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**BUDAPEST**

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## ABSTRACT

The composite particle production in heavy ion collision is calculated in the framework of a hadrochemical model. A critical comparison is performed between the produced entropy and the observables. It is shown that the observed d/p ratio is not the proper quantity to determine the specific entropy, because this ratio strongly depends on the volume of the deuteron.



## АННОТАЦИЯ

С помощью адрохимической модели вычисляется выход дейтронов в столкновении тяжелых ионов. Проводится критическое сравнение конечной энтропии с измеренными данными. Показывается, что число дейтронов сильно зависит от объема дейтронов, поэтому это число не целесообразно применять для точного определения энтропии.



## KIVONAT

Egy hadrokémiai modell segitségével megvizsgáljuk a nehézionütközésben keltett összetett részecskék számát, és összehasonlitjuk a végállapot entrópiáját a megfigyelhető adatokkal. Megmutatjuk, hogy a megfigyelt deuteronszám az összentrópia mellett erősen függ a deuteron térfogatától is, igy az összentrópia mérésére nem a leghasznosabb.

## 1. INTRODUCTION

In the search for exotic, in particular quark-gluon plasma states, the lepton pairs or the strange particles were pointed out as messengers from the early, hot, compressed state of the firecloud formed in heavy ion collisions [1-5] These considerations were motivated by the fact that at lower temperatures (characteristic to the expansion and break-up phase of the fireball) the processes which could change the number of strange particles or the number and spectral distribution of leptons have very small probability.

On the other hand the frequent interaction between hadrons was believed to destroy any possible signature of earlier states in the later thermal history of the firecloud. It has since been surmised, however, that even these hadrons may carry a signature of an earlier phase transition. Namely the phase transition would show up in the total entropy of the firecloud and would be conserved during the later adiabatic expansion. On the other hand the entropy can be read from the ratio of the number of composite particles to the number of nucleons. In fact, the entropy obtained from the experimental deuteron to proton ratio in heavy ion collisions seemed to show an excess over that calculated assuming a hot, hadronic, gaseous phase [6]. This observation led to lengthy discussions.

Our aim in the present work is to analyse carefully the role of entropy in heavy ion collisions. The relation between the specific entropy and observed spectra is discussed in Section 2. A (more or less) consequent description of the expanding fireball in vacuum is given in Section 3. In Section 4 we use these results in a hadrochemical model. The effect of the finite extension of deuterons is dealt with in the van der Waals approximation in Section 5; our results and conclusions are summarized in Section 6.

Throughout the paper we use $\hbar = k = c = 1$.

## 2. ENTROPY AND THE OBSERVED SPECTRA

It was pointed out in Ref. [7] that one can calculate the entropy of the fireball formed in heavy ion collisions from the observed deuteron to proton ratio ($R_{dp}$). On the other hand the authors of that work estimated the entropy

of the fireball at the beginning of the expansion (supposed by them as being adiabatic). Then comparing these two values they concluded that the experiments show an entropy excess during the adiabatic expansion. To avoid this contradiction they suggested some possible reason for this excess, e.g. the phase transition into the quark-gluon plasma or pion condensation, etc.

Because of the importance of such a conclusion, let us look again at these considerations. In Ref. [8] the nonrelativistic thermodynamical treatment of a one component ideal gas was applied to the fireball and it was found that the specific entropy depends only on the ratio $R_{dp}$

$$s_p/n_p = s_N/n_N = 3.95 - \ln R_{dp} \quad . \tag{2.1}$$

But even at this low temperature limit, when the nonrelativistic treatment may be acceptable, the $s_p/n_p$ ratio is not a conserved quantity, which can be compared to its initial value. Only the total entropy of the expanding gas mixture and the total baryon number are conserved during the adiabatic expansion. Denoting by V the actual volume of the fireball one can construct the following conserved quantity:

$$S/N_b = sV/n_bV = \frac{s_N + s_d}{n_N + 2n_d} = 3.95 - \ln R_{dp} - 1.25\ R_{dp}/(1+R_{dp}) \tag{2.2}$$

which is somewhat lower - at the same observed $R_{dp}$ ratio - than the one given by Eq.(2.1) and in Ref. [8].

Although Eq.(2.2) also gives some entropy excess, one has to take into account - before drawing further conclusions - the entropy produced by the chemical equilibration process between nucleons and deuterons. If, after this correction it remains some entropy excess, one may investigate for special sources of that. Some other mechanisms have been considered recently. The replacement of classical statistics by quantum statistics (Ref. [9]) seems not to lower the calculated ratio $R_{dp}$. Stöcker (Ref. [10]) proposed that the number of protons may increase after the break-up of the system by the decay of unstable particles or nuclei. One can also doubt the applicability of the nonrelativistic gas approximation and the assumption of point-like deuterons. To summarize, we conclude that to predict the entropy related to the observed deuteron to proton ratio one must follow up the process of chemical equilibration in a rather relativistic treatment and must also investigate the effects of the finite extension of the deuteron even in the case of the dilute gas. We try to this in the following sections.

## 3. THE QUASIADIABATIC MODEL

For the description of the HIC the adiabatically expanding gas model has been used in several papers [11-13].

The difference between these models and the present treatment is the replacement of the adiabacity hypothesis by a more precise one, which is valid for an arbitrary mixture of gases even if its components transform into each other. The evolution of such a multicomponent perfect fluid (i.e. the viscosity and the heat conductivity are neglected) will be called a quasiadiabatic one. In such systems entropy change may occur because of the chemical transmutations. In the following we shall derive the equations describing this system.

