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THEORY OF THE SPACE-DEPENDENT FUEL MANAGEMENT COMPUTER CODE "UAFCC"

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ABSTRACT

This report displays the theory of the spatial burnup computer code "UAFCC" which has been constructed as a part of an integrated reactor calculation scheme proposed at the Reactors Department of the ARE Atomic Energy Authorty. The "UAFCC" is a single energy-one dimensional diffusion burnup FØRTRAN computer code for well moderated, multi-region, cylindrical thermal reactors. The effect of reactivity variation with burnup is introduced in the steady state diffusion equation by a fictitious neutron source. The infinite muliplication factor, the total migration area, and the power density per unit thermal flux are calculated from the point model burnup code "UABUC" fitted to polynomials of suitable degree in the flux-time, and then used as an input data to the "UAFCC" code. The proposed burnup spatial model has been used to study the different strategemes of the in-core fuel management schemes. The conclusions of this study will be presented in a future publication.

I. INTRODUCTION

A programme to develope an integrated scheme for reactor physics calculations has been proposed at the Reactors Department of the ARE Atomic Energy Authourity. An important part in this scheme is to construct a spatial burnup computer code to predict the space-time variations in the nuclear fuel parameters as burnup proceeds with the object of studying the different in-core fuel managemgnt schemes.

To construct the spatial dependent burnup model, the numerical matching between the point burnup model and the spatial dependent flux distribution is required. The spatial flux distribution can be obtained in principal by solving the transport integral or integro-differential equation using one of the known techniques.[1] In view of the complexity of these techniques which require large computers, the analysis will be restricted to the solution of the one-dimensional diffusion equation for a multiregion-cylindrical reactor which can provide useful information for engineering applications.

In the next section, the theory of the space dependent fuel management computer code "UAFCC" will be given.

2. THEORY OF THE UAFCC-COMPUTER CODE

For a multiplying medium of steady state reactor, the spatial flux distribution requires the solution of the diffusion equation[2].

$$-D \nabla^2 \phi + \Sigma_a \phi - \lambda K_\infty \Sigma_a \phi = 0 \qquad (1)$$

In fuel burn-up problems, however, where the reactor is considered to be operating at a non-steady state, the rate of change of neutron flux with time must be considered in equation[2.1] and the situation becomes more complicated.

A convenint method[3] for treating this case inyloves the use of a factor " λ " which when muliplied by "K_∞" gives a fictitious number of neutrons per fission, which may be adjusted so that, for any composition and configuration, the fission source can be made just to balance the losses, i.e. introducing the effect of compensating automatic control rod to keep the reactor at a critical level via the parameter " λ ". So, " λ" will be the inverse of the effective multiplication factor of the system.

Thus, the diffusion equation for the nonstationary system can be written as:

$$-D \nabla^2 \phi + \Sigma_a \phi - \lambda K_\infty \Sigma_a \phi = 0 \qquad (2)$$

where:

for $\lambda = 1$; the system is critical,
for $\lambda < 1$; the system is supercritical, and
for $\lambda > 1$: the system is subcritical.

For an infinite cylinder of axial symmetry reactor system, containing "n" regions, we can write; for any region "i" at an irradiation time step "j"; the following equation:

$$\left[-D_{ij} \cdot \frac{1}{r} \frac{d}{dr}\left(r \frac{d}{dr} \right) + \Sigma_{ai} - \lambda_j \cdot K_{\infty ij} \Sigma_{aij}\right]\phi(r)=0 \qquad (3)$$

with $i = 1,2,\dots,n$

Considering the nuclear properties included in equation (3) to be space-independent within the region under consideration, it is not difficult to show that the general solution of (3) can be written as:

$$\phi_{ij}(r) = a_{ij}\, X(\alpha_{ij} r) + b_{ij}\, Z(\alpha_{ij} r) \qquad (4)$$

where: a_{ij} and b_{ij} are arbitrary constants,

$$X(\alpha_{ij}r) = I_o(\alpha_{ij}r) \text{ for } \lambda_j K_{\infty ij} < 1$$

$$= 1 \qquad \text{for } \lambda_j K_{\infty ij} = 1 \qquad (5),$$

$$= J_o(\alpha_{ij}r) \text{ for } \lambda_j K_{\infty ij} > 1,$$

$$Z(\alpha_{ij}r) = K_o(\alpha_{ij}r) \text{ for } \lambda_j K_{\infty ij} < 1$$

$$= \ln\left(\frac{1}{r}\right) \qquad \text{for } \lambda_j K_{\infty ij} = 1 \quad (6),$$

