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- **Refined inelastic analysis of piping systems using a beam-type program.**

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1. Introduction

In order to check the compliance with design codes criteria and safety requirements, complete piping systems of Liquid Metal Fast Breeder Reactors (LMFBR) must be analysed using finite elements programs. The difficulty associated with pipings and particularly with elbows is that these structural elements behave simultaneously like beams and shells. A large review of the simplified methods proposed to cope with this problem, has been done by BOYLE and SPENCE [1]. The principal ones are the following two :

- The "primary methods" based on beam-type programs [2] using relevant flexibility factors derived from analytical works [3], [4]. They are economical but supply only global informations. Even if a subsequent determination of local strains is possible [2], [5], they cannot account for local effects like thermal gradients across the thickness of the pipe.
- The "secondary methods" based on shell-type programs, which differ in the choice of discretization of the local displacements fields [6-9]. In most of them, displacements are expanded in Fourier series in the circumferential direction and polynomials in the longitudinal direction. Such methods are rather expensive due to the great number of unknowns and are therefore limited to a few high-stressed components. In compensation, they supply local stresses and strains concentrations and can be used with various loadings.

The best suited method for design purposes should combine the cost-effectiveness of a primary method with the accuracy of a secondary method. This compromise has been attempted by developing a special elbow element, in the frame of the beam program TEDEL [10] of the CEASEMT system [11].

In this program, the Von Mises'criterion together will Hill's principle are assumed, while general creep laws can be implemented.

2. Element formulation in elasticity

2.1 Geometry and hypothesis

The geometry of a typical elbow is shown on fig. 1. The elbow is characterized by its mean surface which is defined by two constant curvature radii and two angular parameters θ (circumferential) and φ (longitudinal). A third parameter z varies across the thickness e.

The Love-Kirchhoff hypothesis for thin shells ($e \ll r$) are assumed.

Therefore, local strains at any point $M(\theta, \varphi, z)$ can be written :

$$\varepsilon_{\alpha\beta}(z) = \varepsilon_{\alpha\beta}(o) + z\, k_{\alpha\beta}(o) \qquad (1)$$

where $\varepsilon_{\alpha\beta}(o)$ et $k_{\alpha\beta}(o)$ are respectively the strain tensor and the curvature variation tensor of the mean surface.

Additional hypothesis first proposed by Von KARMAN [3] are assumed :

- plane sections remain plane
- $\varepsilon_{\theta\theta}(o) = 0$ ("inextensibility" hypothesis)
- $\varepsilon_{\varphi\varphi}(z) = \varepsilon_{\varphi\varphi}(o) \quad \forall z$
- $\varepsilon_{\theta\varphi}(z)$ is negligible
- r is small in front of R.

2.2 Strains - displacements relations

The displacements field of a shell is classically characterized by local displacements (u, v, w) of the mean surface, as shown on fig. 2.

The following strains-displacements relations are derived from Love-Kirchhoff assumptions [12].

$$\varepsilon_{\Theta\Theta}(o) = \frac{1}{r}\left(\frac{\partial v}{\partial \Theta} + w\right)$$

$$\varepsilon_{\varphi\varphi}(o) = \frac{1}{R\,\eta}\left(\frac{\partial u}{\partial \varphi} + w\cos\Theta - v\sin\Theta\right)$$

$$2\,\varepsilon_{\Theta\varphi}(o) = \frac{1}{r}\frac{\partial u}{\partial \Theta} + \frac{1}{R\,\eta}\left(\frac{\partial v}{\partial \varphi} + u\sin\Theta\right)$$

$$k_{\Theta\Theta}(o) = \frac{1}{r^2}\left(\frac{\partial v}{\partial \Theta} - \frac{\partial^2 w}{\partial \Theta^2}\right)$$

$$k_{\varphi\varphi}(o) = \frac{1}{R^2\,\eta^2}\left(\cos\Theta\,\frac{\partial u}{\partial \varphi} - \frac{\partial^2 w}{\partial \varphi^2}\right) + \frac{\sin\Theta}{r\,R\,\eta}\left(\frac{\partial w}{\partial \Theta} - v\right)$$

$$2\,k_{\Theta\varphi}(o) = \frac{1}{r\,R\,\eta}\left(2\,\frac{\partial v}{\partial \varphi} - 2\,\frac{\partial^2 w}{\partial \Theta\partial \varphi} + \frac{\partial u}{\partial \Theta}\cos\Theta - u\sin\Theta\right) + \frac{2\sin\Theta}{R^2\,\eta^2}\left(u\cos\Theta - \frac{\partial w}{\partial \varphi}\right) \quad (2)$$

where $\eta = 1 + \frac{r}{R}\cos\Theta$

According to the first KARMAN's hypothesis, the local displacement u can be expressed in terms of the global in-plane and out-of-plane variations in curvature χ_i and χ_o :

$$\frac{\partial u}{\partial \varphi} = r\cos\Theta\,\chi_i - r\sin\Theta\,\chi_o \quad (3)$$