Let us take an infinitely small volume cell of an expanding fireball, which moves away with the four-velocity $u^i$, and fix our coordinate system to this. We can speak about thermodynamics in this local system after defining internal energy, pressure, particle number densities of the different components of the gas, temperature, chemical potentials; concisely, all the familiar thermodynamical quantities. The internal energy (e) and the pressure (p) of these cells depend on the number densities ($n_a$) of components and the local temperature (T) or its inverse ($\beta$). For ideal gases they can be constructed as the sum of the quantities related to the components:

$$e = \sum_a n_a \cdot e^a \quad , \qquad p = T\sum_a n_a \quad . \tag{3.1}$$

The energy density of a component in the case of the relativistic Boltzmann distribution is given as:

$$e^a = m^a R(m^a/T) \quad , \quad \text{where} \quad R(x) = \frac{3}{x} + \frac{K_1(x)}{K_2(x)} \tag{3.2}$$

and $K_n(x)$ denotes the n-th order modified Bessel function of the imaginary argument. Knowing the local parameters $\{n_a, T\}$ all the other thermodynamical quantities can be expressed, and we can write the energy-momentum tensor of the perfect fluid in the form:

$$T^{ik} = (e + p)u^i u^k - pg^{ik} \quad . \tag{3.3}$$

Here $g^{ik}$ is the metric tensor ($g_{oo} = 1$, $g_{11} = g_{22} = g_{33} = -1$).

We use only the energy-momentum conservation to describe the hydrodynamical features of the expansion in the vacuum. It is given in the form:

$$\partial_i T^{ik} = 0 \quad . \tag{3.4}$$

Supposing a spherically symmetric explosion of the fireball we need only two scalar equations. Let the first be the projection of $T^{ik}$ onto the velocity-field, $u_k$:

$$u_k \partial_i T^{ik} = 0 \quad . \tag{3.5}$$

For the second we shall use the energy conservation, i.e. the timelike component of Eq.(3.4):

$$\partial_i T^{io} = 0 \quad . \tag{3.6}$$

Besides these we have further equations describing chemical processes between the component gases in the familiar way:

$$\partial_i (n_a u^i) = \Psi_a \quad . \tag{3.7}$$

The source term, $\Psi_a$ depends on the temperature and the number densities of each or almost each component. In Section 4 we will describe the "chemical reactions" - decays, collisions, etc. - included in our model, but here we need to know only the fact that the number density of the $a$-th component has a source. The set of Eqs(3.5-3.7) describes the expansion.

To recognize the physical meaning of Eq.(3.5) we transform it to obtain a form of a total four-divergency + another term. After that we substitute Eq.(3.3) into it. We get:

$$u_k \partial_i T^{ik} = \partial_i (u_k T^{ik}) - T^{ik} \partial_i u_k = \partial_i (eu^i) + p \partial_i u^i = 0 \quad . \tag{3.8}$$

If we now apply the First Law of Thermodynamics for the local infinitely small volume cell of the fireball we get:

$$\partial_i (eu^i) = T \partial_i (su^i) - p(\partial_i u^i) + \mu^a \partial_i (n_a u^i) \quad .$$

Replacing this in Eq.(3.8) we get for the entropy the relation:

$$\partial_i (su^i) = -\beta \mu^a \partial_i (n_a u^i) \neq 0 \quad . \tag{3.9}$$

The physical meaning of this equation is to take into account the entropy produced by the chemical processes, i.e. there are possible exothermal and endothermal transmutations. Approximating the chemical equilibrium state the entropy increases according to the Eq.(3.9).

From (3.7) we can express the source of the entropy by the chemical sources and we get the following set of equations to describe the hadrochemistry of an expanding sphere:

$$\partial_i (su^i) = -\beta \mu^a \Psi_a \tag{3.9}$$

$$\partial_i T^{io} = 0 \tag{3.6}$$

$$\partial_i (n_a u^i) = \Psi_a \tag{3.7}$$

We would like to approximate the description of the evolution of the fireball by ordinary differential equations. For this purpose we shall average the spatial dependence of the variables. This averaging can be done in a given coordinate system: we chose the C.M. system of the fireball. In this system we suppose $n_a$ and $\beta$ as being homogeneous.

We need an assumption for the flow velocity-field, too. We have chosen the form of the spatial dependence of it as follows Ref. [11]:

$$\gamma(u)\vec{u} = \vec{r}\cdot\dot{R}(t)/R(t) \quad , \tag{3.10}$$

where $\gamma(u) = (1-u^2)^{-1/2}$ is the Lorentz factor, $R(t)$ is the radius of the fireball at time $t$, which is measured at the centre of the fireball and $\dot{R}(t)$ is its time-derivative.

Using these assumptions we get the following set of averaged equations:

$$\frac{1}{V}\frac{d}{dt}(Vs\langle\gamma\rangle) = -\beta\mu^a\Psi_a \tag{3.11}$$

$$\frac{1}{V}\frac{d}{dt}[V(e\langle\gamma^2\rangle + p\langle\gamma^2u^2\rangle)] = 0 \tag{3.12}$$

$$\frac{1}{V}\frac{d}{dt}[Vn_a\langle\gamma\rangle] = \Psi_a \tag{3.13}$$

Here $V$ denotes the actual - time-dependent - volume of the fireball, and the brackets $\langle\rangle$ mean a spatial averaging. In the present model we supposed $n_a$-s and $\beta$ as being homogeneous, so we cannot avoid the spatial averaging of the expressions depending on the velocity-field. In the following we will call them "kinematic factors". They are:

$$\langle\gamma\rangle \; ; \; \langle\gamma^2\rangle \; ; \; \langle\gamma^2u^2\rangle = \langle\gamma^2\rangle - 1 \quad .$$

Using (3.10) we get:

$$\langle\gamma\rangle = \frac{3}{\dot{R}^3}\int_0^{\dot{R}} x^2\sqrt{1+x^2}\,dx = \frac{3}{8\dot{R}^3}[\dot{R}\sqrt{1+\dot{R}^2}(1+2\dot{R}^2) - \mathrm{Arsh}\dot{R}] \tag{3.14}$$

$$\langle\gamma^2\rangle = \frac{3}{\dot{R}^3}\int_0^{\dot{R}} x^2(1+x^2)\,dx = 1 + \frac{3}{5}\dot{R}^2 \tag{3.15}$$

Finally, from trivial geometry we have the relation:

$$\frac{\dot{V}}{V} = 3\frac{\dot{R}}{R} \quad . \tag{3.16}$$

The initial stage of Eqs(3.10-3.16) is the total overlap of the colliding nuclei, when $V = V_o = r_oA^{1/3}$, $\dot{R} = 0$, and $n_a(0)$, $\beta(0)$ were calculated in the familiar way as the final state of the ignition phase described in Ref. [13].

