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COMMENTS ON GAUGE HIERARCHIES

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Abstract

~~We discuss~~ The problem of gauge hierarchy in a O(N) model is discussed. ~~We~~ It is shown the existence of an upper bound for the hierarchy of order $\alpha^{-1/2}$, as proposed by Gildener. This same constraint appears when the breaking is made by the radiative corrections in a scheme elaborated by Weinberg. It is found ~~We find~~ that fine (tunning) ~~tuning~~ or redefinition of coupling constants to improve hierarchy, as proposed in several papers, cannot be done before the calculation of higher order contributions to the effective potential.

One of the features of Grand Unified Theories (GUTs) (see ref.1 for a review) is to predict new physics at the mass scale of 10^{14}GeV. Those interactions result from the unification in a simple group of the strong interaction described by QCD, with the weak and electromagnetic interactions, wich are described by the Weinberg-Salam model (WSM) and characterized by a mass scale of $\approx O(10^2\text{GeV})$. Such large scale is obtained when some Higgs scalars acquire a vacuum expectation value (VEV) corresponding to the above energies, that is, we need to construct a Higgs potential where the scalar bosons responsible for the breakdown of the grand unified group acquire a VEV of $\approx 10^{14}$GeV and those that break the WSM a VEV of $\approx 10^2$ GeV; this is the so called hierarchy problem. After the pioneer work of Gildener[2], pointing out the existence of a bound of $\alpha^{-1/2}$ on the ratio of the masses of the heavy to light gauge bosons, several authors[3,4] have examined the question and proposed new ways to improve hierarchy, but the actual status of the problem is rather contradictory.

We re-examine the problem in the case of symmetry breaking at tree level and when the breaking is through radiative corrections. We find that hierarchies are possible, but only adjusting the coupling constants after the calculation of several higher order contributions to the effective potential, this happens in all schemes[2-6] without the necessity of imposing extra criteria[7] to determine the existence of hierarchy.

SYMMETRY BREAKING AT THE TREE LEVEL

The model we choose, to discuss the question of gauge hierarchies, has O(N) as the symmetry group. This model has been previously studied by many authors[2,4] and is defined by the Lagrangian

$$\begin{aligned}\mathcal{L} \doteq & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} D^{\mu}\vec{\eta}\, D_{\mu}\vec{\eta} + \frac{1}{2} D^{\mu}\vec{\chi}\, D_{\mu}\vec{\chi} + \frac{1}{2} m_1^2 \vec{\chi}^2 \\ & + \frac{1}{2} m_2^2 \vec{\eta}^2 - \frac{1}{4} f_1 (\vec{\chi}^2)^2 - \frac{1}{4} f_2 (\vec{\eta}^2)^2 - \frac{1}{2} f_3 \vec{\chi}^2 \vec{\eta}^2 \\ & - \frac{1}{2} f_4 (\vec{\chi}\cdot\vec{\eta})^2 \end{aligned} \qquad (1)$$

where $\vec{\chi}$ and $\vec{\eta}$ are two N-vector scalar fields, and a reflection symmetry $(\vec{\eta}(\vec{\chi}) \leftrightarrow -\vec{\eta}(-\vec{\chi}))$ is imposed upon these fields (hereafter, for simplicity, we will drop the vector symbol over $\vec{\chi}$ and $\vec{\eta}$).

We will always assume conditions, such that the VEV $\langle\chi\rangle$ will be much larger than $\langle\eta\rangle$. In fact, $\langle\chi\rangle$ will cause the symmetry breaking of the "grand unified group" O(N) down to O(N-1), while $\langle\eta\rangle$ will break O(N-1) down to O(N-2) at a scale of $O(10^{-12})$ compared with $\langle\chi\rangle$. This last step in the symmetry breaking pattern corresponds to the electroweak symmetry breaking in a realistic grand unified model.

The stationary points of the scalar potential that lead to a gauge hierarchy are

$$\chi^2 = \frac{m_1^2 f_2 - m_2^2 f_3}{f_1 f_2 - f_3^2} \quad , \quad \eta^2 = \frac{m_2^2 f_1 - m_1^2 f_3}{f_1 f_2 - f_3^2} \qquad (2)$$

and they will be a global minimum when $f_4 > 0$ (this is the condition of perpendicularity of the fields in the stationary points[2]), so that hereupon we assume $f_4 > 0$. We impose also the conditions $f_2 \geq f_1$ and $m_1 \geq m_2$ to insure that $\langle\chi\rangle/\langle\eta\rangle >> 1$ and $-(f_1 f_2)^{1/2} < f_3 < f_1 \frac{m_2^2}{m_1^2}$ so that the Higgs bosons masses are real.