The other hypothesis lead to the following strains-displacements relations:

$$\varepsilon_{\varphi\varphi}(o) = r\cos\Theta\,\chi_i - r\sin\Theta\,\chi_o + \frac{1}{R}(w\cos\Theta - v\sin\Theta)$$
$$\varepsilon_{\Theta\Theta}(o) = 0 = \varepsilon_{\Theta\varphi}(o)$$
$$k_{\Theta\Theta}(o) = \frac{1}{r^2}\left(\frac{\partial^2 w}{\partial \Theta^2} + w\right) \quad (4)$$
$$k_{\varphi\varphi}(o) = 0 = k_{\Theta\varphi}(o)$$

Local displacements v and w are related through the inextensibility hypothesis :

$$w = -\frac{\partial v}{\partial \Theta} \quad (5)$$

2.3 Shape functions - Strain energy

Following a displacements-type method, the normal displacement w is expanded in Fourier serie, the coefficients of which become the new unknowns. For sake of simplicity, only the even terms have been kept in the serie :

$$w(\Theta, \varphi) = \sum_n a_n(\varphi)\cos 2n\Theta + b_n(\varphi)\sin 2n\Theta \quad (6)$$

The coefficients a_n and b_n, as well as the curvature variations χ_i and χ_o can be chosen linear on the element. In the following, they will be taken as constant for sake of clarity. Strains can be written in a matrix form :

$$\varepsilon = Bq \quad (7)$$

where q is a 2n+2 sized vector of the unknowns :

$$q^t = [X_i \; X_o \; a_1 \; \ldots \; a_n \; b_1 \; \ldots \; b_n]$$

and B is matrix the terms of which depend on the parameters (Θ, φ, z).

By separating the global and local variables, the matrix B is expressed in a block form :

$$\begin{pmatrix} \varepsilon_{\varphi\varphi}(z) \\ \\ \varepsilon_{\Theta\Theta}(z) \end{pmatrix} = \begin{pmatrix} B_{11} & \vdots & B_{12} \\ & \vdots & \\ 0 & \vdots & B_{22} \end{pmatrix} \begin{pmatrix} X \\ --- \\ a \end{pmatrix} \qquad (8)$$

The elastic strain energy per unit length of the elbow, assuming a plane state of stresses, is :

$$U = \frac{1}{2} \int_0^{2\pi} \int_{-\frac{e}{2}}^{+\frac{e}{2}} \frac{E}{1-\nu^2} \left[\varepsilon_{\varphi\varphi}^2 + \varepsilon_{\Theta\Theta}^2 + 2\,\nu\,\varepsilon_{\Theta\Theta}\,\varepsilon_{\varphi\varphi}\right] r d\Theta dz \qquad (9)$$

where E is Young's modulus and ν Poisson's ratio, or :

$$U = \frac{1}{2} \int_S q^t\, B^t\, D\, B\, q\, dS = \frac{1}{2}\, q^t\, K\, q \qquad (10)$$

where D is the Hooke matrix and K is the stiffness matrix of the elbow.

2.4 Condensation of the local unknowns

The elbow is loaded by inplane and out-of-plane moments M_i and M_o. Their work can be written :

$$W_e = M_i\, X_i + M_o\, X_o = (X \,\vdots\, a)^t \times (-\tfrac{M}{0}-) = q^t\, F \qquad (11)$$

The minimization of the total potential energy of the elbow leads to a linear system :

$$Kq = F \quad \text{i.e.} \quad \begin{pmatrix} K_{11} & \vdots & K_{12} \\ \hline K_{21} & \vdots & K_{22} \end{pmatrix} \begin{pmatrix} X \\ --- \\ a \end{pmatrix} = \begin{pmatrix} M \\ --- \\ 0 \end{pmatrix} \qquad (12)$$

In fact, owing to the orthogonality of the trigonometric shape functions, the inplane bending and out-of-plane bending equations are uncoupled and the associated stiffness coefficients are identical.

In order to come back to a global stress-strain law for the elbow, between the moment and the curvature variation, the local parameters a are condensed :

$$a = -\,K_{22}^{-1}\, K_{21}\, X \;, \quad \text{which gives}$$

$$M = (K_{11} - K_{12}\, K_{22}^{-1}\, K_{21})\, X = \frac{EI}{k}\, X \qquad (13)$$

where I is the moment of inertia of the cross section and k is the well known flexibility factor which accounts for the increased flexibility of the elbow when compared with a straight pipe of the same characteristics.

3. Element formulation in plasticity

3.1 State of the problem

Due to an increasing loading, plastic areas appear across the wall thickness, thus modifying the linear distribution of stresses and strains. The non-linearity of the problem leads to an iterative system of the following form [13-14] :

$$K \, \Delta q_{i+1} = \Delta F + \Delta F_i^p \tag{14}$$

where ΔF is the load increment, Δq is the unknown displacement increment and K is the elastic stiffness matrix. The problem consists in the determination of the equilibrating forces ΔF_i^p due to plasticity. It requires the calculation of the serie coefficients a_n, b_n at iteration i, thus supplying an accurate estimate of the flexibility factor taking account of the state of plasticity. A similar technique is used for creep problems.