To predict the experimentally observed inclusive cross-section data we need to choose a time point (break up time) when the expansion and hadrochemistry "freeze out", namely the chemical composition of the mixture and momentum distribution of the components do not change from this moment till the detection of particles.

There are several criteria for the "freeze out" of fireball models [11, 14]. In the present calculation we use the following one: if the average collision number per particle in the total volume during a characteristic cooling time is less than one, then the hydrodynamic description must loose its validity. Here we only check for the self-consistency of application of the hydrodynamic description in the present model: we calculate the averaged number of collisions per baryon simultaneously with the prediction of the inclusive p, d, π spectra.

We define the characteristic cooling time by the change of the thermal energy:

$$\frac{1}{\tau} = \frac{|\partial_i (e_{th} u^i)|}{e_{th}} \quad , \tag{3.17}$$

where $e_{th} = e - \sum_a m^a n_a$. The averaged number of collisions per particle is calculated supposing collisions between nucleons as being independent:

$$a = \tau \langle \sigma_{pp}^{tot} v_{rel} \rangle n_{baryon} \quad . \tag{3.18}$$

Here the brackets <> mean the average over the momentum space supposing relativistic Boltzmann distribution. (See Section 4.)

## 4. HADROCHEMISTRY

In this Section we concentrate on the right hand side of Eq.(3.7), viz. on the $\Psi_a$ chemical source term.

The different processes of chemical reactions are grouped into the following types:

| | | | |
|---|---|---|---|
| 1. | "one to two" | $A \rightleftharpoons B + C$ | e.g. $\Delta \rightleftharpoons N$ |
| 2. | "two to two" | $A + B \rightleftharpoons C + D$ | e.g. $NN \rightleftharpoons D$ |
| 3. | "two to three" | $A + B \rightleftharpoons A + C + D$ | e.g. $ND \rightleftharpoons NNN$ |

The first type, the well-known decay, changes the number density of the a-th component as:

$$\dot{n}_a = -\Gamma_a n_a \tag{4.1}$$

where $\Gamma_a$ is the width of the decaying resonance.

In the co-moving frame of a fluid cell the source term gives the change of $n_a$ in the proper time, so $\Psi_a(1) = -\Gamma_a n_a$. But since the energy and momentum distribution of the decaying particles is Boltzmannian, the average can be written as:

$$\Psi_a(1) = -\langle\Gamma_a\rangle n_a = -\Gamma_a n_a \langle 1/\gamma_{th}\rangle = -\Gamma_a n_a \frac{K_1(\beta m_a)}{K_2(\beta m_a)} \quad . \qquad (4.2)$$

Here $K_n(x)$ denotes the n-th order Bessel function of the imaginary argument and $\gamma_{th}$ is the Lorentz factor corresponding to the thermal motion.

The second type is the two-body collision. If each colliding set of particles has Boltzmann distribution in the momentum space, then the averaged rate factor is:

$$\langle\sigma v_{rel}\rangle = \frac{\beta \int_{(m_3+m_4)^2}^{\infty} \sigma(s)\lambda(s)K_1(\beta\sqrt{s})\frac{1}{2\sqrt{s}}ds}{4m_1^2m_2^2K_2(\beta m_1)K_2(\beta m_2)} \quad , \qquad (4.3)$$

where $\lambda(s) = (s-m_1^2-m_1^2)^2 - 4m_1^2m_2^2$; $s = (p_1+p_2)^2$ represents the invariant C.M. energy squared; $m_1 \ldots m_4$ are the rest masses of particles A...D; $\sigma(s)$ is the cross-section of the process investigated and $\beta = 1/T$ is the inverse temperature. In the case of the process A + B $\rightleftharpoons$ C + B the chemical source term is proportional to the rate factor and the number densities of the colliding particles:

$$\Psi_A(2) = -\langle\sigma_{AB}v_{rel}\rangle n_A n_B \quad . \qquad (4.4)$$

The third type of hadrochemical processes is very similar to the second one: only the difference is shown by the reverse process, which is a three-body collision. But we generally take into account the reverse processes by means of the equilibrium ratio. For example, for A + B $\rightleftharpoons$ A + C + D the total change of B is:

$$\Psi_B(3) = -\langle\sigma_{AB}v_{rel}\rangle n_A(n_B - \rho_B^{CD} n_C n_D) \quad . \qquad (4.5)$$

Similar expressions are written for the reverse processes of first and second type:

$$\Psi_B(1) = \langle\Gamma_A\rangle(n_A - \rho n_B n_C) \quad , \qquad (4.6)$$

$$\Psi_B(2) = -\langle\sigma_{AB}v_{rel}\rangle(n_A n_B - \rho n_C n_D) \quad . \qquad (4.7)$$

The equilibrium ratios are determined by the following restriction: in chemical equilibrium the source term has to be equal to zero. On the other hand we know from the Boltzmann distribution that $n_a = e^{\beta\mu^a}Q(\beta,m_a)$, where $Q(\beta,m_a) = \frac{d_a}{2\pi^2\beta}m_a^2K_2(\beta m_a)$ is the familiar canonical partition function, and $d_a$ denotes the spin and isospin degeneracy factor of the particle of type a. Hence the equilibrium ratio, $\rho$, is given as:

$$\rho = \left(\frac{n_A n_B}{n_C n_D}\right)_{eq} = \frac{e^{-\beta(\mu_A+\mu_B)} Q_A Q_B}{e^{-\beta(\mu_C+\mu_D)} Q_C Q_D} = \frac{Q_A Q_B}{Q_C Q_D} \quad . \tag{4.8}$$

One can get the equilibrium ratios for each type of process in a similar way.

