

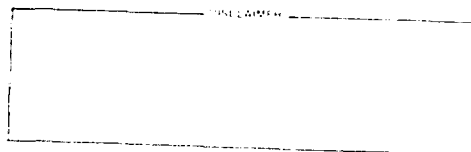
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TITLE: E2 transitions in deformed nuclei and the IBA

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E2 transitions in deformed nuclei and the IBA

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1. Introduction

The application of the Interacting Boson Approximation to the region of well deformed nuclei is a topic of considerable current interest in nuclear structure, as evidenced by the two invited talks already devoted to the subject at this conference. Such interest is hardly surprising, since a detailed study of the model in this region offers the best opportunity to compare and contrast its predictions with those of the best understood of the geometrical concepts, the deformed rotor. Indeed both similarities and crucial differences between the two approaches have been pointed out by Iachello (1981) and Casten and Warner (1981), and many of the latter center around the relative E2 strengths predicted in the IBA for a Hamiltonian which is near the SU(3) limit of the model. It is the purpose of this paper to study in more detail the mechanism which determines the relative E2 strengths in the IBA, and, more specifically, to investigate the structure of the E2 operator necessary to reproduce the empirical $B(E2)$ values in deformed even-even nuclei in the rare earth region.

In the IBA-1 basis, the SU(3) Hamiltonian can be written as (Arima and Iachello, 1978)

$$H = -\kappa Q \cdot Q - \kappa' L \cdot L \quad (1)$$

The resulting level scheme for $N=16$ is shown in Fig. 1. The IBA bands are grouped into different representations of SU(3), states of the same spin being degenerate within a given representation. The corresponding E2 operator is

$$T(E2) = \alpha [(s^{\dagger}d + d^{\dagger}s)^{(2)} - \sqrt{7}/2 (d^{\dagger}d)^{(2)}]; \quad (2)$$

This operator, when applied to the wave functions of the SU(3) Hamiltonian, results in a selection rule which forbids transitions between different SU(3) representations. Thus, in the rigorous SU(3) limit, transitions from, for instance, the first excited $K=0^+$ or 2^+ bands to the ground band are forbidden, while transitions between the $K=0^+$ and 2^+ bands are allowed. This situation is essentially the inverse of the starting point of the geometrical description where the lowest excited $K^{\pi}=0^+(\beta)$ and $2^+(\gamma)$ excitations would be expected to have relatively strong E2 transitions to the ground band, and where $\beta \rightarrow \gamma$ transitions would be forbidden, since they require the destruction of one type of phonon and the creation of another. Empirically, the existence of strong E2 transitions between γ and ground bands is a well established feature of deformed nuclei and hence it is necessary to perturb the rigorous SU(3) limit in the IBA to reproduce this

feature at least. Such a perturbation can, in principle, be introduced via the Hamiltonian or the E2 operator, and we will begin by studying the effects of the latter alone.

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2. Inter-representation B(E2) values in the SU(3) limit of the Hamiltonian

The most general form of the IBA-1 E2 operator can be written as

$$T(E2) = \alpha \{ (s^{\dagger} d^{\dagger} + d^{\dagger} s)^{(2)} + R/\sqrt{5} (d^{\dagger} d)^{(2)} \} \quad (3)$$

Comparison with eq. (2) indicates that the SU(3) form arises when $R = -\sqrt{35}/2 = -2.958$. (The factor of $\sqrt{5}$ has been introduced to ensure compatibility with the appropriate computer program.) In the SU(5) or O(6) limits, R is zero, and hence its range of values can be expected to lie between 0 and -2.958. The reduced matrix elements from the two terms of the operator can be defined as

$$\begin{aligned} M_1 &= (2J_i + 1)^{-1/2} \langle \chi_f J_f || (s^{\dagger} d^{\dagger} + d^{\dagger} s) || \chi_i J_i \rangle \\ M_0 &= (2J_i + 1)^{-1/2} \langle \chi_f J_f || 1/\sqrt{5} (d^{\dagger} d) || \chi_i J_i \rangle \end{aligned} \quad (4)$$