Strains and stresses are determined at some points regularly distributed all over the cross section, see fig. 3,and inelastic strains are computed when criteria are met, assuming a plane state of stresses. Then numerical integration is performed over the cross section, leading to the various matrices.

3.2 Incremental equations

Starting from an equilibrium state of stresses, σ_Θ^o, σ_φ^o, the strain energy increment can be written :

$$\Delta U = \int_V \left(\int_o^{\Delta\varepsilon} \sigma \, d\varepsilon \right) dV \tag{15}$$

where $\Delta\varepsilon$ is the total strain increment.

Assuming a linear variation of the strain increment on the step, and noting $\Delta\varepsilon^p$ for the plastic strain increment, it comes :

$$\Delta U = \int_V \Delta q^t \, B^t \, D \, B \, \Delta q \, dV - \int_V \Delta q^t \, B^t \, (D \, \Delta\varepsilon^p - \sigma^o) \, dV \tag{16}$$

i.e., after integration :

$$\Delta U = \frac{1}{2} \Delta q^t \, K \, \Delta q - \Delta q^t \, (\Delta F^p - F^o) \tag{17}$$

The external work increment is :

$$\Delta W_e = \Delta q^t \, (F^o + \Delta F) \tag{18}$$

The minimization of the total potential energy on the increment leads to a system which can be partitionned into :

$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} \Delta X \\ \Delta a \end{pmatrix} = \begin{pmatrix} \Delta M \\ o \end{pmatrix} + \begin{pmatrix} \Delta m^p \\ \Delta f^p \end{pmatrix} \tag{19}$$

The Δa unknows are condensed as before :

$$\Delta a = K_{22}^{-1} \left[- K_{21} \, \Delta\chi + \Delta f^P \right]$$

$$(K_{11} - K_{12} K_{22}^{-1} K_{21}) \, \Delta\chi = \Delta M + \Delta M^P \qquad (20)$$

with $\Delta M^P = \Delta m^P - K_{12} K_{22}^{-1} \Delta f^P$

Thus, the formulation is identical to the one used in a classical beam program.

3.3 Thermal gradient loads

The previous formulation enables accounting for temperature gradients across the pipe wall thickness. Indeed, if the temperature distribution is known at each integration point, the corresponding Duhamel forces can be computed :

$$\Delta F^{th} = \int_V B^t \, D \, \Delta\varepsilon^{th} \, dV \qquad (21)$$

where $\Delta\varepsilon^{th}$ are the thermal strain variations.

Besides, the temperature variation causes a mean circomferential strain of the pipe, which is given by :

$$\overline{\Delta\varepsilon}_{\Theta\Theta} = \overline{\Delta\varepsilon}^{th}_{\Theta\Theta} - \overline{\Delta\varepsilon}^{p}_{\Theta\Theta} - \nu \left[\overline{\Delta\varepsilon}_{\varphi\varphi} - \overline{\Delta\varepsilon}^{th}_{\varphi\varphi} - \overline{\Delta\varepsilon}^{p}_{\varphi\varphi} \right] \qquad (22)$$

where the bar denotes a mean value over the section S :

$$\overline{\Delta\varepsilon}_{\Theta\Theta} = \frac{1}{S} \int_S \Delta\varepsilon_{\Theta\Theta} \, dS$$

This additional strain must be accounted for in the calculation of the stresses.

4. Short examples

Two short examples of application are given below. An industrial application of the method is presented in another paper [15].

4.1 Limit load of elbows

The limit moment has been calculated for different elbows made of elastic perfectly plastic material, with the following characteristics :

$$r = 300 \text{ mm}, \quad R = 900 \text{ mm}, \quad e = \begin{vmatrix} 10 \text{ mm} \\ 20 \text{ mm} \\ 40 \text{ mm} \end{vmatrix} \quad \lambda = \begin{vmatrix} 0.1 \\ 0.2 \\ 0.4 \end{vmatrix}$$

where λ is the characteristic parameter of the elbow, i.e. :

$$\lambda = \frac{e \, R}{R^2}$$

Young's modulus and yield stress are :

$$E = 20{,}000 \text{ Kgf/mm}^2 \quad , \quad Sy = 20 \text{ kgf/mm}^2$$

Figure 4a shows the variation of the ratio between the limit moment M_L of the elbow and the yield moment My for which Sy is reached in the pipe.

Figure 4b shows the variation of the ratio between the limit moment M_L and the limit moment M_{L_S} of a straight pipe of the same characteristics.

The results are in quite good agreement with the upper bound as calculated by SPENCE [16], using a creep law $\dot{\varepsilon} = B\,\sigma^n$ with $n \rightarrow \infty$.