In a central heavy ion collision a thermalization process goes on. This can be thought of as a sort of "chemical reaction" when the stages of this "reaction chain" are the more and more Maxwell-Boltzmann like distributed particles. In the present model we describe only two stages of this thermalization process: the original component having sharp energy and momentum distribution (named "cold" nucleons, $N_o$), and the Boltzmannian components ("hot" particles, N, Δ, π...). It has been pointed out that the Boltzmann-distribution is almost reached after 2-3 collisions per particle [15]. Now we model the thermalization as a one step process: $N_o + N_o \to A + B$ or $N + X \to N + X$.

The source terms related to these processes contain the rate factor for the elastic nucleon-nucleon scattering averaged over the momentum distribution of the cold nucleons. For $N_o + N_o$ collision it is trivial:

$$\langle \sigma v \rangle_{oo} = \sigma(s_o)\frac{\lambda^{1/2}(s_o)}{2E_o^2} = \sigma(m_o \sinh\zeta_o)\cdot 2\tanh\frac{\zeta_o}{2} \quad . \tag{4.9}$$

For an $N_o + X$ collision the rate factor is:

$$\langle \sigma v \rangle_{o1} = \frac{\int_{(m_3+m_4)^2}^{\infty} \sigma(s)\lambda^{\frac{1}{2}}(s)\left[e^{-\beta\left[\frac{E_o h^{\frac{1}{2}}(s)+P_o\lambda^{\frac{1}{2}}(s)}{2m_o^2}\right]} - e^{-\beta\left[\frac{E_o h^{\frac{1}{2}}(s)-P_o\lambda^{\frac{1}{2}}(s)}{2m_o^2}\right]}\right] ds}{8m_1^2 P_o E_o K_2(\beta m_1)} \tag{4.10}$$

In these equations $\zeta_o$ is the rapidity of the cold nucleon; $E_o$ and $P_o$ are its energy and momentum respectively; $s = (p_o+p_1)^2$ is the invariant C.M. energy squared, $s_o$ is the same for cold nucleons, the functions h(s) and λ(s) are given as

$$h(s) = (s - m_o^2 - m_1^2)^2 \quad , \tag{4.11}$$

$$\lambda(s) = h(s) - 4m_o^2 m_1^2 \quad . \tag{4.12}$$

Finally, we list the hadrochemical processes included in the present model:

| | |
|---|---|
| First type | $\Lambda \leftrightarrow N\pi$, $\rho \leftrightarrow \pi\pi$ |
| Second type | $N_oN_o \to NN$ or $N\Delta$, $N_oN \to N\Delta$, $N_oX \to NX$ (X = N, D, π, Λ), $NN \leftrightarrow N\Delta$, $NN \leftrightarrow D\pi$ |

Third type $N_0D \rightarrow NNN$, $ND \rightarrow NNN$

The cross-section for these processes are taken from experimental data (Ref. [16]).

## 5. THE EXTENDED DEUTERON

To investigate the effect of the extended size of the deuteron on the observable quantities, especially the entropy related to the ratio $R_{dp}$ we suggest that the van der Waals approximation for the description of the fireball be applied. This treatment does not give the real equation of state but gives a more precise one than the ideal gas approximation. In fact, it will be shown in this section that even in the case of dilute gases the finite extension of deuteron causes appreciable change in the deuteron to proton ratio, $R_{dp}$.

For simplicity here we regard a two component gas as one that includes deuterons and nucleons. In the van der Waals approximation one should correct the internal energy because of the interactions and the pressure because of both the finite extensions of the particles and the interactions between them. The pressure p and the internal energy e of this system can be written as follows:

$$p(n_a, T) = \frac{p_{ideal}}{1 - \sum_a n_a V^a} + \pi(n_a) \quad , \tag{5.1}$$

$$e(n_a, T) = e_{ideal} + W(n_a) \quad . \tag{5.2}$$

The index "ideal" marks the quantities related to the ideal gas and $V^a$ is the volume occupied by a particle of the a-th component. From simple thermodynamic relations we get for the pressure correction:

$$\pi(n_a) = \sum_a n_a \frac{\partial W}{\partial n_a} - W(n_a) \quad .$$

Hence the free energy density f and the chemical potentials $\mu^a$ are given as:

$$f = f_{ideal} - nT\ln(1 - \sum n_a V^a) + W(n_a) \quad , \tag{5.3}$$

$$\mu^a = \mu^a{}_{ideal} - T\ln(1 - \sum n_r V^r) + \frac{nTV^a}{1 - \sum n_r V^r} + \frac{\partial W}{\partial n_a} \quad , \tag{5.4}$$

where $n = \sum n_a$ denotes the total number density.

If we are in the dilute gas limit, which is defined by the relation:

$$\sum n_r V^r << 1 \tag{5.5}$$

then the second term in expression (5.4) is negligible. Besides this the fourth term of (5.4) does not influence the chemical equilibrium if $w = w(n_o)$, i.e. if there are only two-body interactions between the particles. Nevertheless, the third term contains $n \cdot V^a$, which is not negligible even in the dilute gas limit because the fireball may contain many nucleons and appreciably extended - although few - deuterons.

In the chemical equilibrium between nucleons and deuterons

$$\mu_D = 2\mu_N \quad . \tag{5.6}$$

If we now use approximation (5.5) and insert the form (5.4) of the chemical potentials into Eq.(5.6) we get the following:

$$\frac{n_D}{n_N^2} = \left(\frac{n_D}{n_N^2}\right)_{ideal} \cdot e^{-(V^D-2V^N)(n_N+n_D)} \quad . \tag{5.7}$$

This means that even in the dilute gas limit, the extended size of the deuteron may cause a relatively strong decrease in the deuteron to proton ratio $R_{dp}$, and it leads - according to Eq.(2.2) - to some entropy "excess". The characteristic specific volume $V^D$ is not necessarily the geometrical volume of the deuteron but, however, it is expected to be in the same order of magnitude. So we calculated $V^D$ as

$$V^D = \frac{4\pi}{3} \langle r_D^2 \rangle^{3/2} = 20 \text{ fm}^3 \quad , \tag{5.7a}$$

where $\langle r_D^2 \rangle$ was obtained from the wave function of Ref. [17] which contained a repulsive core. The difference $\delta V = V^D - 2V^N$ occurring in Eq.(5.7) will have the value $\delta V = 10 \text{ fm}^3$ using $V^N = \frac{4\pi}{3} r_o^3 = 5 \text{ fm}^3$.