so that

$$B(E2; J_i \rightarrow J_f) = \alpha^2 (M_1 + R M_0)^2 \quad (5)$$

Note that M_0 and M_1 define the $\Delta J = 0$ and ± 1 terms respectively. Thus, the selection rule which results in rigorously zero B(E2) values for transitions between SU(3) representations can arise in two possible ways; either

$$M_1 = 2.958 M_0 \quad (6) \quad \text{or} \quad M_1 = M_0 = 0 \quad (7)$$

In fact, the condition of (7) occurs only when $\Delta \lambda > 4$. In the other cases, the zero B(E2) values arise because of the condition of eq. (6), namely,

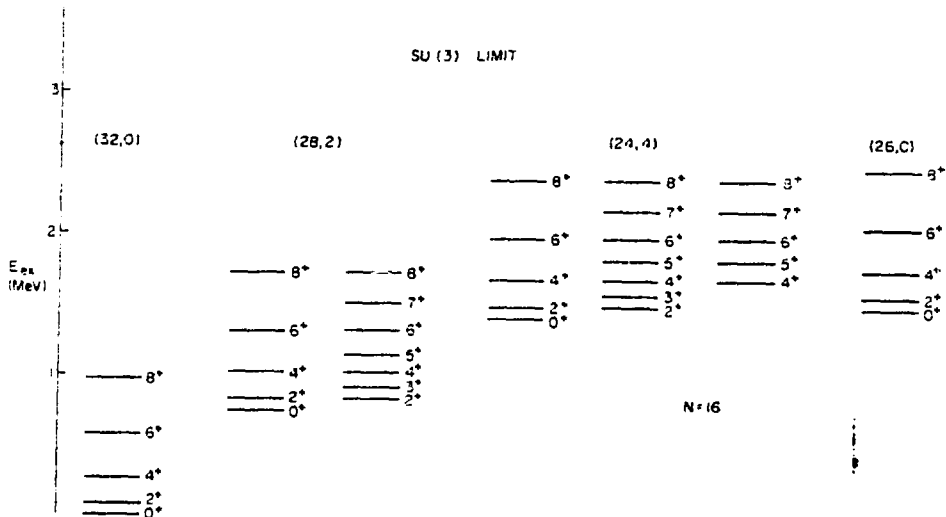


Fig. 1 The first four representations of the SU(3) limit in the IBA.

an exact cancellation of the two contributions to the E2 matrix element for the SU(3) value of R. Then the E2 matrix element can be written as

$$\langle X_f J_f || T(E2) || X_i J_i \rangle = \alpha(2J_i + 1)^{1/2} M_1 (1 + R/2.958) \quad (8)$$

It is now evident from eq. (8) that, in the SU(3) limit of the Hamiltonian, the ratio of any two inter-representation matrix elements with the same initial spin is a constant, independent of R. More specifically

$$\frac{\langle X_f J_f || T(E2) || X_i J_i \rangle}{\langle X'_f J'_f || T(E2) || X'_i J'_i \rangle} = \frac{M_1}{M'_1} \quad (9)$$

Thus, for the SU(3) Hamiltonian, the result of eq. (9) implies that the B(E2) strengths of all inter-representation transitions can be put on a single relative scale. The correspondence between the geometrical and IBA descriptions can then be studied via the E2 strengths predicted in each model, which are frequently a very sensitive structure probe.

The non-zero inter-representation transitions and their relative strengths are shown in fig. 2 for the first four representations of SU(3). Several crucial features emerge from these results. Firstly, considering the (28,2) representation, which contains the analogues of the classical β and γ vibrations, the $\beta \rightarrow \gamma$ strength is seen to be considerably weaker than the $\gamma \rightarrow g$ strength. This point has already been discussed earlier by Iachello (1981) and, coupled with the existence of strong $\gamma \rightarrow \beta$ transitions, which were discussed in detail by Casten and Warner (1981), indicates a