4.2 In plane bending of an elbow in the creep regime

This example is taken from reference [17] : An elbow with characteristics : $R = 3r$ and $\lambda = 0.1$ or 0.2, is loaded by a constant in plane bending moment. The tensile strength curve is shown on fig. 5. The following creep law has been adopted :

$$\dot{\varepsilon}^c = B\,\sigma^n \quad \text{with} \quad n = 5 \quad \text{and} \quad B = 6{,}026.10^{-12}$$

A sector of 90° was studied using 6 elements, with 3 terms in the Fourier serie.

Figure 6 shows the ratio $\frac{\omega-\omega_o}{\omega_o}$ where ω is the end rotation and ω_o is the end rotation obtained for the elastic limit moment M_o taken as initial load.

It may be noticed that the stationary rotation rate is in good agreement.

Figure 7 shows the inelastic flexibility factor calculated as the ratio between stationary rotation rate of the bend and that of a straight pipe of same characteristics.

Figure 8 shows the circumferential stress factor

$$\sigma_\Theta^* = \frac{\sigma_\Theta}{M_o \frac{r}{I}}$$ at the initial and stationary states.

5. Conclusion

A finite element for inelastic piping analysis has been presented, which enables accounting for local effects like thermal gradients and supplies local states of stresses and strains, while keeping all the advantages of a classical beam type program (easy to use, simple boundary conditions, cost effectiveness). Thanks to the local description of the cross section, geometrical non-linearity due to inertia modification can be introduced together with material non-linearity. The element can also be degenerated into a straight pipe element.

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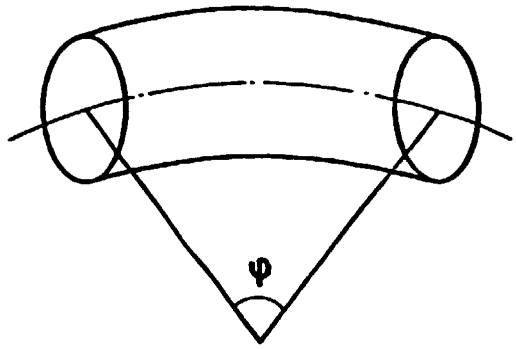