One can observe that a relatively small change in the volume occupied by a deuteron causes a large change in the equilibrium ratio of deuterons to protons.

The specific entropy of this two-component van der Waals gas (in the limit defined by Eq.(5.5)) is given as follows:

$$s_B/n_B = 3.95 - \ln R_{dp} - 1.25\, R_{dp}/(1+R_{dp}) - (n_N+n_D)(V^D-V^N) \tag{5.8}$$

## 6. RESULTS AND CONCLUSIONS

In this paper we calculate such symmetric collisions where the total baryon number is 80 or 40, mostly at 0.8 GeV/nucleon bombarding energy. This choice describes a situation very similar to the experiments on KCl+Ar, nevertheless slight differences in the final chemical composition can be expected because there is a 6 % neutron excess in the experimental situation.

Regarding the hadrochemical processes, *Fig. 1* shows that using $\delta V = 10$ fm$^3$ and E/A = 0.8 GeV, the final composition is as follows:

$$N_o = 28, \quad N = 40, \quad D = 6, \quad \pi = 8 \quad .$$

The processes can also be followed in *Fig. 1*. The break on the N curve is an artifact from a simplification in the model, namely that the $N_o + N_o \rightarrow N + N$ process is neglected after the total overlap. Nevertheless, one can conclude that there remains a cold subsystem. The equilibration of the deuteron number is very rapid, in accordance with Kapusta's calculation, while the pion and delta numbers have monotonous trends because the equilibrium ratios in the $\Delta \rightleftharpoons N\pi$ process depend on the density. Observe, however, that the sum of the number of deltas and pions, i.e. the number of pions which will be detected soon reaches a constant value. In view of this, the prediction of the measurable pion to nucleon ratio is largely insensitive to the choice of the break up time. With increasing bombarding energy the final pion number increases and the deuteron number decreases, because of the higher temperature.

After the total overlap a one-fluid model was used. The evolution of some hydrodynamic and thermodynamic parameters is displayed in *Fig. 2*. Obviously, the thermal equilibrium, cannot, however, be valid when the cooling is too fast compared with the equilibrating processes. The quantity "a" defined by Eq.(3.18) shows the relation between the characteristic time of these processes. Obviously the critical value of "a" is about 1, and in this model we assume that for $a < a_{cr}$ the particles are free. Nevertheless, the a(t) curve is very flat at a = 1, thus the breakup time is not well-defined. For this reason we regard $a_{cr}$ as a free parameter to be fitted to the slope of the proton spectrum. However, *Fig. 2* shows that the breakup density is about 1/4 normal nuclear density, $T_{br} \sim 40$ MeV and the velocity of the surface is about c/2. The entropy produced by the chemical processes is about 1 %, definitely less than the neglected contribution of the viscosity.

The calculated p, $\pi$ and d spectra are shown in *Fig. 3*. It is expected that the slopes should be different because of the different masses. The breakup time has been chosen within the breakup interval in such a way that the high energy proton slope be correct (79 MeV). Then even the initial low energy part of the proton spectrum is correctly given, and similarly the $\pi$ slope is near to the experimental value (64 MeV instead of 66 MeV). The calculated deuteron spectrum definitely differs from the exponential one in the displayed energy region due to the larger rest mass of the deuteron.

In calculating the spectra we used $\delta V = 0$, i.e. we neglected the volume of deuteron. The value of $\delta V$ causes only secondary effects in the shapes of the spectra but, of course, it is very important for the final number of deuterons. Unfortunately, as we emphasized in Section 5, until we know the correct equation of state of the interacting n-d system, one cannot determine the actual value of $\delta V$. Comparing the calculated and experimental ratios one can see that $\delta V \gtrsim 10$ fm$^3$ explains the detected deuteron number up to E/A $\lesssim$

$\lesssim$ 1.4 GeV/nucleon (*Fig. 4*). This result clearly demonstrates that the deuteron deficiency, alternatively interpreted as entropy excess, is not an evidence for phase transition or strong attractive forces. In *Fig. 5* we display the entropy calculated from the experimental d/p ratios with the help of expressions for ideal one-component (Eq.(2.1)), ideal two-component (Eq. (2.2)), and two-component van der Waals (Eq.(5.3)) gases. The predictions of Siemens and Kapusta [7] for the entropy is also shown in the figure.

We can conclude that the entropy calculated from the experimental deuteron to proton ratio is within the normal hadronic matter range predicted by Siemens and Kapusta [7], if we take into account the finite volume of the deuteron. Therefore, the present experimental data do not force the assumption of the formation of some exotic state. Moreover, the chemical composition of the reaction products in a central heavy ion collision can be understood within the framework of the present hadrochemical model for moderate energies E $\lesssim$ 1.5 GeV/nucleon. At the same time it is important to realize that - due to the otherwise not well measurable volume excess $\delta V$ of deuterons - the deuteron number is not the proper quantity for "measuring" the entropy produced in the collision, i.e. for finding a piece of evidence of exotic states of matter. The more compact composite particles such as $He^3$, t and $\alpha$ may be more appropriate for that purpose.