$$= Y_o(\alpha_{ij}r) \qquad \text{for } \lambda_j K_{\infty ij} > 1$$

J_o and Y_o are the zero order bessel functions of the first and second kind respectively and that denoted by the symbols I_o and K_o are the modified zero-order bessel functions of the first and second kind respetively;

$$\alpha_{ij} = \sqrt{\frac{|\lambda_j K_{\infty ij} - 1|}{L^2_{ij}}} \qquad (7)$$

$$L^2_{ij} = \frac{D_{ij}}{\Sigma_{aij}} \qquad (8)$$

and the region "i" is of inner radius "r_{i-1}", and outer radius "r_i", i.e.

$$r_{i-1} \leqslant r \leqslant r_i \qquad (9)$$

Considering the modified one-group theory, "L^2_{ij}" can be replaced by "M^2_{ij}" ($=L^2_{ij} + \tau_{ij}$); the total migration area of a neutron from birth until it is absorbed at some lower energy; and equation (7) can be written as:

$$\alpha_{ij} = \sqrt{\frac{\left|\lambda_j K_{\infty ij} - 1\right|}{M_{ij}^2}} \qquad (10)$$

Since the flux must vanish at the extrapolated boundary of the "n^{th}" region, we have the condition that:

$$a_{nj} \cdot X_{nj} \left(\alpha_{nj} \; r_n \right) + b_{nj} \cdot Z(\alpha_{nj} \; r_n) = 0 \qquad (11)$$

Where "r_n" is the extrapolated outer radius.

In fact; equation (11) is a transcedental equation in the eigenvalue "λ_j" ; which is a measure of the reactivity of the system at the irradiation-time step "j". The first eigenvalue will determine the shape of the first fundamental mode of the flux distribution. In order to compute "λ_j", it is necessary to calculate "a_{nj}" and "b_{nj}" as fo follows:

Applying the continuity condition of flux and current at the boundary of the two regions "i" and "i-1", we get the following recurrence relation connecting "a_{ij}" and "b_{ij}" with "$a_{(i-1)j}$", and "$b_{(i-1)j}$":

$$a_{ij} = \mathcal{L}_1 \; a_{(i-1)j} + \mathcal{L}_2 \; b_{(i-1)j} \qquad (12)$$

$$b_{ij} = \mathcal{L}_3 \; a_{(i-1)j} + \mathcal{L}_4 \; b_{(i-1)j} \qquad (13)$$

where:

$$\mathcal{L}_1 = \frac{Z(\alpha_{ij} r_{i-1}) . X' \left(\alpha_{(i-1)j} r_{i-1}\right) - Z' \left(\alpha_{ij} r_{i-1}\right) X \left(\alpha_{(i-1)j} r_{i-1}\right)}{Z(_{ij} r_{i-1}) . X' \left(\alpha_{ij} r_{i-1}\right) - Z' \left(\alpha_{ij} r_{i-1}\right) X \left(\alpha_{ij} r_{i-1}\right)}$$

(14)

$$\mathcal{L}_2 = \frac{Z(\alpha_{ij}r_{i-1})\, Z'(\alpha_{(i-1)j}r_{i-1}) - Z'(\alpha_{ij}r_{i-1})\, Z(\alpha_{(i-1)}r_{i-1})}{Z(\alpha_{ij}r_{i-1})\, X'(\alpha_{ij}r_{i-1}) - Z'(\alpha_{ij}r_{i-1})\, X(\alpha_{ij}r_{i-1})} \tag{15}$$

$$\mathcal{L}_3 = \frac{X(\alpha_{ij}r_{i-1})\, Z'(\alpha_{(i-1)}r_{i-1}) - X'(\alpha_{ij}r_{i-1})\, Z(\alpha_{(i-1)j}r_{i-1})}{X(\alpha_{ij}r_{i-1})\, Z'(\alpha_{ij}r_{i-1}) - X'(\alpha_{ij}r_{i-1}) Z(\alpha_{ij}r_{i-1})} \tag{16}$$