Fig. 1a

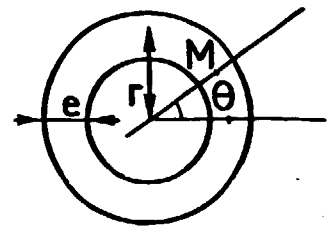


Fig. 1b

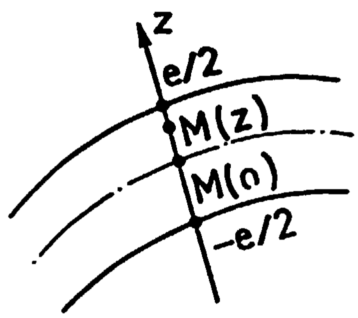


Fig. 1c

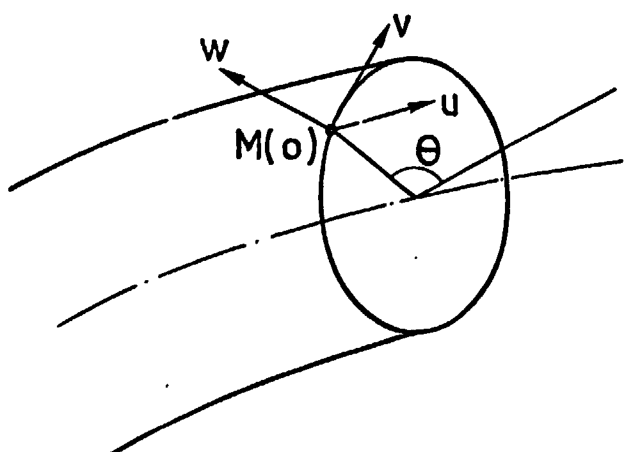


Fig. 2

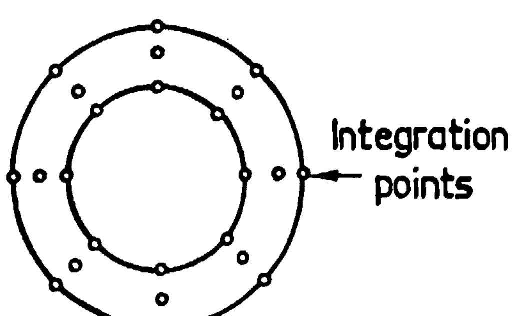
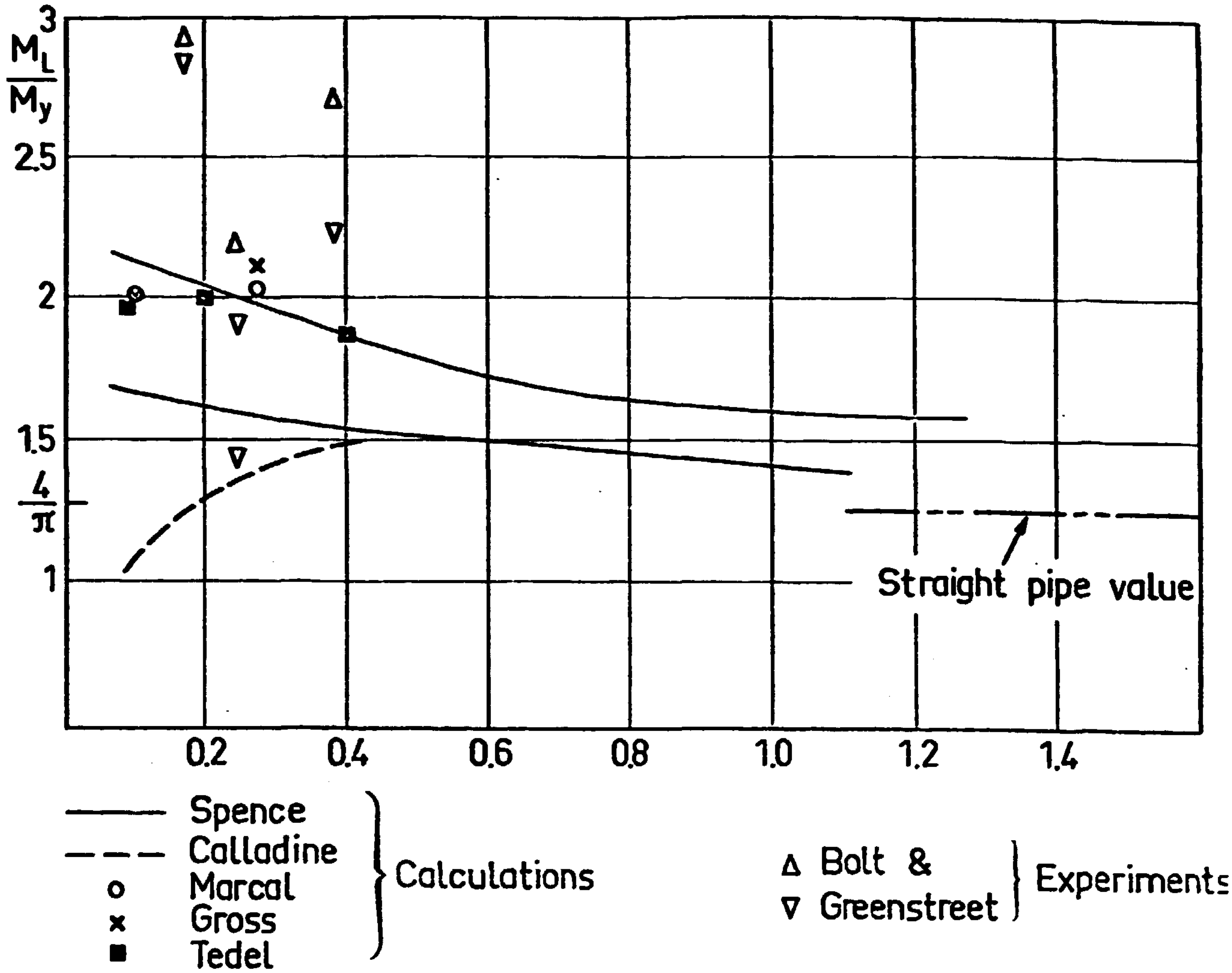
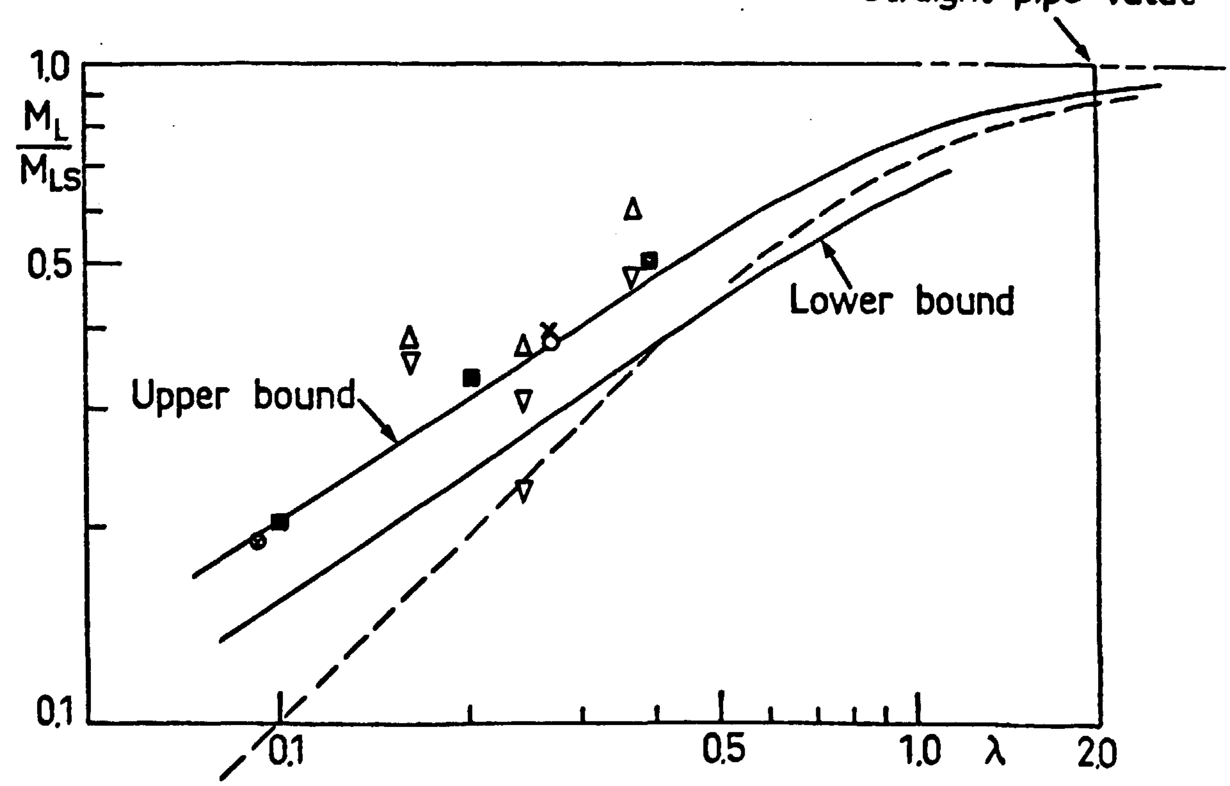