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*Fig. 1a*

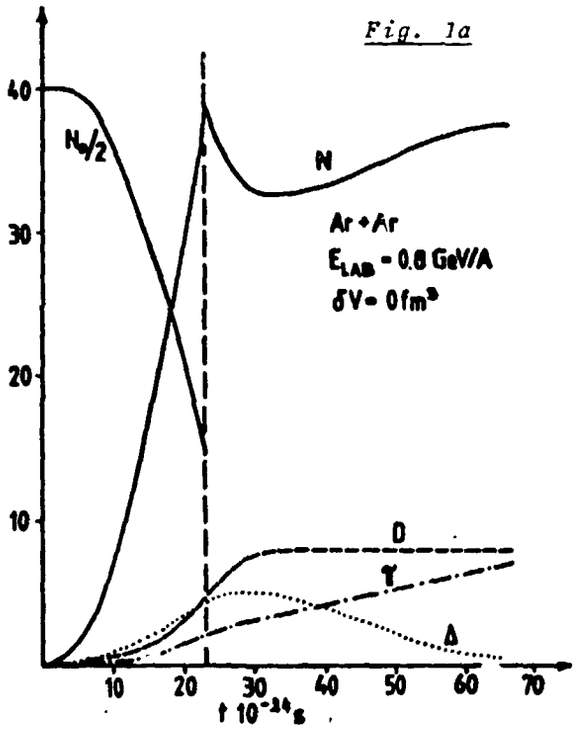


*Fig. 1b*

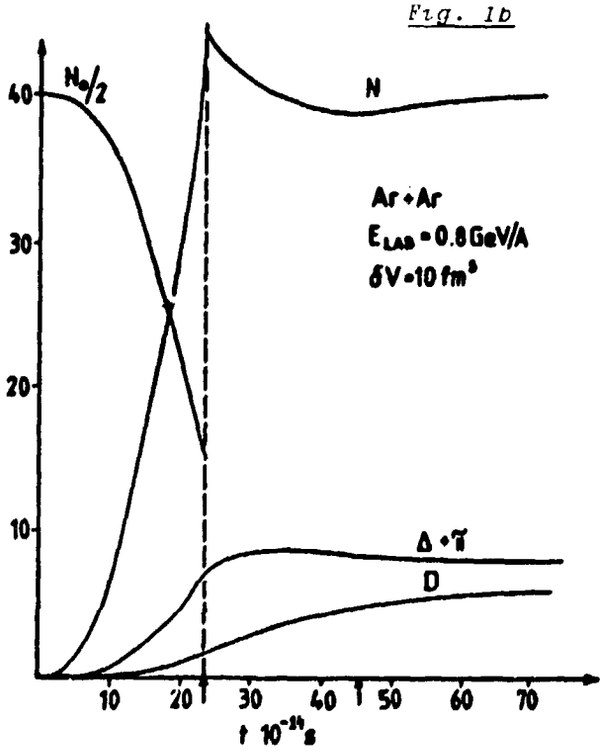


*Fig. 1*

*Number of different kind of particles is shown as a function of time for the reaction Ar+Ar at $E_{LAB}$ = 800 MeV/A. Figures 1a and 1b were obtained without and with van der Waals correction (δV = 10 fm³). The dashed vertical line marks the total overlap of the colliding heavy ions. Observe that the Δ+π number, i.e. the number of the detectable pions very soon reaches the final value. The deuteron number on Fig. 1b is monotonously increasing because of the van der Waals correction.*