There are many suggestions on ways to implement an hierarchy at the tree level in this problem[3,4] like $M_H^2/M_L^2 \approx \langle\chi^2\rangle/\langle\eta^2\rangle \approx 10^{24}$ (hereafter a big ratio of mass between the heavy (M_H) and light (M_L) gauge bosons will display what we mean by a great hierarchy), all one has to do is adjust the parameters in

$$\frac{M_H^2}{M_L^2} \approx \frac{\langle\chi^2\rangle}{\langle\eta^2\rangle} = \frac{m_1^2 f_2 - m_2^2 f_3}{f_1 m_2^2 - f_3 m_1^2} \qquad (3)$$

If we take $f_3 \to 0$ then $M_H^2/M_L^2 \simeq (f_2 m_1^2)/(f_1 m_2^2)$, and by making f_1 and m_2^2 sufficiently smaller than f_2 and m_1^2, one has the desired hierarchy. However, as Gildener has repeatedly pointed out[2,8], even if $f_3 = 0$ at the tree level, it still has a value of $O(\alpha^2)$ coming from one loop corrections, and we have $M_H^2/M_L^2 \simeq f_2/O(\alpha^2)$. So, if f_2 is $O(\alpha)$ we have

$$\left.\frac{M_H}{M_L}\right|_{TREE} < \alpha^{-1/2} \qquad (4)$$

In order to overcome this bound on the hierarchy one adjusts the parameters up to the one loop level, but then there will be a bound at the two loop level so that this class of contributions must be taken into account, and this go on up to $O(\alpha^{L+1}) \simeq 10^{-24}$ where $L\ (\simeq 10)$ is the number of loops of the graphs that must be taken into account, in order to achieve the desired hierarchy. This can be shown explicitly at the one loop level through a calculation of the effective potential.

We calculate the effective potential using standard techniques, as formulated by Jackiw[9]. We study the case where the symmetry is broken down to O(N-2) so that in deriving the effective propagator we set all fields to zero except two orthogonal components of χ and η, let us say $\langle \eta_i \rangle = v\, \delta_{i\,N-1}$ and $\langle \chi_i \rangle = \epsilon\, \delta_{iN}$, and write the effective potential for η and χ (ϵ^2 and v^2 at the tree level are given by (2)). The effective potential, calculated in the Landau gauge, in the one loop approximation is

$$V_1(\eta,\chi) = -\frac{1}{2} m_1^2 \chi^2 - \frac{1}{2} m_2^2 \eta^2 + \frac{1}{4} f_1 \chi^4 + \frac{1}{4} f_2 \eta^4 + \frac{1}{2} f_3 \chi^2 \eta^2$$
$$+ \frac{1}{2} f_4 (\chi \cdot \eta)^2 + \frac{1}{32\pi^2} \Big[\frac{3}{2}(N-2)\Big(g^4 \eta^4 \ln \frac{\eta^2}{M^2} + g^4 \chi^4 \ln \frac{\chi^2}{M^2} \Big)$$
$$+ \frac{3}{2} g^4 (\chi^2+\eta^2)^2 \ln \frac{\chi^2+\eta^2}{M^2} + \sum_{i=1}^{4} \frac{1}{2} M_i^4(\chi,\eta) \ln \frac{M_i^2(\chi,\eta)}{M^2} \tag{5}$$
$$+ \frac{(N-2)}{2} \left(-m_1^2 + f_1 \chi^2 + f_3 \eta^2\right)^2 \ln \frac{-m_1^2 + f_1 \chi^2 + f_3 \eta^2}{M^2}$$
$$+ \frac{(N-2)}{2} \left(-m_2^2 + f_2 \eta^2 + f_3 \chi^2\right)^2 \ln \frac{-m_2^2 + f_2 \eta^2 + f_3 \chi^2}{M^2} \Big]$$

with

$$M_1^2(\chi,\eta) = (-m_2^2 + 3 f_2 \eta^2 + f_3 \chi^2)\cos^2\alpha + (-m_1^2 + 3 f_1 \chi^2 + f_3 \eta^2)\sin^2\alpha$$
$$- 4 f_3 \eta \chi \sin\alpha \cos\alpha \tag{6a}$$

$$M_2^2(\chi,\eta) = (-m_2^2 + 3 f_2 \eta^2 + f_3 \chi^2)\sin^2\alpha + (-m_1^2 + 3 f_1 \chi^2 + f_3 \eta^2)\cos^2\alpha$$
$$+ 4 f_3 \eta \chi \sin\alpha \cos\alpha \tag{6b}$$

$$M_3^2(\chi,\eta) = (-m_1^2 + f_1 \chi^2 + (f_3+f_4)\eta^2)\cos^2\beta + (-m_2^2 + f_2 \eta^2 + (f_3+f_4)\chi^2)\sin^2\beta$$
$$- 2 f_4 \chi \eta \sin\beta \cos\beta \tag{6c}$$

$$M_4^2(\chi,\eta) = \left(-m_1^2 + f_1\chi^2 + (f_3+f_4)\eta^2\right)\sin^2\beta + \left(-m_2^2 + f_2\eta^2 + (f_3+f_4)\chi^2\right)\cos^2\beta \tag{6d}$$
$$+\, 2f_4\,\chi\eta\,\sin\beta\cos\beta$$

where the angles α and β , due to the scalars mass matrix diagonalization are

$$\tan 2\alpha = \frac{4 f_5\, \epsilon\sigma}{(m_2^2 - m_1^2) + \epsilon^2(3f_1 - f_5) - \sigma^2(3f_2 - f_5)} \tag{7a}$$

and

$$\tan 2\beta = \frac{2 f_4\, \epsilon\sigma}{(m_1^2 - m_2^2) + (f_3+f_4)(\epsilon^2 - \sigma^2) + f_2\sigma^2 - f_1\epsilon^2} \tag{7b}$$

the last two terms of eq.(5) correspond to the contribution of 2(N-2) unphysical scalar bosons. There is also another Goldstone boson and three physical Higgs scalars, that are combinations of masses M_1 to M_4 , m_1^2 is the mass scale introduced by the renormalization conditions.