Fig. 3



$\frac{M_L}{M_y}$
3
2.5
2
1.5
$\frac{4}{\pi}$
1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Straight pipe value
Spence
Calladine
Marcal
Gross
Tedel
Calculations
Bolt &
Greenstreet
Experiments

Fig.4 a



Straight pipe value
1.0
$\frac{M_L}{M_{LS}}$
0.5
0.1
Upper bound
Lower bound
0.1
0.5
1.0
λ
2.0

Fig.4 b

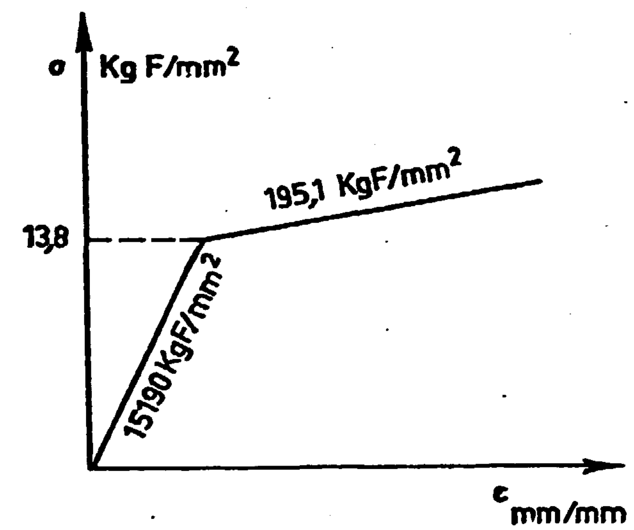


Fig. 5

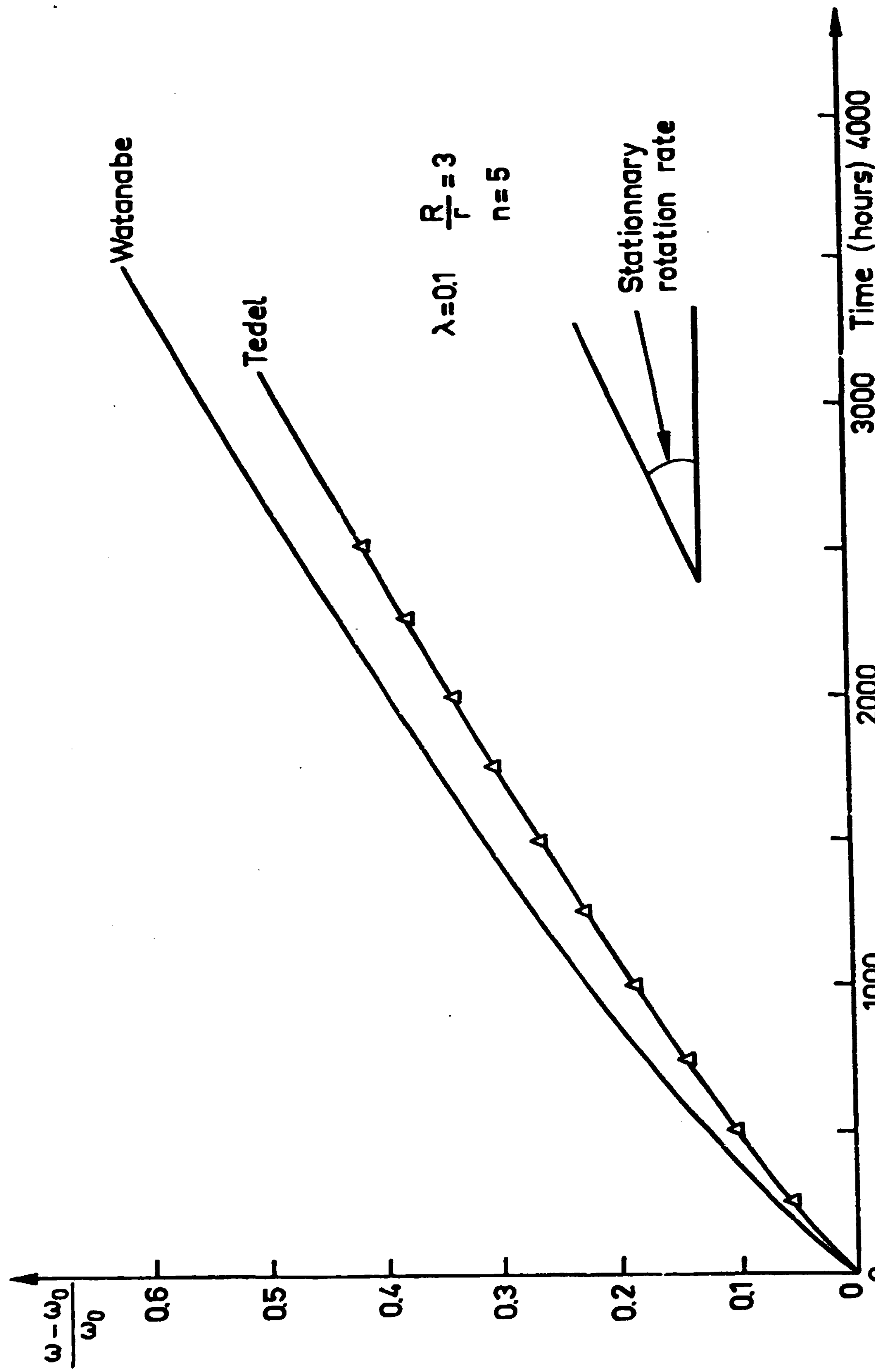
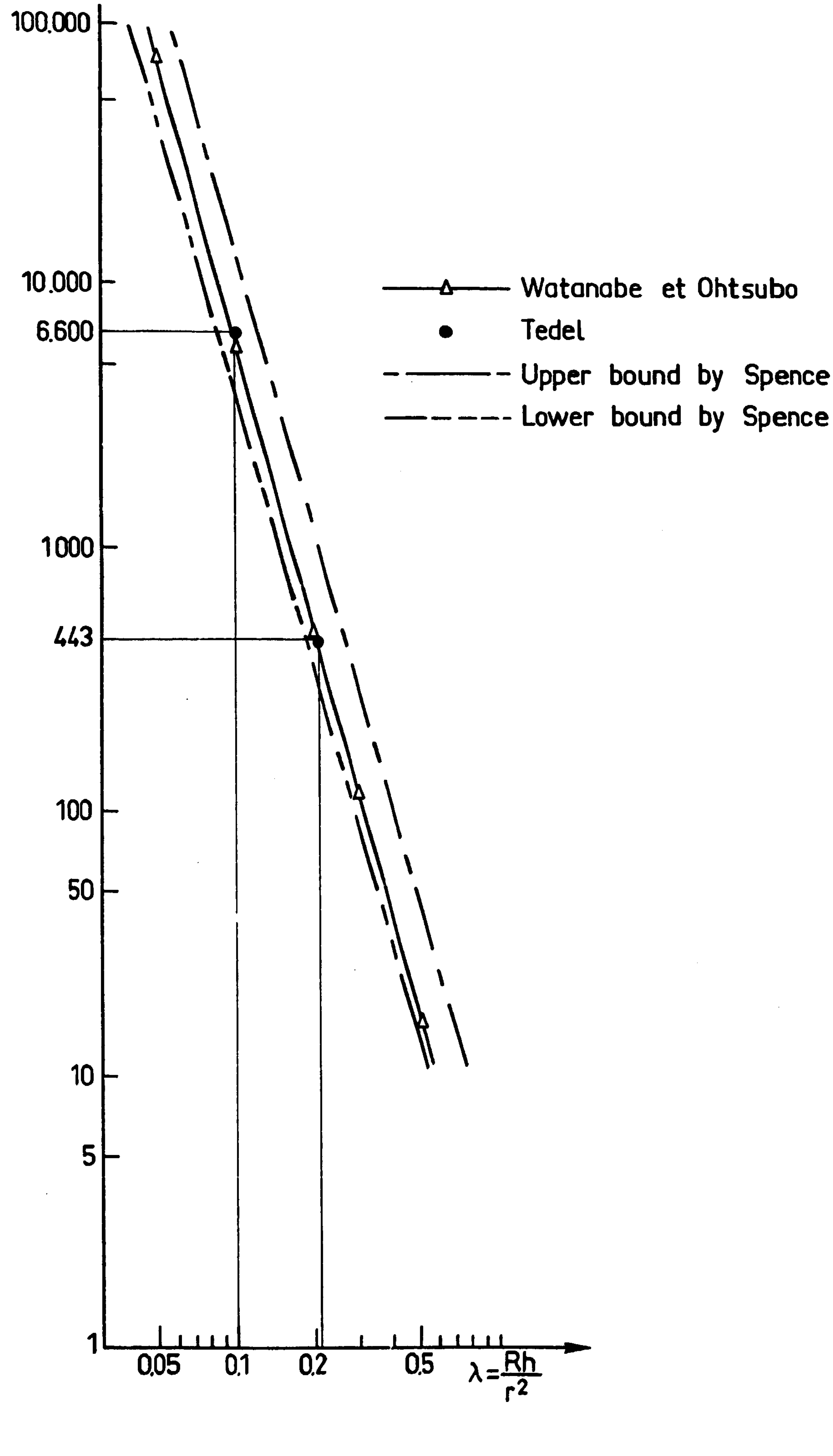