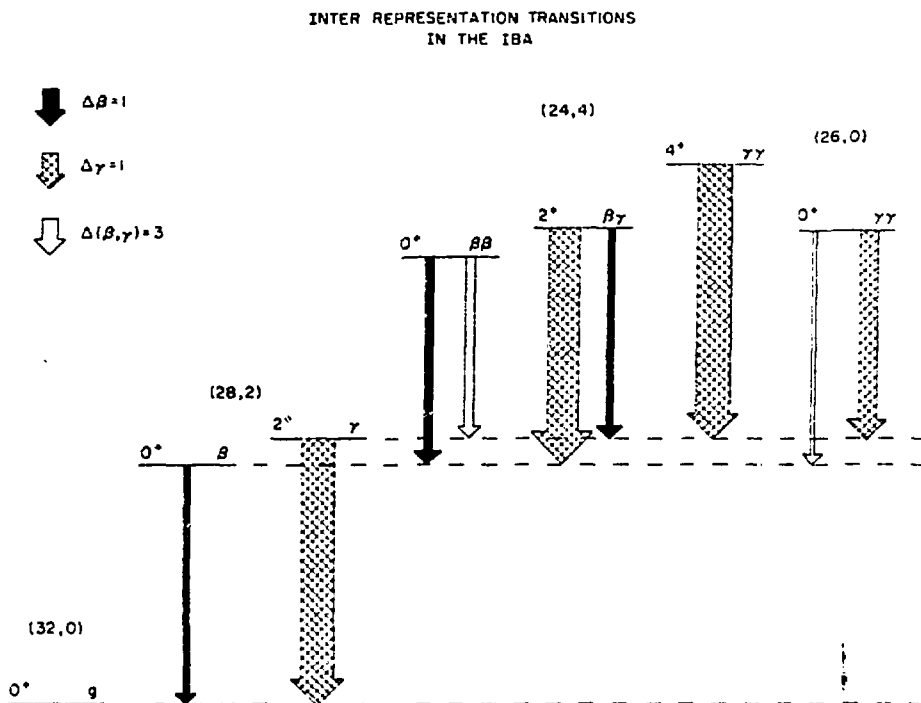


Fig. 2 Relative B(E2) strengths of $2^+ \rightarrow 2^+$ transitions for the SU(3) Hamiltonian. Only bandheads are shown.

markedly different structure for the β band in the IBA relative to that which is normally assumed from the geometrical framework. It is now evident that these features are both inherent in the SU(3) Hamiltonian, being independent of the specific parameterization chosen for the E2 operator, and it will be shown later that they remain in the presence of a perturbed Hamiltonian.

Turning now to the higher SU(3) representations, in the geometrical model, the $\gamma\gamma \rightarrow g$ transitions would be expected to equal the $\gamma \rightarrow g$, while the $\beta\beta \rightarrow \beta$ would be twice the $\beta \rightarrow g$. The relative B(E2) strengths for the SU(3) Hamiltonian are shown in fig. 2 and the identification of the analogues of the $K^\pi=2^+ \beta\gamma$ and $4^+ \gamma\gamma$ excitations is straightforward. However, in considering the remaining two $K^\pi=0^+$ excitations, the candidate for the $\gamma\gamma$ mode has only half its expected strength to the γ band, while both the $\beta\beta \rightarrow \gamma$ and $\gamma\gamma \rightarrow \beta$ branches, forbidden in the geometrical model, are significant in the IBA scheme.

Table 1

Relative B(E2) values from the $K^\pi=0^+$ double β and γ vibrations

Transition	$\beta \rightarrow g$	$\gamma \rightarrow g$	$\beta\beta \rightarrow \beta$	$\beta\beta \rightarrow \gamma$	$\gamma\gamma \rightarrow \gamma$	$\gamma\gamma \rightarrow \beta$
IBA	0.15	1.0	0.20	0.27	0.55	0.10
Geometrical	0.15	1.0	0.30	0.0	1.0	0.0