$$\mathcal{L}_4 = \frac{X(\alpha_{ij}r_{i-1}) X'(\alpha_{(i-1)j}r_{i-1}) - X'(\alpha_{ij}r_{i-1}) Z(\alpha_{(i-1)j}r_{i-1})}{X(\alpha_{ij}r_{i-1})\, Z'(\alpha_{ij}r_{i-1}) - X'(\alpha_{ij}r_{i-1}) Z(\alpha_{ij}r_{i-1})} \tag{17}$$

$$X'(\alpha_{ij}r_i) = \frac{d}{dr} X(\alpha_{ij}r)\, ; \quad r=r_i \tag{18}$$ and

$$Z'(\alpha_{ij}r_i) = \frac{d}{dr} Z(\alpha_{ij}r)\, ; \quad r=r_i \tag{19}$$

It can be noticed that at the innermost region of the reactor system (region 1), the coefficients "a_{1j}" and "b_{1j}" are given by:

$a_{1j} = 1$, since the flux is measured in units of its value at the centre of the reactor; "ϕ_j^{cL}"; and

$b_{1j} = 0$, since the flux should be finite at the reactor centre.

Starting with these initial values for "a_{1j}" and "b_{1j}" and advancing using the recurrence relations(12) and (13), the cofficients "a_{nj}" and "b_{nj}" can be easily determined.

In order to couple the above discussed spatial dependence with the point burn-up model; to evaluate "λ_j" and hence the effective multiplication factor "K_{effj}" of the system as irradiation proceeds; it is necessary to apply the point burn-up code "UABUC"[4] at each space region of the core (with assumed space average flux and uniform nuclear properties for each space region) as well as at each irradiation time step. To simplify the numerical procedure, a gross assumption will be introduced as follows.

In computing the local changes in nuclide compositions during irradiation; using the burnup kinetic equations; there are two independent variables: namely; the thermal flux-time and the thermal neutron flux. However; in power reactors; the concentration changes are assumed to be strongly dependent upon flux-time, and approximately independent upon the changes in the neutron flux. Therefore, the fuel composition and nuclear proparties at any location in the reactor core are known once the cumulative flux-time at that location has been computed. Correspondingly, the movement of fuel in the reactor can be simulated by shifting of "flux-times". This property makes it easy to study the different fuel management techniques[5].

Accordingly, the irradiation-dependent parameters that are needed for the space-dependent burnup analysis will be calculated by the point burnup computer program "UABUC" at different flux-time values and then fitted to polynomials of suitable degree in flux-time "θ"; using a least square routine; and then used as an input data to the "UAFCC" code.

These peremeters are:

- The infinite multiplication factor, "K_{∞}"
- The total migration area, M^2 (cm^2)
- The power density produced per unit thermal flux which by definition is given by[6]

$$\mathcal{E}(\theta)\ (\mathrm{MeV/Cm}) = \sum_{\overline{\overline{xy}}} N_{\overline{\overline{xy}}}(\theta) \cdot \hat{\sigma}_f^{\overline{xy}} \cdot E_{\overline{\overline{xy}}} \qquad (20)$$

where:

$N_{\overline{xy}}(\theta)$ is the concentration of fissile isotope $\overline{xy}$ at flux-time "θ" (nuclides/b.cm),

$\hat{\sigma}_f^{\overline{xy}}$ is the effective microscopic fissions cross-section(barn),

$E_{\overline{xy}}$ is the energy released per fission of the fissile isotope $\overline{xy}$ (Mev/fission), and "θ" expressed in neutrons per barn.

The constructed "FØRTRAN" computer program "UAFCC" is based on the idea that at any irradiation-time step "j" and region "i" having a flux-time "θ_{ij}", the calculations will be advanced in the following way:

1- The quantities "$K_{\infty ij}$", "M^2_{ij}", and "ε_{ij}" can be calculated using the polynomials previously described.

2- A guessed value of "λ_j" is assumed from which "α_{ij}" is calculated using equation (10), then the coefficients "a_{ij}" and "b_{ij}" are calculated using the recurrence relations(12,13). If the proposed value of "λ_j" does not satisfy equation(11), an incremental deviation in "λ_j" takes place and the process continues until the polarity of equation(11) changes, so, the value of "λ_j" is specified between to limits and the "bi-section" numerical method(7,8) is then used to get a refined value for "λ_j" that satisfies equation (11).