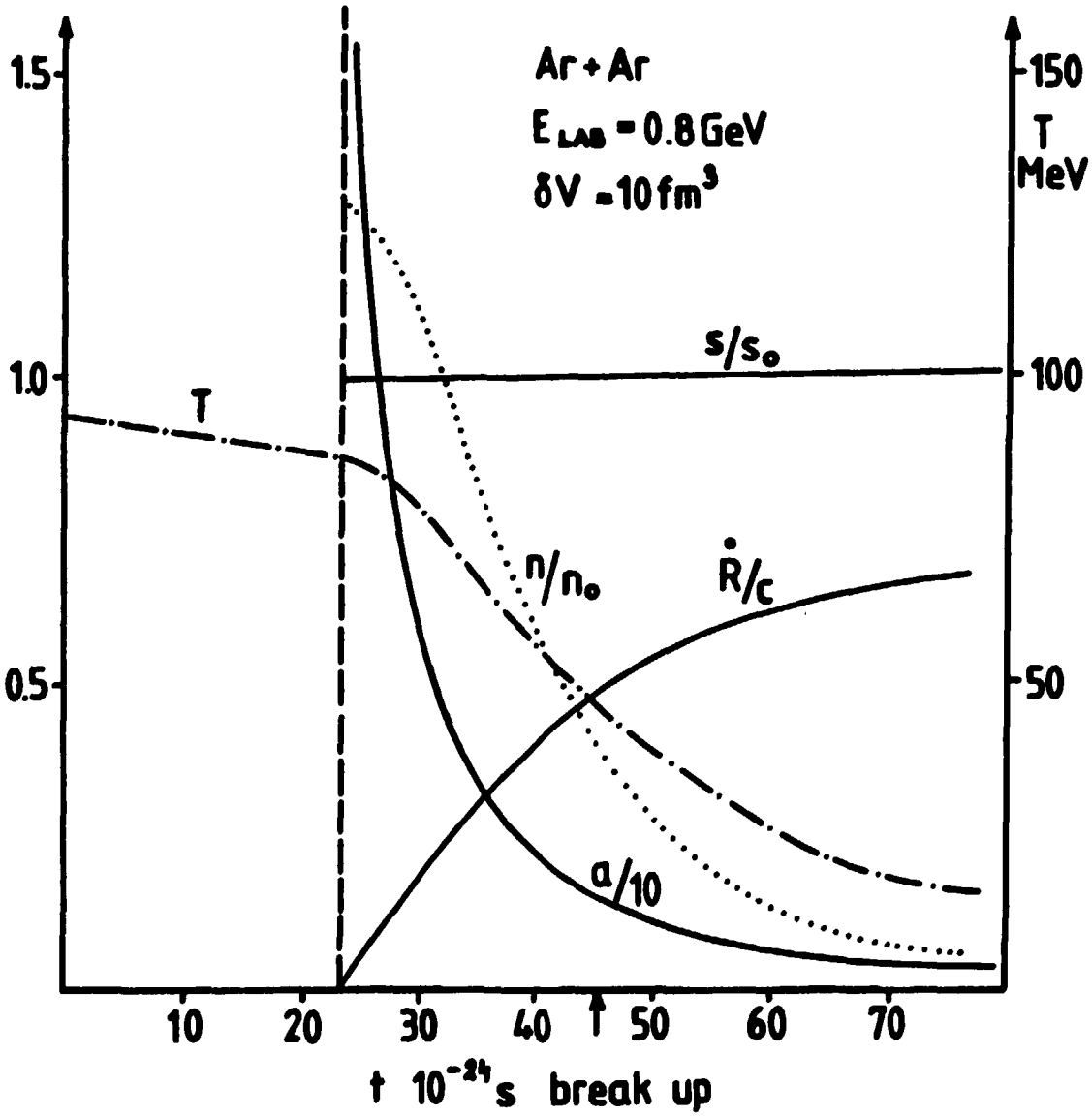


*Fig. 2*

*Thermodynamical quantities versus time calculated with van der Waals correction ($\delta V$ = 10 fm$^3$) for the heavy ion collision Ar+Ar at $E_{LAB}$ = 800 MeV/A. S denotes the total entropy, $S_0$ is its value at the total overlap; T and n are the temperature and number density, respectively; $n_0$ is the normal nuclear matter density. $\dot{R}$ is the expansion velocity of the surface; a is the average collision number per particle during a characteristic cooling time (Eq. (3.18)). The broken line marks the total overlap. The arrow denotes the break up time at which the effective proton temperature fits the experimental one. This corresponds to $a_{eff}$ = 1.6.*

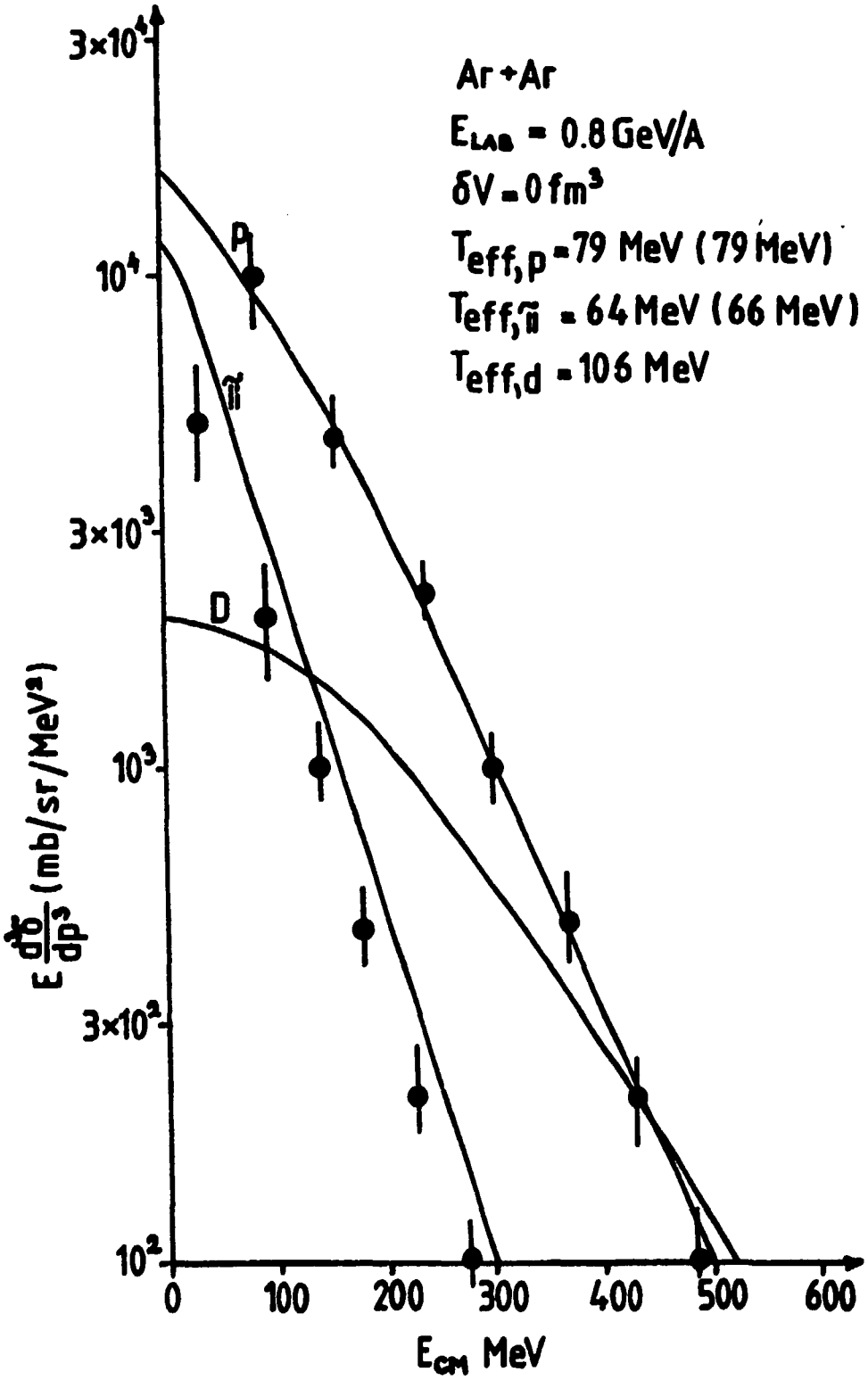


*Fig. 3*

*Invariant p, π and d cross-sections calculated for the Ar+Ar reaction at $E_{LAB}$ = 800 MeV/A. The effective temperatures correspond to the slope factors. In parenthesis the experimental values (Ref. [18]) are given.*

