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THE COUPLED ADIABATIC APPROXIMATION IN THE THREE BODY PROBLEM

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Abstract

In the framework of the hyperspherical formalism, we present a study of the Coupled Adiabatic Approximation (CAA) for the case of three nucleons interacting via central spin-dependent two-body potentials. We analyse the convergence of the ground state eigenvalues versus the grand orbital quantum number (2K) and compare the results to that of the Coupled Equations (CE). We also compare with two simpler but less accurate approximations : the Uncoupled Adiabatic Approximation (UAA) and the Extreme Adiabatic Approximation (EAA). The former provides an upper and the latter provides a lower bound to the ground state energy.

1. Introduction

In recent years the ground state of the three nucleon systems has been analysed by many authors using different methods which were stimulated by the feasibility of solving numerically in various ways the few particle equations. Among these approaches we may distinguish between the Faddeev integral equation methods [1-6] and the Schrödinger variational methods [7-16] which have yielded in the last decade successful calculations in the bound state problem of the ^{3}He and ^{3}H nuclei.

In the present work we use an adiabatic approximation, proposed by one of us ten years ago [17], to find the triton binding energy and wave function for central two-body potentials. The idea of the adiabatic approximation was introduced in the past by Born and Oppenheimer to solve the bound of electrons and nuclei in a diatomic molecule. That is, the electronic wave function and eigenvalue U are calculated for frozen internuclear coordinate R. This eigenvalue U(R) is then used to determine the vibrational and rotational level of the molecule. This extreme adiabatic approximation was later on proved by Born [19,20] and the reader can find a discussion of these approximations by Kolos [21].

Recently this method was used by authors treating two electron and three-electron problems with hyperspherical harmonics expansions [22-26]. The extreme adiabatic approximation (EAA) was used both for the ground state and for the continuum state. More recently the extreme adiabatic approximation and the uncoupled adiabatic approximation (UAA) were used [27,28] to find numerical values for the triton binding energy for different choices of central two body potential and the accuracy was judged by comparison with the corresponding value of the triton binding energy calculated as the solution of coupled differential equations (CE) [10,12].

In section 2 we present the mathematical development of the coupled adiabatic approximation and demonstrate the basic inequalities :

$$E(EAA) \leqslant E(CE) \leqslant E(CAA) \leqslant E(UAA)$$

where the binding energies for fixed grand orbital quantum number (2K) are those in the EAA, the coupled differential equation (which gives the exact result for a given value of K), the coupled adiabatic approximation (CAA) and the UAA respectively.

In section 3 we apply our three forms of the adiabatic approximation to find the triton energy and wave function for five different two-body potentials. The accuracy of the approximation is found by comparison with the results of the coupled differential equations [12] for the same two-body potential.

In the last section we estimate accuracy with the coupled adiabatic approximation and also with the EAA and UAA. We discuss possible applications of these adiabatic approximations to several other problems.

2. The adiabatic approximation

The Schrödinger equation for a wavefunction $\Psi(r,\Omega)=r^{-(D-1)/2}\,U(r,\Omega)$ in hyperspherical coordinates is as follows

$$(2.1)\quad \left\{-\frac{\hbar^2}{m}\left[\frac{d^2}{dr^2}+\left(L^2(\Omega)-(D-1)(D-3)/4\right)r^{-2}\right]+V(r,\Omega)-E\right\}U(r,\Omega)=0\ .$$

$D = 3(A-1)$ is the space dimensionality for a system of A particles after elimination of the center of mass X. $L^2(\Omega)$ is the grand orbital operator while $V(r,\Omega)$ is the potential.

The idea leading to the adiabatic approximation is the following. Instead of using the h.h basis in order to solve the Schrödinger equation one chooses the basis $B_\lambda(r,\Omega)$. This set consists of the eigenfunctions of the angular operator

$$(2.2)\quad \left\{-\frac{\hbar^2}{m}\left[L^2(\Omega)-(D-1)(D-3)/4\right]r^{-2}+V(r,\Omega)\right\}B_\lambda(r,\Omega)=U_\lambda(r)B_\lambda(r,\Omega)$$

where r is a parameter.

The eigenvector $B_\lambda(r,\Omega)$ is for the eigenpotential $U_\lambda(r)$. In using the complete B_λ basis for the expansion of the wave function $U(r,\Omega)$ the equations obtained in projecting (2.1) on this basis become coupled by the first and second derivatives of $B_\lambda(r,\Omega)$ with respect to r. When the variation of B_λ with r is small we expect to generate nearly decoupled equations which can be easily solved using the eigenpotential $U_\lambda(r)$.

For improving the solution in which only one equation is taken into account one uses the concept of optimal subset : i.e., one assumes that only the equations directly coupled to the main equation contribute significantly to the solution.

Let us write equation (2.1) using $u(r,\Omega)=B_\lambda(r,\Omega)\,u_\lambda(r)$

$$\left\{-\frac{\hbar^2}{m}\frac{d^2}{dr^2}+U_\lambda(r)-E\right\}B_\lambda(r,\Omega)\,u_\lambda(r)=0 \tag{2.3}$$

where

$$\int\left|B_\lambda(r,\Omega)\right|^2 d\Omega=1 \ . \tag{2.4}$$

Taking the derivatives of $B_\lambda(r,\Omega)$ one generates two new orthogonal elements $B_\lambda^{(1)}$ and $B_\lambda^{(2)}$:

$$\begin{aligned}\frac{dB_\lambda}{dr}&=C_1B_\lambda^{(1)}\\ \frac{d^2B_\lambda}{dr^2}&=C_1C_2B_\lambda^{(2)}+\frac{dC_1}{dr}B_\lambda^{(1)}-C_1^2B_\lambda\end{aligned} \tag{2.5}$$

where the normalization constants are chosen in such a way that

$$\left\langle B_\lambda^{(i)}(r,\Omega)\,\middle|\,B_\lambda^{(j)}(r,\Omega)\right\rangle=\delta_{ij} \qquad \left(B_\lambda^{(0)}\equiv B_\lambda\right) \tag{2.6}$$

The bracket means an integration over the surface of the unit sphere in the D dimensional space as in eq.(2.4).