3- The spatial flux distribution "$\phi_{ij}(r)$"; for region "i" at the irradiation-time step "j"; is then obtained from equation (4).

4- The space average flux in units of the flus at the core centre, "$\bar{\phi}_{ij}$" ; for each region "i" is obtained from the relation:

$$\bar{\phi}_{ij} = \frac{\int_{r_{i-1}}^{r_i} \phi_{ij}(r).2\pi r \, dr}{\int_{r_{i-1}}^{r_i} 2\pi r \, dr} \qquad (21)$$

It can be shown that:

a. for $\lambda_j K_{\infty ij} > 1$:

$$\overline{\phi}_{ij} = \frac{2}{\alpha_{ij}} \cdot \frac{1}{r_i^2 - r_{i-1}^2} \left\{ a_{ij} \left[r_i J_1 (\alpha_{ij} r_i) - r_{i-1} J_1 (\alpha_{ij} r_{i-1}) \right] - b_{ij} \left[r_i Y_1 (\alpha_{ij} r_i) - r_{i-1} Y_1 (\alpha_{ij} r_{i-1}) \right] \right\} \quad (22),$$

b. for $\lambda_j K_{\infty ij} < 1$:

$$\overline{\phi}_{ij} = \frac{2}{\alpha_{ij}} \cdot \frac{1}{r_i^2 - r_{i-1}^2} \left\{ a_{ij} \left[r_i I_1 (\alpha_{ij} r_i) - r_{i-1} I_1 (\alpha_{ij} r_{i-1}) \right] + b_{ij} \left[r_i K_1 (\alpha_{ij} r_i) - r_{i-1} K_1 (\alpha_{ij} r_{i-1}) \right] \right\} \quad (23),$$ and

c. for $\lambda_j K_{\infty ij} = 1$:

$$\overline{\phi}_{ij} = a_{ij} + b_{ij} \left[0.5 - \frac{r_i^2 \cdot \ln(r_i) - r_{i-1}^2 \cdot \ln(r_{i-1})}{r_i^2 - r_{i-1}^2} \right] \quad (24)$$

5- The core-centre flux value "ϕ_J^{cL}" ; assuming a constant average power of the reactor, can be calculated from the relation:

$$\phi_j^{cL} = \frac{P}{k \cdot \sum_{i=1}^{n} \overline{\phi}_{ij} \varepsilon_{ij} A_i} \quad (25)$$

where:

P : is the system power level per unit length of height (watt/cm),
A_i : is the cross-sectional area of region "i" (Cm^2), and
k : is a conversion factor = 1.60206×10^{-13} watt. sec./Mev.

6. The incremental change in the flux-time " $\Delta\theta_{ij}$ "; in the space-time mesh "ij", the incremental burn-up value " $\Delta\bar{B}_{ij}$", and the regional average power density "$\bar{Q}_{ij}$" can be simply calculated from:

$$\Delta\theta_{ij} = h.\ \phi^{cL}_{j}.\Delta t\ .\ \bar{\phi}_{ij} \qquad (26)$$

$$Q_{ij} = k.\ \phi^{cL}_{j}.\ R_{i}\ .\ \bar{\phi}_{ij}.\mathcal{E}_{ij} \qquad (27)$$

$$\Delta\bar{B}_{ij} = \frac{\bar{Q}_{ij}\ .\ \Delta t}{\rho} \qquad (28)$$

where

h	is a conversion factor equals to 0.864×10^{-19} barn. sec./cm^2. day,
R_i	is the fuel-to-cell volume ratio of region "i",
$\bar{Q}_{ij}$	is the power density (watt/cm^3 of fuel),
Δt	is the irradiation-time increment (days), and
ρ	is the density (gm/cm^3 of fuel).

7. The calculations continue to a new space-time mesh "i(j+1)", and starts with the new value of the flux-time "$\theta_{i(j+1)}$" given by:

$$\theta_{i(j+1)} = \theta_{ij} + \Delta\theta_{ij} \qquad (29)$$

8. When the value of " λ_j" reaches unity the fuel is rearranged according to the scheme under investigation.

The "UAFCC" code has been used to conduct a comparative study between the "In-Out", "Out-In" and "Batch" loading schemes for a pressurized heavy water reactor using natural uranium fuel. The results of this study will be presented in a future publication.

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