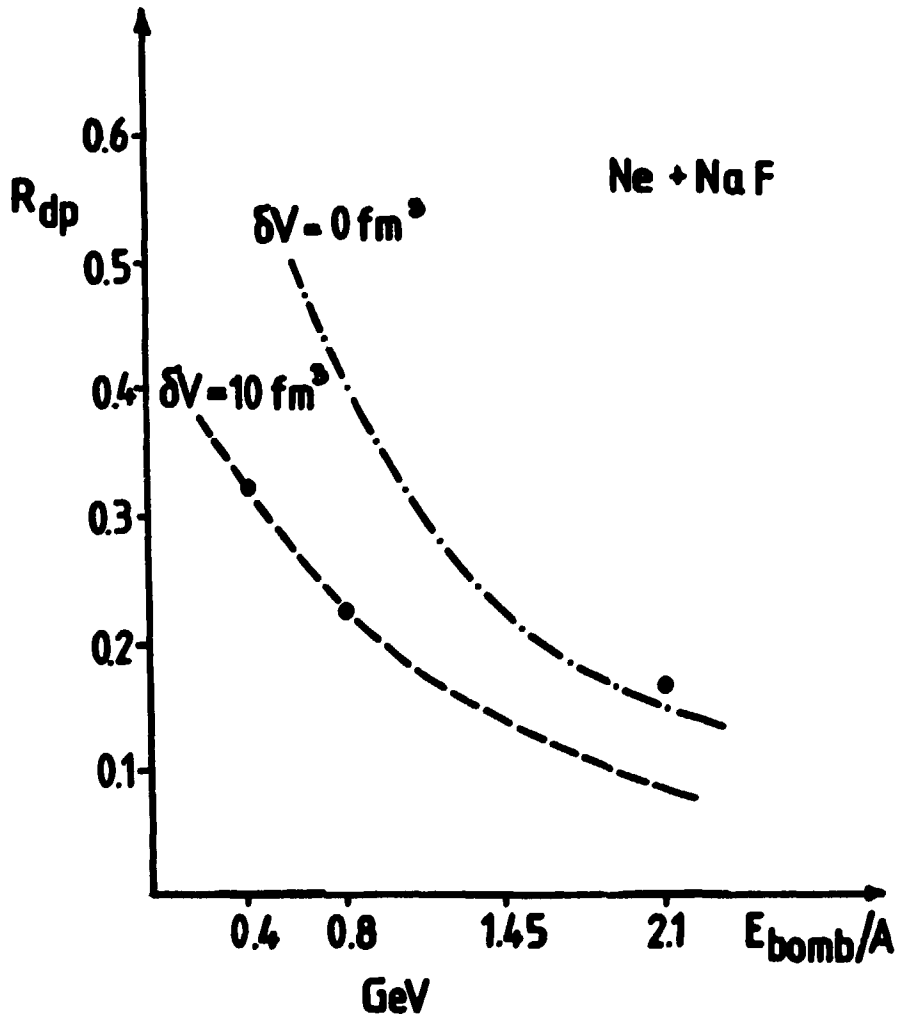


*Fig. 4*

*The d/p ratios calculated without and with van der Waals correction ($\delta V$ = 10 $fm^3$). The dots are the experimental values (Ref. [18]) for the Ne + NaF reaction.*

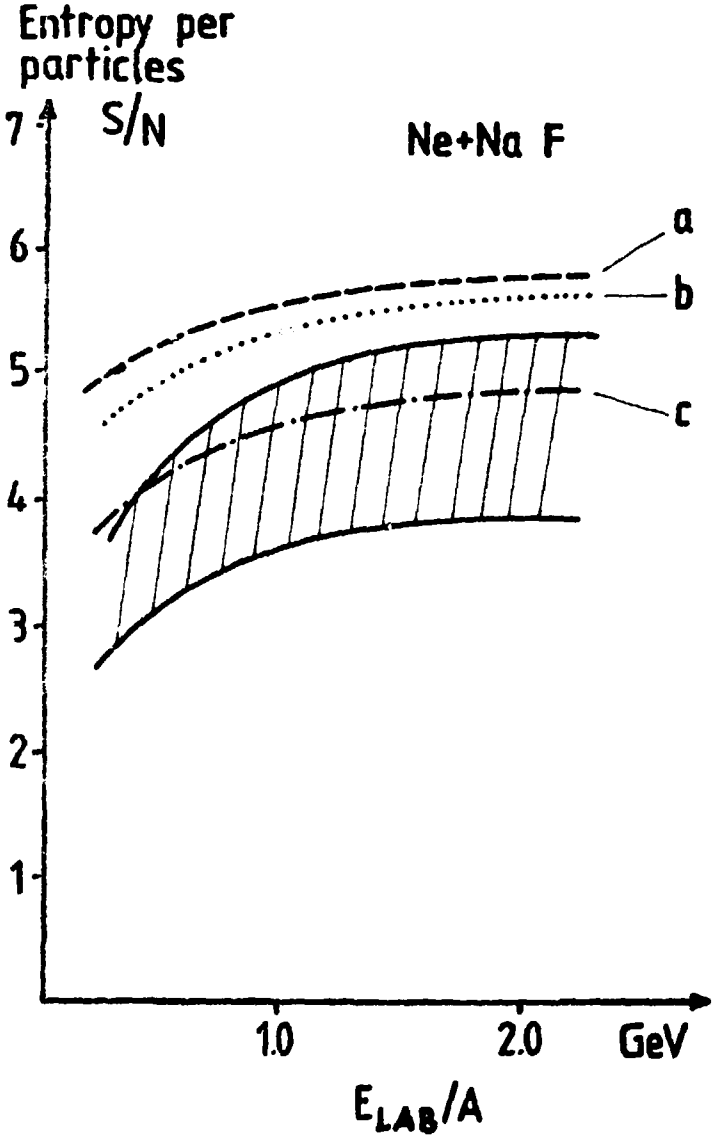


Fig. 5

*The entropy per particle, S/N, versus laboratory bombarding energy per nucleon for the Ne + NaF reaction. The curves a, b and c are the S/N values obtained from the measured deuteron/proton ratios ($R_{dp}$). The curve a shows the $S_{neutron}/N_{neutron}$ in an ideal gas (Eq. (2.7)). The curve b gives $S_{total}/N_{baryon}$ for an ideal two-component gas (Eq.(2.8)). The curve c display $S_{total}/N_{baryon}$ for a van der Waals gas with $\delta V = 10$ fm$^3$ (Eq.(5.8)). The shaded area corresponds to the prediction of Siemens and Kapusta (Ref. [7]) for hadronic matter.*

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