One expands the wave function according to

$$\Psi(r,\Omega)=r^{-(D-1)/2}\sum_{i=0}^{2}B_\lambda^{(i)}(r,\Omega)\,u_\lambda^{(i)}(r) \tag{2.7}$$

The Schrödinger equation (2.1) is transformed into a set of three coupled equations :

$$(2.8)\quad -\left[\frac{\hbar^2}{m}\frac{d^2}{dr^2}+E\right]u_\lambda^{(i)}(r)+\sum_{j=0}^{2}U_\lambda^{(i,j)}(r)\;u_\lambda^{(j)}(r)=0\,,$$

where

$$U_\lambda^{(i,j)}(r)=\left\langle B_\lambda^{(i)}\right|-\frac{\hbar^2}{mr^2}\left(L^2(\Omega)-\frac{(D-1)(D-3)}{4}\right)+V(r,\Omega)\left|B_\lambda^{(j)}\right\rangle$$

$$(2.9)\qquad +\,W_\lambda^{(i)}\,\delta_{ij}+\frac{\hbar^2}{m}\,P_j^{(i)}\,,$$

$W_\lambda^{(i)}$ is given in terms of the normalization functions $C_i(r)$

$$(2.10)\quad W_\lambda^{(0)}=\frac{\hbar^2}{m}C_1^2\;,\;W_\lambda^{(2)}=\frac{\hbar^2}{m}C_2^2\;,\;W_\lambda^{(1)}=W_\lambda^{(0)}+W_\lambda^{(2)}\;,$$

while $P_j^{(i)}$ are the following differential operators

$$\begin{aligned}&P_1^{(0)}=-P_0^{(1)}=2\sqrt{C_1}\,\frac{d}{dr}\sqrt{C_1}=\frac{dC_1}{dr}+2C_1\frac{d}{dr}\\(2.11)\quad&P_2^{(1)}=-P_1^{(2)}=2\sqrt{C_2}\,\frac{d}{dr}\sqrt{C_2}\\&P_2^{(0)}=P_0^{(2)}=-C_1C_2\;.\end{aligned}$$

The elements $B(r,\Omega)$ orthogonal to $B_\lambda^{(i)}$ $(i = 0,1,2)$ do not give an equation coupled to the main $(i = 0)$ equation and are neglected in the optimal subset approximation.

The $B_\lambda^{(i)}$ constitute the optimal adiabatic subset. The main equation without coupling is called the uncoupled adiabatic approximation (UAA)

$$(2.12)\quad \left\{-\frac{\hbar^2}{m}\frac{d^2}{dr^2}+U_\lambda^{(0,0)}(r)-E\right\}u_\lambda^{(0)}(r)=0\;.$$

Here

$$U_\lambda^{(0,0)}(r) = U_\lambda(r) + \frac{\hbar^2}{m} C_1^2(r) \quad . \tag{2.13}$$

The neglect of C_1^2 in (2.13) leads to the extreme adiabatic approximation (EAA).

The extreme adiabatic approximation provides a lower limit to the ground state binding energy, as shown below.

Let us assume that the exact solution of (2.1) is given by :

$$u_X(r,\Omega) = B_X(r,\Omega)\, u_X(r)$$

The projection of the Schrödinger equation on any $B(r,\Omega)$ orthogonal to $B_X(r,\Omega)$ disappears and the Schrödinger equation becomes :

$$\left\{ -\frac{\hbar^2}{m}\frac{d^2}{dr^2} + U_X(r) + \frac{\hbar^2}{m} C_X^2(r) - E_X \right\} u_X(r) = 0$$

where $U_X(r)$ is the matrix element (2.9) in which $\lambda = X$ for $i = j = 0$ while C_X is taken from (2.5). Let us label $\lambda = 0$ the ground state.

The inequality $U_0(r) < U_X(r)$ results from the property that $B_0(r,\Omega)$ is associated with the lowest eigen potential $U_0(r)$.

Therefore the extreme adiabatic equation

$$\left\{ -\frac{\hbar^2}{m}\frac{d^2}{dr^2} + U_0(r) - E_\varepsilon \right\} u_\varepsilon(r) = 0$$

provides a lower limit to the binding energy. We have the following sequence :

$$E_E < E_X < E_C < E_U$$

where the ground state binding energies E refer respectively to the extreme adiabatic (E) exact (X), coupled adiabatic (C) and uncoupled adiabatic (U) equations. Of course the inequalities for E_C and E_U follow immediately from the variational principle.

3. Determination of the eigen basis $B_\lambda(r,\Omega)$

Let us assume that we have to solve a problem in which the wave function is expanded on the h.h. basis $\mathcal{P}_{2K}(\Omega)$:

$$U(r,\Omega) = \sum_{K=0}^{K_{max}} \mathcal{P}_{2K}(\Omega)\, U_{2K}(r) \tag{3.1}$$

where K varies from zero to K_{max}.