Fig. 6

Normalized rotation ω against time



100,000
10,000
6,600
1000
443
100
50
10
5
1
0,05
0,1
0,2
0,5
$\lambda = \frac{Rh}{r^2}$
Watanabe et Ohtsubo
Tedel
Upper bound by Spence
Lower bound by Spence

Fig 7

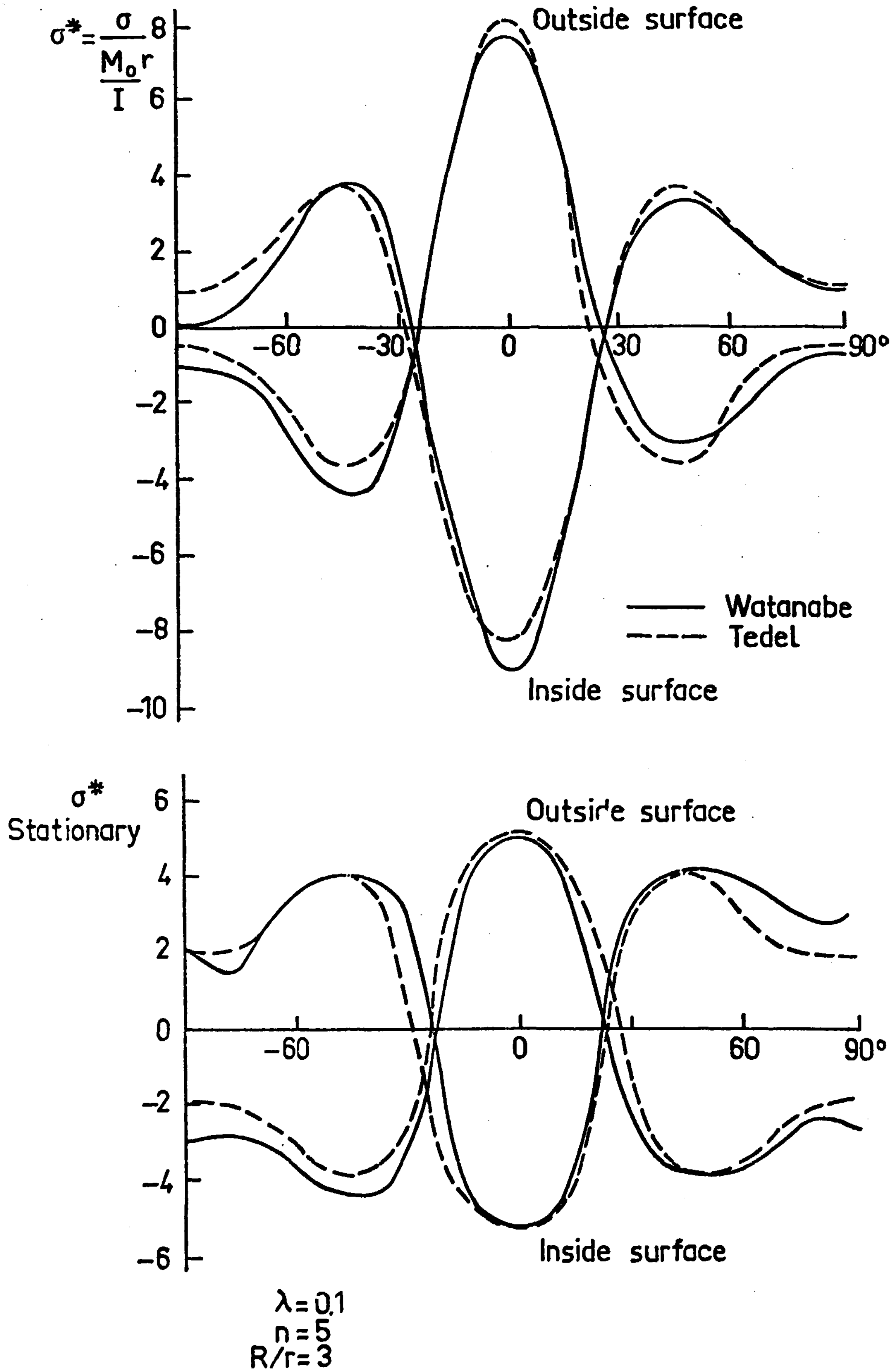


Fig.8

Circumferential stress factor at initial and stationary states