These results can be understood if the two higher lying $K^\pi=0^+$ bands in the IBA are no longer associated with pure $\beta\beta$ or $\gamma\gamma$ modes in the geometrical basis. The relative strengths of the transitions from these bands and the β and γ bands are given in Table 1. If the IBA band is described as $|\lambda, \mu; K^\pi\rangle$, then

$$|24, 4; 0^+\rangle_{\beta\beta} = a|\beta\beta\rangle + b|\gamma\gamma\rangle \quad (10)$$

$$|26, 0; 0^+\rangle_{\gamma\gamma} = b|\beta\beta\rangle - a|\gamma\gamma\rangle \quad (11)$$

where the subscripts on the left refer to the labeling of fig. 2 and Table 1. Considering first the "forbidden" $\gamma\gamma \rightarrow \beta$ transitions, eq. (11) yields

$$\langle 26, 0; 0^+ |_{\gamma\gamma} |E2| |\beta\rangle = b \langle \beta\beta | |E2| |\beta\rangle \quad (12)$$

It has been assumed that the β and γ bands are equivalent in the IBA and geometrical descriptions. Thus, the values of Table 1 give

$$b^2 = 0.1/0.3 = 0.33; \quad a^2 = 0.67 \quad (13)$$

Then use of these values in eqs. (14) and (15) gives

$$B(E2; \beta\beta' \rightarrow \beta) = 0.20; \quad B(E2; \beta\beta' \rightarrow \gamma) = 0.33; \quad B(E2; \gamma\gamma' \rightarrow \gamma) = 0.67 \quad (14)$$

The primed notation signifies the IBA bands of eqs. (10) and (11). While these numbers do not correspond precisely to those of Table 1, indicating additional small admixtures in the two bands, it is nevertheless clear that these two bands must be considered as predominantly a mixture of the pure $\beta\beta$ and $\gamma\gamma$ configurations, the $(26, 0) 0^+$ band being approximately 67% the $\gamma\gamma$ configuration and 33% $\beta\beta$, with the $(24, 4) 0^+$ band being the inverse.

3. Empirical determination of the IBA E2 operator

In principle, the constants α and R of the E2 operator of eq. (3) can be considered as free parameters in the description of a particular nucleus. Nevertheless it is of interest to investigate whether the global empirical systematics on $B(E2)$ values in the rare earth region can be used to limit the possible range of each. To this end, it is necessary only to look at transitions involving β , γ and ground bands, since $B(E2)$ data for higher bands is scarce. Furthermore, since the parameter R affects only relative $B(E2)$ values, while α affects only the absolute scale, the former can first be studied by setting $\alpha=1$ in eq. (3) and looking at $B(E2)$ ratios. This is done for the SU(3) Hamiltonian in fig. 3, where relative $B(E2)$ values are plotted as a function of R in fig. 3a, and the corresponding $B(E2)$ ratios are shown in fig. 3b. As expected from the discussion of section 2, the ratio of the two inter-representation transitions, $\beta \rightarrow g / \gamma \rightarrow g$, is constant, independent of R . This is not true for the intra-representation transitions, since in this case, the two contributions to the E2 matrix element, M_1 and M_0 , have opposite phase and hence add for negative values of R . However, it can be seen that the change of the intra-representation $B(E2)$ values with R is small, and this feature arises because the M_0 contribution to the matrix element is considerably smaller than the M_1 contribution. Thus the quantities most sensitive to the value of R are the ratios of inter to intra-representation transitions.

In a realistic calculation it is clearly necessary to perturb the IBA Hamiltonian in order to reproduce the observed energies, which would otherwise follow the pattern of fig. 1. In a recent detailed study of ^{168}Er (Warner et al, 1980 and 1981) it was shown that use of an additional term $\kappa''\text{P}\cdot\text{P}$ in the Hamiltonian of eq. (1) was sufficient to reproduce the overall features of the experimental level scheme. Such a perturbation has