Eq.(2.2) is solved in using the same truncated h.h. basis for the expansion of $B_\lambda^{(i)}$:

$$B_\lambda^{(i)}(r,\Omega) = \sum_{K=0}^{K_{max}} b_{K,\lambda}^{(i)}(r)\, \mathcal{P}_{2K}(\Omega) \tag{3.2}$$

The set $b_{K,\lambda}^{(0)}(r)$ is a solution of the system of linear equations :

$$\left[\frac{\hbar^2}{m} L_K(L_K+1)r^{-2} - U_\lambda(r)\right] b_{K,\lambda}^{(0)}(r) + \sum_{K'=0}^{K_{max}} U_K^{K'}(r)\, b_{K',\lambda}^{(0)}(r) = 0 \tag{3.3}$$

$L_K = 2K + \frac{D-3}{2}$ for bosons ground state and D = 6 for the triton.

$U_K^{K'}(r)$ is the potential matrix

$$U_K^{K'}(r) = \langle P_{2K}(\Omega)|V(r,\Omega)|P_{2K'}(\Omega)\rangle = 3\sum_{K''}(-1)^{K''}\langle K|K''|K'\rangle V_{2K''}(r)$$

where $V_{2K''}(r)$ are the multipole of the potential (see ref.[17]).

According to eqs.(2.4) and (2.5) the $b^{(i)}_{K,\lambda}(r)$ and the normalization constants are given by

$$b^{(1)}_{K,\lambda}(r) = C_1^{-1}\,\frac{d\,b^{(0)}_{K,\lambda}(r)}{dr}$$

(3.4)

$$b^{(2)}_{K,\lambda}(r) = (C_1C_2)^{-1}\left[\frac{d^2}{dr^2} - \frac{1}{C_1}\frac{dC_1}{dr}\frac{d}{dr} + C_1^2\right] b^{(0)}_{K,\lambda}(r)$$

with $\sum_K b^{(i)}_{K,\lambda}\, b^{(j)}_{K,\lambda} = \delta_{ij}$, $C_1^2 = \sum_K \left[\frac{d}{dr} b^{(0)}_{K,\lambda}(r)\right]^2$

$$[C_1C_2]^2 = \sum_K \left\{\left(\frac{d^2}{dr^2} - \frac{1}{C_1}\frac{dC_1}{dr}\frac{d}{dr} + C_1^2\right) b^{(0)}_{K,\lambda}(r)\right\}^2 .$$

Using the expansion (3.2) the bracket in eq. (2.9) becomes

$$\langle B_\lambda^{(i)}| -\frac{\hbar^2}{m}\left(L^2(\Omega) - \frac{(D-1)(D-3)}{4}\right)r^{-2} + V(r,\Omega)|B_\lambda^{(j)}\rangle$$

(3.5)

$$= \sum_{KK'} b^{(i)}_{K,\lambda}(r)\, b^{(j)}_{K,\lambda}(r)\left\{\frac{\hbar^2}{mr^2} L_K(L_K+1)\delta_{KK'} + U_K^{K'}(r)\right\} .$$

4. Numerical results

In order to get some idea how the results of the different adiabatic approximations should look like with different kind of interactions, we perform calculations on the triton binding energy with five central two-body potentials having typical behaviour : Volkov (V) [31], Afnan-Tang (S3) [33] , Eikemeier-Hachenbroich (S4) [32] , Bell-Delves (B.D) [34] and G2 [12].

For a given potential we calculate the matrix elements of the bracket of the formula (3.3) which reads :

$$M_K^{K'}(r) = \left(2K+\frac{3}{2}\right)\left(2K+\frac{5}{2}\right)r^{-2} + 3\sum_{K''}(-1)^{K''}\langle K|K''|K'\rangle V_{2K''}(r) ,$$

where $V_{2K''}(r)$ and $\langle K|K''|K'\rangle$ stand for the multipoles of the potential and the three-body hyperspherical coefficients [17] respectively. We diagonalize the matrix $M_K^{K'}(r)$ for every value of r. To construct the eigenpotential, we select after the diagonalization at the lowest r value r_0 and among the eigenvectors $b_{0\lambda}^{(0)}(r_0)$ that one which is completely decoupled and characterized by the unity for $b_{0\lambda}^{(0)}(r_0)$ and zero for all the $b_{K\lambda}^{(0)}(r)$ with $K \neq 0$. We select the corresponding eigenvalue which is the value of our eigenpotential at r_0 . Then for all the next step in r we have to follow this eigenvalue by continuity and select its corresponding eigenvector. Indeed we have to be careful with the crossing of the eigenvalues in the successive diagonalizations, also the choice at each r step of the lowest eigenvalue may be dangerous for instance if we introduce the mixed symmetry state with the G2 potential.

If we compare the selected eigenpotentials $U_\lambda(r)$ in accordance with the analyzed potential, one can see on the figure 1, that eigenpotentials $U_\lambda(r)$ have very different asymptotic behaviour according to the hardness of the interaction but all of them go through zero near r = 1.5 fm.

The set of eigenfunctions $b^{(0)}_{K\lambda}(r)$ shows structure at short distance and have different straight asymptotes for large r values as shown on the figures 2a and 2b. The determination of the basis $b^{(1)}_{K\lambda}(r)$ and $b^{(2)}_{K\lambda}(r)$ is done numerically with a five points differentiation formula from the basis $b^{(0)}_{K\lambda}(r)$ according to the equations (3.4). At short distance up to r = 0.8 fm we use a Stoer-Padé extrapolation and interpolation of the basis $b^{(1)}_{K\lambda}(r)$ and $b^{(2)}_{K\lambda}(r)$ to remove spurious numerical oscillations. Then each basis $b^{(1)}_{K\lambda}(r)$ and $b^{(2)}_{K\lambda}(r)$ is reorthogonalized to obtain accurate orthogonality relations (eq.(2.6)). These two last procedures are necessary to minimize the errors in the determination of the coupling amplitude $U^{(1,2)}_{\lambda}(r)$ (eq.(2.9)).

