n_{\bar q} = Q(\beta,0)\, e^{-\beta\mu} ,$$

where μ is the chemical potential. Because of the baryon number conservation, $(n_q - n_{\bar q}) = b$ = constant, hence

$$n_q = \frac{b}{2}\cdot\left(1+\sqrt{1+(2Q(\beta,0)/b)^2}\right) ,$$
$$n_{\bar q} = \frac{b}{2}\cdot\left(-1+\sqrt{1+(2Q(\beta,0)/b)^2}\right) \qquad (4)$$

and

$$n_q \cdot n_{\bar q} = Q^2(\beta,0) . \qquad (5)$$

Because eqs.(1) conserve the strangeness density, $(n_s - n_{\bar s})$, it remains equal to the initial zero value.

On account of these considerations the system of eqs.(1) can be reduced to one ordinary first order differential equation. For the number density of strange quarks it reads as:

$$\dot n_s = \lambda \cdot Q^2(\beta,0) - \lambda \cdot r \cdot n_s^2 . \qquad (6)$$

The particular solution of this equation with initial condition $n_s(0)=0$ *has the following analytical form:*

$$n_s(t) = Q(\beta,m_s)\cdot \tanh\left(\lambda\cdot\frac{Q^2(\beta,0)}{Q(\beta,m_s)}\cdot t\right) . \qquad (7)$$

From eq. (7) one can read the characteristic time for achieving the equilibrium value of strange quarks:

$$\tau = 2\,\frac{Q(\beta,m_s)}{Q^2(\beta,0)}\cdot\frac{1}{\lambda} = \frac{2d_s}{d_q^2}\cdot\frac{\pi^2\beta^3}{\lambda}\cdot\frac{1}{2}(\beta m_s)^2\cdot K_2(\beta m_s) . \qquad (8)$$

The number densities of strange quarks and anti-quarks reach 76% of their equilibrium value at time $t=\frac{\tau}{2}$, and 96% of this amount at time $t=\tau$; so one may certainly suppose chemical equilibrium τ time after the formation of the quark-gluon plasma. Note that τ depends on the coupling constant as $\propto 1/\alpha^2$.

To estimate this time we have to calculate the cross section of the process $q\bar q \to s\bar s$ (see Fig.1a), appearing in expression (2) for λ. Supposing the applicability of the perturbative QCD we treat this process as an analogue of the muon pair creation in QED (see Fig. 1b). In the *expression for the cross section of the* $e^- e^+ \to \mu^- \mu^+$ process [18] we substitute the strange quark mass for muon mass to obtain the cross section for the $q\bar q \to s\bar s$ process, as

$$\sigma_0(s) = \frac{4\pi}{3}\cdot\frac{\alpha^2}{s}\cdot\left(1+\frac{2m_s^2}{s}\right)\sqrt{1-\frac{4m_s^2}{s}} . \qquad (9)$$

Here $\alpha = g^2/4\pi$ is the coupling constant of the strong interaction. We suppose it to be 0.2 [17] . The s is again the square of the C.M. energy of the collision.

The $q\bar{q} \rightarrow s\bar{s}$ process is, however, "colored" and its cross section is different from the cross section of muon pair creation. 3 x 3 = 9 kinds of colored $q\bar{q}$ collision are possible, but only 8 can be transformed to a member of the gluon-octet, so the cross section is 8/9 of its uncolored analogue. We use in (2)

$$\sigma(s) = \frac{8}{9}\,\sigma_0(s) \quad .$$

The characteristic time, τ , was determined by performing numerically the integral in expression (2). The τ values so obtained at different temperatures are plotted in Fig.2.

The total reaction time for the U + U collision at 2.1 GeV/A bombarding energy in the LAB system is about 50×10^{-24} sec (dotted line in Fig.2). By inspecting Fig.1a of Ref. [15] one may guess the possible lifetime of the very compressed and hot stage to be about 5-10 x 10^{-24} sec (dashed line in Fig.2).

It is clear from Fig.2 that the possible lifetime of quark-gluon plasma itself is far less than the time necessary for chemical equilibration between strange and non strange quarks.

An interesting feature of the relaxation time, τ , given in eq. (8) is that it does not depend on the density but on the temperature of the quark-gluon plasma ball only.

Finally we want to emphasize again that all of our considerations are based on the applicability of the perturbative QCD for the description of the plasma ball. Even in this case the relaxation time may change appreciably from the value given here if one uses other values for the rest mass of the strange quark and /or for the coupling constant. Here τ is inversely proportional to α^2 and it shows a qualitatively exponential dependence on the strange quark, mass, as $\exp(\frac{1}{T}m_s)$.

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FIGURE CAPTIONS

Fig.1. The lowest order graphs corresponding to the process $q\bar{q} \rightarrow s\bar{s}$ (a) and the analogue one $e^-e^+ \rightarrow \mu^-\mu^+$ (b), respectively.

Fig.2. The characteristic time, τ, for chemical equilibration of strange quarks is plotted as a function of temperature, T, of the quark-gluon plasma. The total collision time and the guessed lifetime of the quark phase for a central U + U collision at E_{LAB}= 2.1 GeV/A energy are also marked.