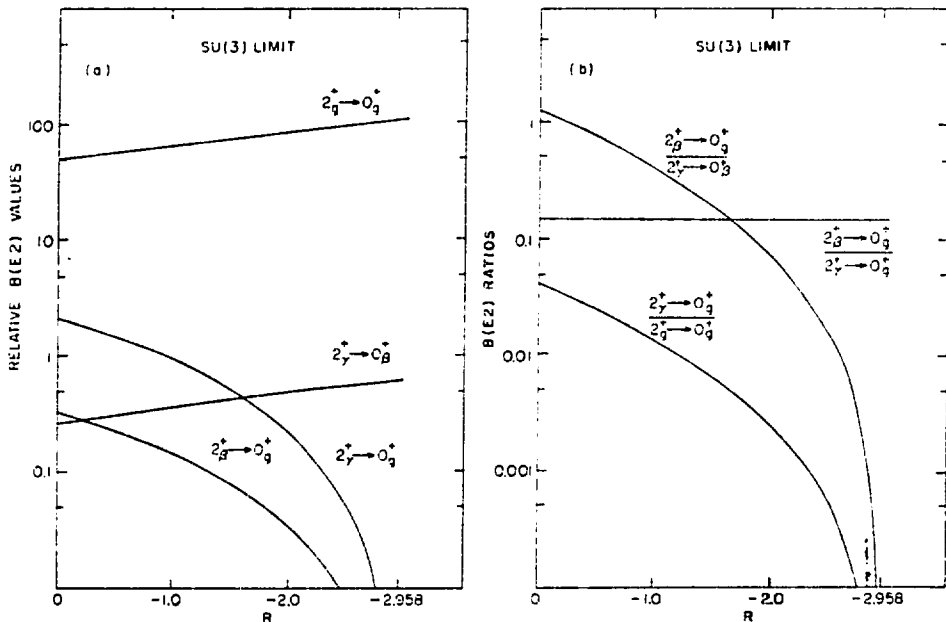


Fig. 3 (a) Relative $B(E2)$ values and (b) $B(E2)$ ratios for the SU(3) Hamiltonian with $N=16$.

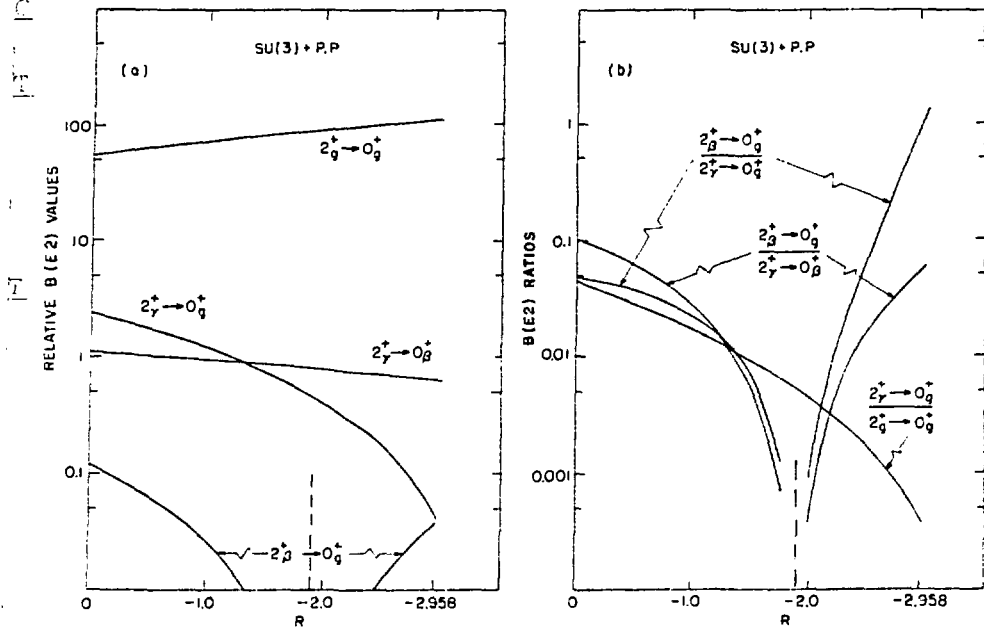


Fig. 4 (a) Relative B(E2) values and (b) B(E2) ratios for a perturbed SU(3) Hamiltonian with $N=16$, $\kappa=0.004$ MeV, $\kappa'=-0.0105$ MeV and $\kappa''=0.015$ MeV.

the effect of pushing the β band above the γ band in energy, as well as breaking the degeneracies in the higher representations. Since the majority of nuclei in the deformed rare earth region exhibit this feature, the current study will be limited to a perturbation of this form.