In tables (1-5) we give the results obtained for the binding energy of the triton for each adiabatic approximation EAA, UAA, CAA, the accuracy of which is found by comparison with the coupled equation (CE) results for the same two-body potential. The UAA results agree with independent calculations by Das et al. [27] . The extreme adiabatic approximation gives an energy below that of the coupled equations because in the EAA one defines a lower limit of the eigenpotential. The energy difference between the EAA and the UAA depends on the hardness of the interaction and it increases with the strength of the core. The UAA effective potential differs from the EAA one by the normalization function $C^2_1(r)$. This function contains all the information about the variation of the eigenfunction $b^{(0)}_{K\lambda}(r)$ and its amplitude increases with the hardness of the interaction. In the figure 3 we show the shape of $C^2_1(r)$ for three different potentials. The great difference between the amplitude of $C^2_1(r)$ for Volkov and G2 potentials accounts for the energy difference between the EAA and UAA of these two potentials. This energy gap is characteristic of the hardness of the interaction. Using the value of this gap it is possible to estimate the CE energy with the very simple rule :

$$E(CE) \simeq E(UAA) + 0.20 \times \left[E(EAA) - E(UAA) \right] \quad .$$

The normalization function $\left[C_1 C_2 \right]^2$ has a shape similar to that of $C_1^2(r)$ (fig.4) and a more important amplitude, however its role in the coupled equations (1.8) is decreased by the presence of the coupling interactions $P_j^\ell(r)$. The uncoupled and the coupled adiabatic approximation both give energies above the coupled equation energy because we are using trial function with the complete hamiltonian. The CAA energy is lower than UAA since CAA trial function has greater flexibility. Lower limits that are reasonably close to the exact energy are hard to find in quantum mechanics. For the triton problem we do have the use of Hall-Post lower bound. But even for a simple two body potential . Humberston [29] finds that the Hall-Post lower bound falls one MeV below the triton energy. However Brady [30] finds that the Hall-Post lower bound misses by tens or even hundreds of MeV for more realistic potentials. The EAA should prove useful both for its simplicity and for providing a lower bound relatively close to the true energy.

In table 6 we compare the percentages of the partial waves obtained for the five potentials in the uncoupled and coupled adiabatic approximation compared to the coupled equation ones. We see significant differences among the percentages for the first partial waves despite the fact that all the energies are very close. This difference is due to a slight change of the partial wave amplitude but not a change of the shape, then it does not modify the shape of the charge form factor $F_{CH}(q^2)$ (fig.5).

In table 7 we give for comparison the percentage of the three partial waves occuring in the expansion (1.7). As expected the most important is the first wave $B_{\lambda}^{(0)}(r)$ which account more than 99.9 % in the wave function, the amplitude of the two others $B_{\lambda}^{(1)}(r)$ and $B_{\lambda}^{(2)}(r)$ increasing with the hardness of the interaction S3 and G2.

Summary and outlook

The ground state of three particles interacting with central forces is susceptible to be found with a discrepancy about 0.5 % by means of the coupled adiabatic approximation. The amount of computer time used is rather modest and it is reasonable to view realistic calculations with this method, or at least to use coupled adiabatic wave function as a trial wave function for the coupled differential equations. At present an extension of the CAA method using higher number of coupled equations is not interesting, because of the complicated nature of coupling amplitudes. Of course the CAA is quite general and can be applied to a large variety of problems using coupled differential equations in single variable.

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Table Captions

Table 1	Convergence of the triton binding energy in terms of K for the Volkov potential for each adiabatic approximation EAA, UAA, CAA and the corresponding coupled equation (CE) results.
Table 2	Convergence of the triton binding energy in terms of K for the Afnan-Tang (S3) potential for each adiabatic approximation EAA, UAA, CAA and the corresponding coupled equation (CE) results.
Table 3	Convergence of the triton binding energy in terms of K for the Eikemeier-Hakenbroich (S4) potential for each adiabatic approximation EAA, UAA, CAA and the corresponding coupled equation (CE) results.
Table 4	Convergence of the triton binding energy in terms of K for the Bell-Delves (B.D) potential for each adiabatic approximation EAA, UAA, CAA and the corresponding coupled equation (CE) results.
Table 5	Convergence of the triton binding energy in terms of K for the G2 potential for each adiabatic approximation EAA, UAA, CAA and the corresponding coupled equation (CE) results compared to the interpolated formula values.
Table 6	Comparison of the percentages of the partial waves obtained for the five potentials [V,S3,S4,BD,G2] in the uncoupled and coupled adiabatic approximation with the coupled equations (CE) ones.

VOLKOV POTENTIAL [31]

K	EAA	UAA	CAA	.CE
2	8.190	8.036	8.074	8.079
3	8.475	8.278	8.330	8.330
4	8.524	8.324	8.375	8.376
5	8.568	8.363	8.414	8.416
6	8.598	8.389	8.440	8.443
7	8.607	8.397	8.448	8.451
8	8.615	8.402	8.453	8.457
9	8.619	8.406	8.457	8.461
10	8.621	8.408	8.458	8.463
11	8.623	8.4086	8.4591	8.464
12	8.624	8.4092	8.4597	8.465

- Table 1 -

S3 POTENTIAL [33]

K	EAA	UAA	CAA	CE
2	2.396	2.050	2.112	2.12
3	4.976	4.480	4.593	4.594
4	5.602	5.072	5.191	5.196
5	6.252	5.655	5.785	5.775
6	6.733	6.076	6.211	6.208
7	6.899	6.219	6.355	6.355
8	7.033	6.334	6.471	6.470
9	7.130	6.417	6.554	6.557
10	7.172	6.453	6.589	6.592
11	7.203	6.480	6.615	6.620
12	7.226	6.501	6.634	6.640