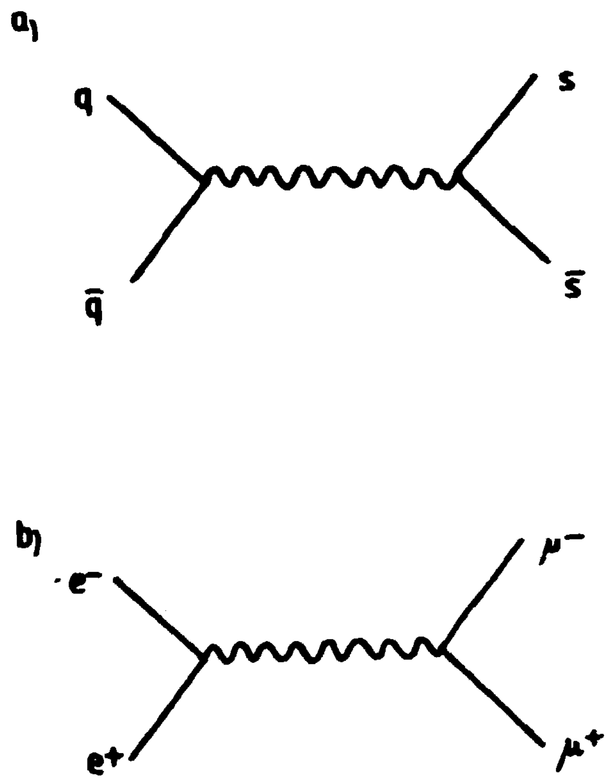

Fig. 1.

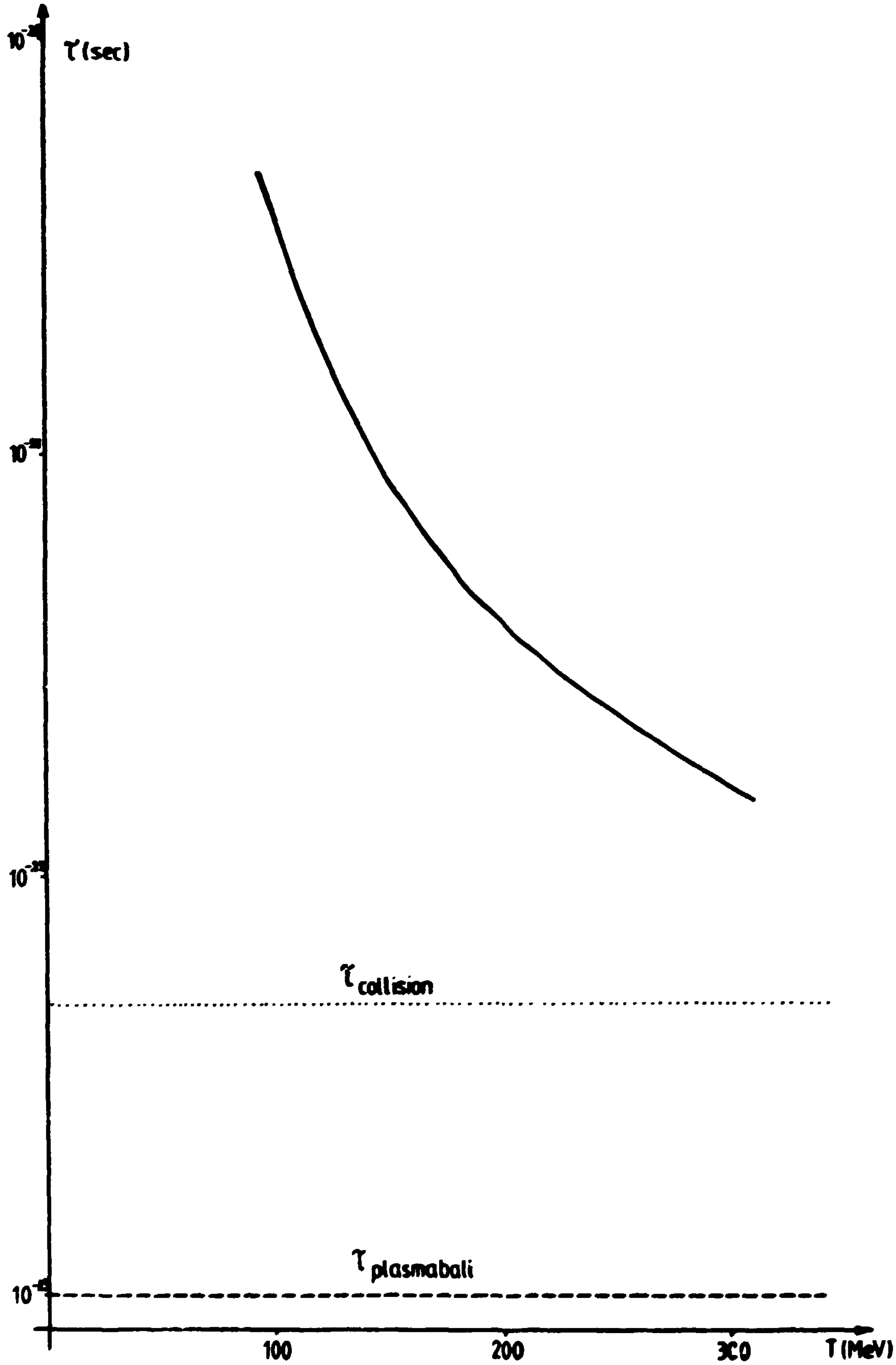

Fig. 2

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Szegő Károly
Szakmai lektor: Lovas István
Nyelvi lektor: Harvey Shenker
Példányszám: 450 Törzsszám: 81-482
Készült a KFKI sokszorosító üzemében
Felelős vezető: Nagy Károly
Budapest, 1981. augusztus hó