The relative B(E2) values and corresponding ratios are plotted as a function of R in fig. 4, for a κ'' value of 15 keV. It can be seen in fig. 4a that the $g \rightarrow g$ and $\gamma \rightarrow g$ transitions are relatively little affected by the perturbation to the Hamiltonian, although the latter no longer goes to zero at $R=-2.958$. Not surprisingly, since the P·P term principally affects the energy of the β band, it is the E2 matrix elements involving this band which are also changed the most. More specifically, the values of $B(E2; 2_{\beta}^+ \rightarrow 0_g^+)$ have decreased significantly, the value now going to zero at $R=1.9$, while the overall strength of the $2_{\gamma}^+ \rightarrow 0_{\beta}^+$ transition is increased, and the slope of the line changes, corresponding to a change in the relative phases of M_1 and M_0 . In fig. 4b, the ratio of $\beta \rightarrow g$ to $\gamma \rightarrow g$ transitions is, of course, no longer a constant but follows the behavior of the $\beta \rightarrow g$ transition. A further increase in the size of the P·P term again leaves the $\beta \rightarrow g$ and $\gamma \rightarrow g$ transitions virtually unaffected while decreasing the magnitude of the $\beta \rightarrow \beta$ transitions and increasing the $\gamma \rightarrow \beta$ transitions.

It is now apparent that the ratio $B(E2; 2_{\gamma}^+ \rightarrow 0_g^+) / B(E2; 2_{\beta}^+ \rightarrow 0_g^+)$ is very little affected by the perturbation to the SU(3) Hamiltonian, while varying considerably as a function of R. Empirical values for this ratio are plotted in fig. 5a for the nuclei for which the P·P perturbation can be expected to be appropriate, and it can be seen that the data fall in a relatively narrow band which, in fact, implies a range of -0.5 to -1.2 for the constant R. Most importantly, it is evident that the SU(3) value of $R(-2.958)$ predicts

values of the ratio two orders of magnitude smaller than observed in experiment. Since the strength of transitions involving the β band are rather sensitive to the size of the P-P perturbation in the Hamiltonian, it is not possible to obtain a precise prediction for them from the extracted value of R . Nevertheless, two crucial features can be deduced. Firstly, $\beta \rightarrow g$ transitions are predicted to be considerably weaker than $\gamma \rightarrow g$ transitions, the ratio $\beta \rightarrow g / \gamma \rightarrow g$ varying from ~ 0.15 in the SU(3) limit to progressively smaller values as κ increases. Experimental values of this ratio are plotted in fig. 5b, and it is clear that this prediction is borne out. Secondly, $\gamma \rightarrow \beta$ transitions are predicted to dominate over $\beta \rightarrow g$ transitions. This feature has been dealt with in some detail in the previous talk, and so will not be discussed further here.

Finally, the possible range of values of α in eq. (3) can be determined from the absolute empirical $B(E2; 2^+_g \rightarrow 0^+_g)$ values. This has been done by taking the range of $B(E2; 2^+_g \rightarrow 0^+_g)$ values implied by the deduced R value ($\alpha=1$) for $N=16$ and correcting them for the expected boson number dependence of N^2 . The IBA E2 operator appropriate to deformed nuclei in the rare earth region can then be written as

$$T(E2) \sim (0.145 \pm 0.15) \{ (s^{+\nu}_d + d^+_s)^{(2)} - (0.85 \pm 0.35) / \sqrt{5} (d^{+\nu}_d)^{(2)} \} \quad (15)$$