- Table 2 -

S4 POTENTIAL [32]

K	EAA	UAA	CAA	CE
0	4.596	4.596	4.596	3.667
2	4.919	4.697	4.738	4.741
3	6.106	5.842	5.891	5.891
4	6.400	6.133	6.181	6.182
5	6.722	6.443	6.493	6.490
6	6.980	6.686	6.736	6.735
7	7.075	6.773	6.824	6.824
8	7.158	6.846	6.899	6.898
9	7.222	6.902	6.955	6.957
10	7.251	6.927	6.980	6.982
11	7.275	6.946	7.000	7.002
12	7.293	6.961	7.015	7.018

- Table 3 -

BELL-DELVES POTENTIAL [34]

K	EAA	UAA	CE
0			2.541
2	3.148	3.137	3.140
3	3.559	3.539	3.542
4	3.652	3.629	3.633
5	3.739	3.713	3.717
6	3.808	3.780	3.785
7	3.835	3.806	3.810
8	3.858	3.829	3.833
9	3.878	3.848	3.852
10	3.888	3.857	3.862
11	3.897	3.865	3.871
12	3.905	3.873	3.878

\- Table 4 -

G2 POTENTIAL [12]

K	EAA	UAA	CAA	CE	Interpolated (formula X)
0	-	-	-	-	
2	-	-	-	-	
3	3.098	2.303	2.502	2.504	2.46
4	4.864	3.870	4.116	4.125	4.07
5	6.517	5.293	5.572	5.589	5.54
6	7.618	6.254	6.534	6.560	6.52
7	7.976	6.564	6.844	6.873	6.846
8	8.253	6.808	7.084	7.117	7.097
9	8.447	6.982	7.253	7.290	7.275
10	8.527	7.056	7.323	7.362	7.35
11	8.588	7.113	7.377	7.417	7.408
13	8.633	7.156	7.416	7.458	7.451

- Table 5 -

	K	0	2	3	4	5	6	7	8	9	10	11	12
S4	UAA	98.317	1.0680	.4143	.0623	.0533	.0396	.0142	.0119	.0092	.0040	.0032	.0024
	CAA	98.70	.8075	.3248	.0535	.0468	.0336	.0114	.0091	.0067	.0028	.0022	.0016
	CE	98.82	.6899	.3162	.0532	.0478	.0359	.0126	.0103	.0079	.0034	.0027	.0020
S3	UAA	97.418	1.4533	.7116	.1220	.1149	.0872	.0307	.0251	.0186	.0078	.0059	.0042
	CAA	97.868	1.1441	.6346	.1134	.1022	.0718	.0235	.0179	.0126	.0051	.0038	.0027
	CE	98.07	.9753	.5947	.1066	.0990	.0748	.0255	.0201	.0152	.0061	.0046	.0034
V	UAA	98.802	.874	.261	.0285	.0174	.0097	.0028	.0019	.0013	.0005	.0003	.0002
	CAA	99.186	.61	.1649	.0183	.0112	.0059	.0015	.0099	.0006	.0002	.0001	.0001
	CE	99.29	.5036	.1618	.0198	.0132	.0077	.0022	.0015	.0009	.0004	.0002	.0002
B.D	EAA	98.134	1.1817	.5025	.0754	.0518	.0307	.0090	.0064	.0043	.0017	.0013	.0009
	UAA	98.129	1.839	.5038	.0756	.0519	.0308	.0090	.0065	.0044	.0017	.0013	.0009
	CE	98.34	1.047	.4484	.0677	.0468	.0267	.0082	.0059	.0040	.0016	.0012	.0009
G2	UAA	95.554	2.1716	1.343	.2655	.2652	.2013	.0694	.0549	.0393	.0159	.0116	.0081
	CAA	96.079	1.795	1.333	.2569	.2337	.1614	.0516	.0382	.0261	.0105	.0077	.0055
	CE	96.30	1.659	1.252	.2472	.2286	.1641	.0542	.0416	.0291	.0117	.0085	.0059

- Table 6 -

	$P(U^{\circ})$	$P(U^1)$	$P(U^2)$
V	99.954	.039	.007
S3	99.951	.037	.011
S4	99.962	.032	.006
G2	99.919	.062	.018

- Table 7 -

Figure Captions

Fig.1 Comparison of the eigenpotentials $U_\lambda(r)$ determined from the two-body interactions : Volkov (continuous line), G2 (dashed line), S4 (dotted-line) and S3 (dot-dashed line).

Fig.2a Comparison of the eigenfunctions $b_0^{(o)}(r)$ and $b_2^{(o)}(r)$ determined from two-body interactions : Volkov (continuous line), G2 (dashed line), S3 (dot-dashed line).

Fig.2b Comparison of the eigenfunction $b_3^{(o)}(r)$ determined from two-body potentials : Volkov (continuous line), G2 (dashed line), S4 (dotted line) and S3 (dot-dashed line).

Fig.3 Shape of the normalization function $C_1^2(r)$ for three potentials : Volkov (continuous line), G2 (dashed line) and S3 (dot-dashed line).

Fig.4 Shape of the normalization function $[C_1C_2]^2$ for three potentials : Volkov (continuous line), G2 (dashed line) and S3 (dot-dashed line).

Fig.5 The ^{3}H charge form factor calculated with three wave functions CE (continuous line), CAA (dashed line) and UAA (dotted-line) determined from the G2 potential.

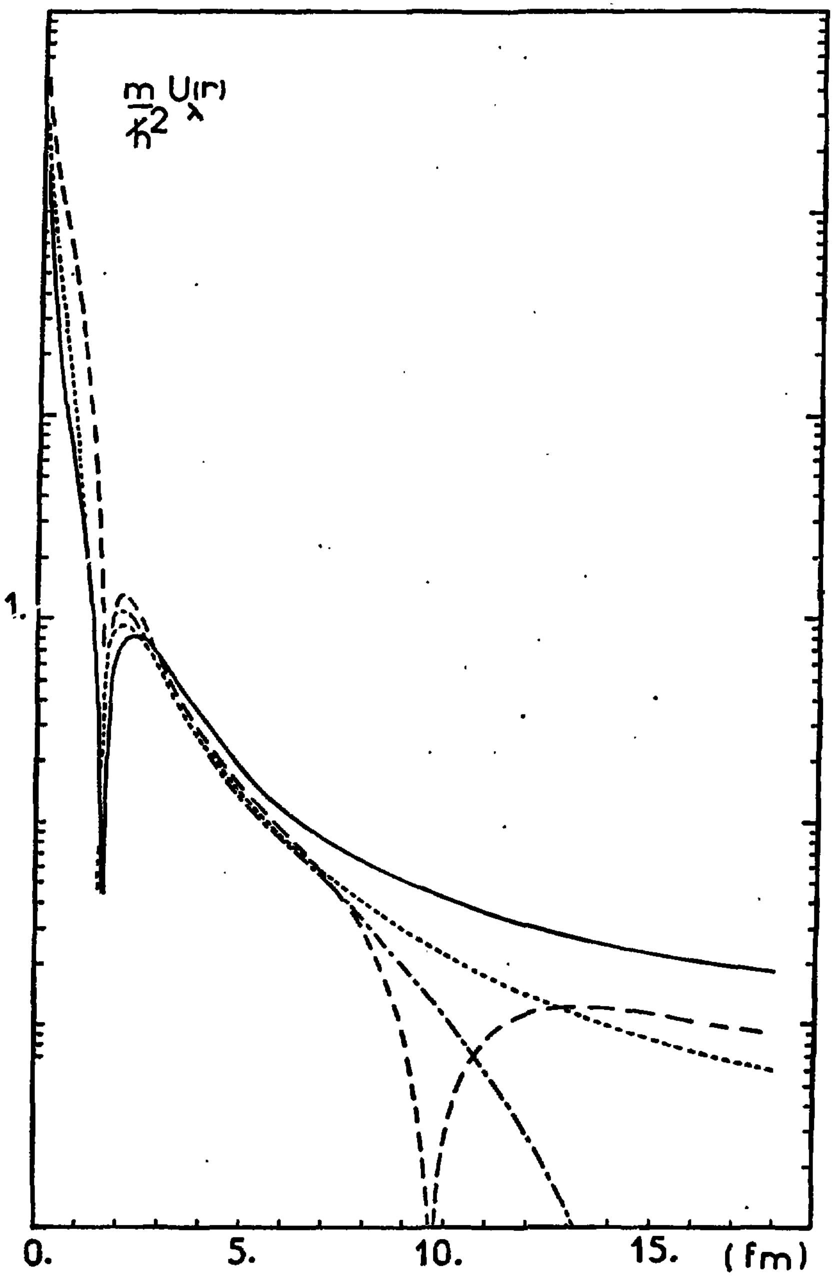


fig.1

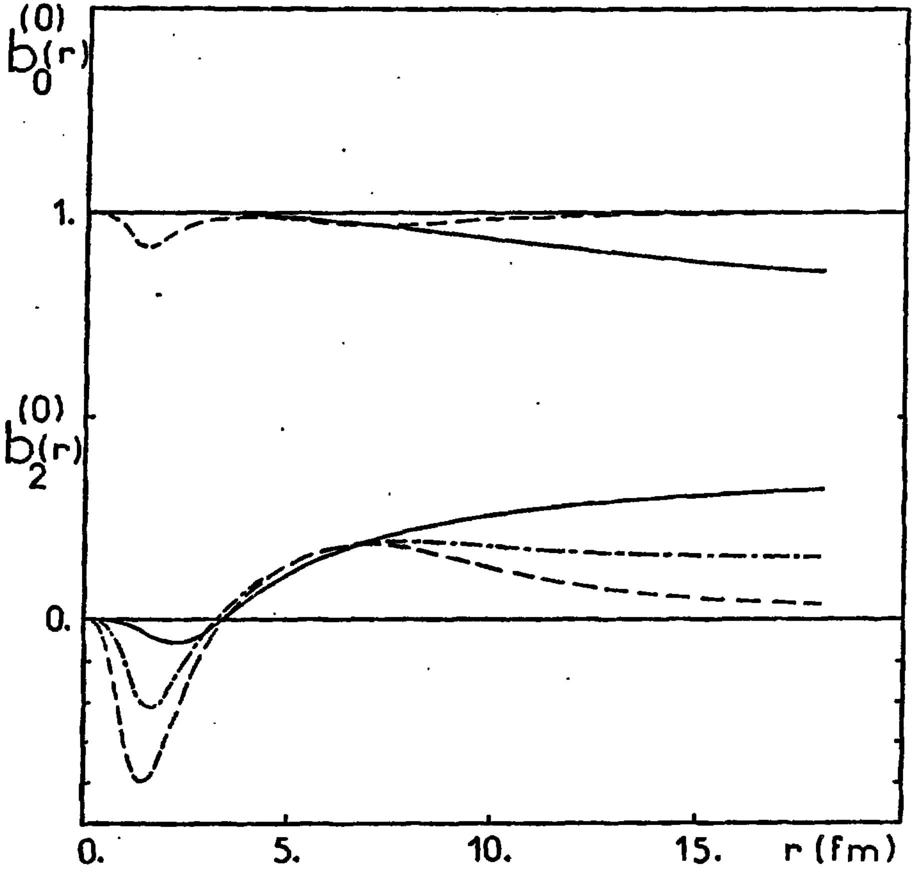
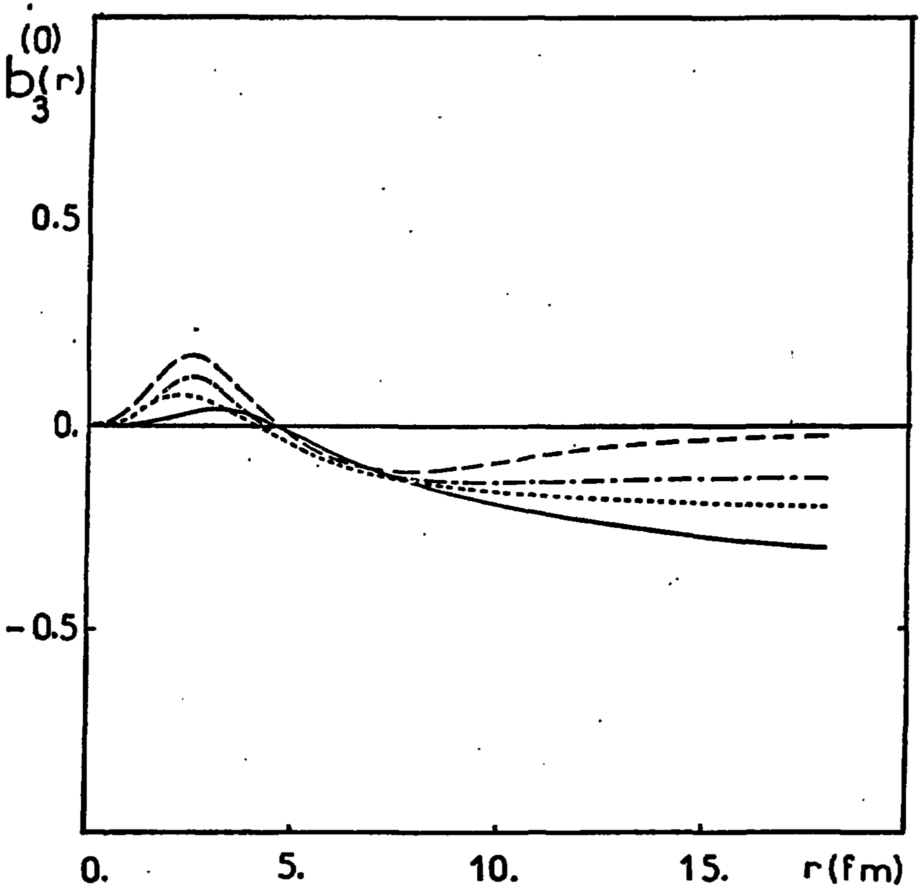