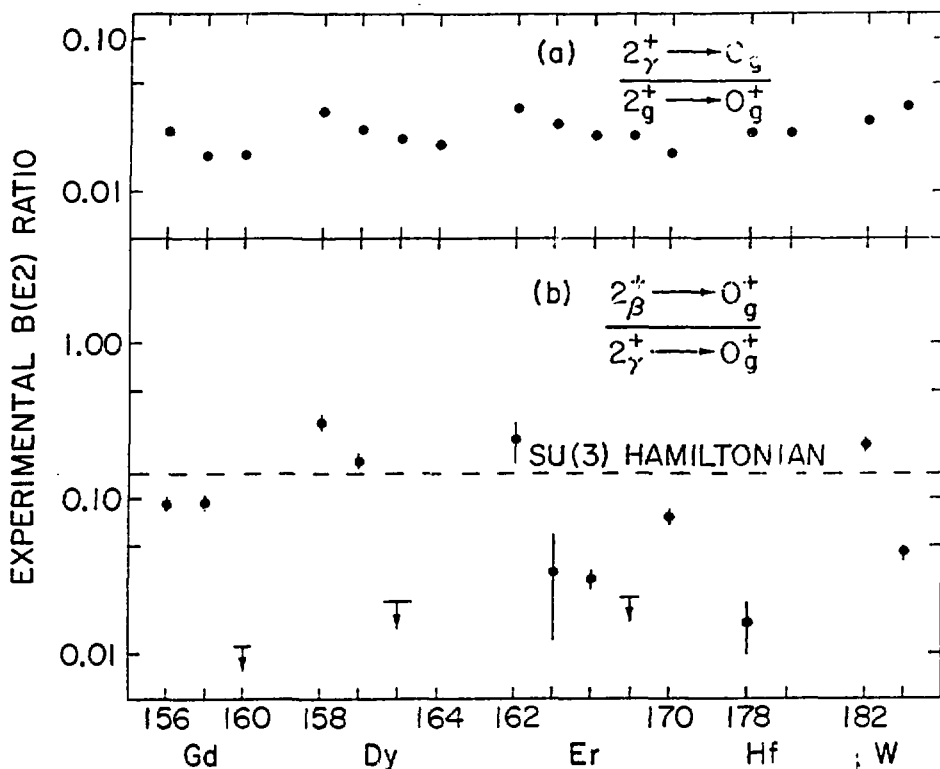


Fig. 5 Empirical values of $B(E2)$ ratios for deformed rare earth nuclei where the assumed β band is above the γ band in energy. The references are listed separately at the end of the text.

(4. Conclusions should begin writing on this line

The recognition that the SU(3) wave functions lead to fixed ratios of $B(E2)$ strengths for inter-representation transitions allows the E2 matrix elements to be used as a sensitive probe in determining the structure of the IBA bands in the geometrical basis. The existence of this tool has been hidden to date by the fact that all such transitions are rigorously forbidden for the SU(3) value of R normally adopted. Thus it has been shown that the higher lying 0^+ excitations in the IBA do not correspond to the pure multiphonon excitations of the geometrical model, but rather to mixtures of them.

The deduced empirical values of the constants α and R in the E2 operator lead automatically to a prediction that $\gamma \rightarrow g$ and $\gamma \rightarrow \beta$ transitions will dominate $\beta \rightarrow g$ transitions in deformed nuclei. Neither of these features can be deduced a priori from the geometrical model, and yet both seem to be verified by the data, although further experimental studies are necessary to confirm the latter throughout the region. Thus, the characteristics of the first excited 0^+ band in the IBA, while corresponding well with those found empirically, are very different from those normally associated with the classical β vibration.

Finally the need for an E2 operator whose structure is very different from that suggested for the SU(3) limit seems to indicate a certain inconsistency in the current application of the IBA in this region, since the quadrupole operator in the Hamiltonian is always assumed to take the SU(3) form. It would be interesting to attempt calculations in which the Hamiltonian and E2 operator were required to use the same form for Q. The constant R of eq. (3) affects the quadrupole term in the Hamiltonian as

$$Q \cdot Q \sim M_2 + 2RM_1 + R^2M_0 \quad (16)$$

where $\alpha=1$ has been assumed, and M_2, M_1, M_0 represent $\Delta n_d = +2, +1, 0$ terms respectively. It can thus be seen that a reduction of R will lead to an increase in the importance of the $\Delta n_d = +2$ term, relative to the others. Since this is the major effect resulting from the introduction of the P-P perturbation, it seems likely that the two effects will be similar.

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