fig.2a



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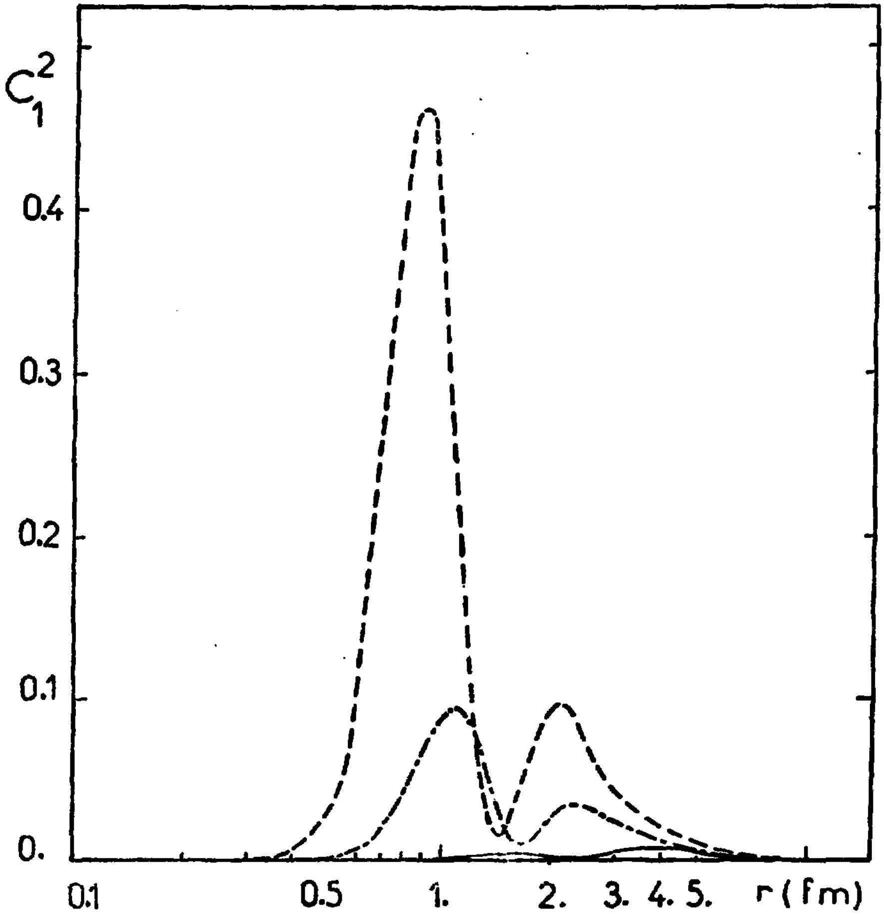

fig.3

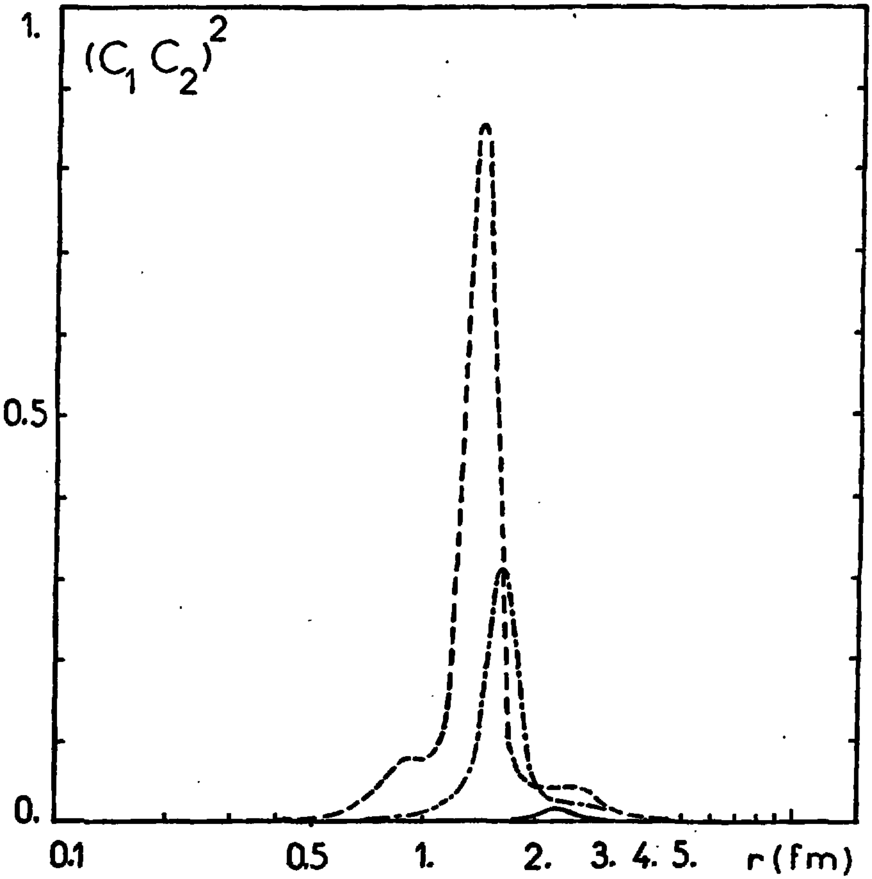

fig.4

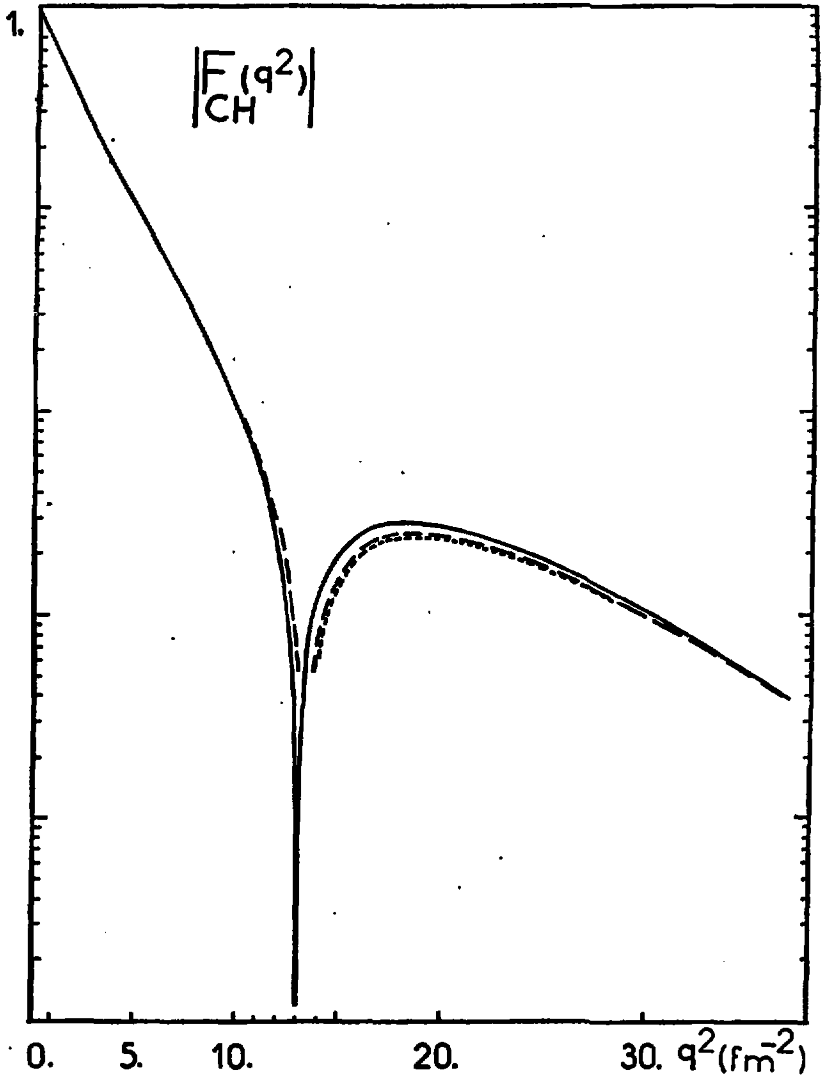

fig.5