The minimum of the effective potential is obtained from the solution of the equations $\partial V_1/\partial \chi = 0$ and $\partial V_1/\partial \eta = 0$. If we neglect the scalar contributions to (5)[10], one can write the following relation at the minimum

$$\frac{M_H^2}{M_L^2} \approx \frac{\chi^2}{\eta^2} = \frac{m_1^2\left[f_2 + A(N-2)\left(1+2\ln\frac{\eta^2}{v^2}\right) + G(\chi,\eta)\right] - m_2^2\left(f_3 + G(\chi,\eta)\right)}{m_2^2\left[f_1 + A(N-2)\left(1+2\ln\frac{\chi^2}{\epsilon^2}\right) + G(\chi,\eta)\right] - m_1^2\left(f_3 + G(\chi,\eta)\right)} \qquad (8)$$

where $A = \dfrac{3g^4}{32\pi^2}$, and

$$G(\chi,\eta) = A\left(1 + 2\ln\frac{\chi^2+\eta^2}{\epsilon^2+v^2}\right)$$

In writing eq.(8) we have chosen the renormalization condition, using the classical VEV of η and χ as renormalization point, instead of M_1^2 as in (5). We can simplify eq.(8) disregarding the logarithmic terms, so that

$$\frac{M_H^2}{M_L^2} \simeq \frac{m_1^2\left[f_2+\frac{3g^4}{32\pi^2}(N-1)\right]-m_2^2\left(f_3+\frac{3g^4}{32\pi^2}\right)}{m_2^2\left[f_1+\frac{3g^4}{32\pi^2}(N-1)\right]-m_1^2\left(f_3+\frac{3g^4}{32\pi^2}\right)} \qquad (9)$$

We can see from this equation that even if we set $f_3=0$, there still is a contribution $\Delta f_3 = \frac{3g^4}{32\pi^2}$ proportional to m_1^2 in the denominator. When analysing the hierarchy at the tree level we required $m_2^2 \ll m_1^2$, but if m_1^2 is too much larger than m_2^2 , M_L becomes imaginary. Mohapatra and Senjanović[4] impose the condition $m_1^2 \simeq (N-1)\, m_2^2$, $f_3=0$ and $f_1 \ll f_2$, so that $M_H^2/M_L^2 \simeq f_2/f_1$ and obtain the desired hierarchy. Exactly here lies the misunderstanding[8] common ~~to~~ several of the discussions on hierarchy. There is a natural scale proceeding from two loop terms of the order g^6 , limiting the adjustment of the denominator in eq.(8) on the range given by $[1, \alpha^3]$. Finally, if f_2 is $O(\alpha)$ (as we have assumed in order to obtain (4)), we arrive at the ratio

$$\left.\frac{M_H}{M_L}\right|_{1\,loop} < \alpha^{-1} \qquad [illegible]$$

So, one can adjust [illegible] the coupling constants, so that the denominat[illegible] becomes very small, but then terms in g^6 mu[illegible]. The most one can obtain is an improv[illegible] at each order in the loop expansion of [illegible] eq.(9). To obtain an hierarchy like [illegible] 10^{-24}; one must include the contributions [illegible] graphs up to ten or more loops, and only th[illegible] constant adjustment. We can also observe [illegible] limit on hierarchy, there is no much mea[illegible] new constraints to be imposed upon coupl[illegible][7] in order to prevent the hierarchy conditi[illegible] to a zero mass condition for light bosons, [illegible] where the hierarchy is obtained the ne[illegible] natural limit on the mass of the light [illegible]

SYMMETRY BREAKING THROUGH RADIATIVE CORRECTIONS

Recently Weinberg[8] showed that when there is a symmetry breaking induced by radiative corrections[9] to the potential, and some of the scalars remain massless (or with a small mass), then it is possible to attain as large an hierarchy as required by GUTs.

In a effective theory with the potential[5]

$$V(\phi) = V(0) + \frac{1}{4!} f_{abcd}(K)\, \phi^a \phi^b \phi^c \phi^d \qquad (11)$$

at the tree level, the minimum of the potential such that $\phi \neq 0$ is given by

$$\min_{n^a n^a = 1} \left[f_{abcd}(K_0)\, n^a n^b n^c n^d \right] = 0 \qquad (12)$$

where $\phi^a = \phi n^a$. The condition (12) is exactly the condition for $f(K_0) = 0$. K_0 is found by applying the renormalization group equations to $f(M)$, defined at some grand unified mass scale, with suitable initial conditions (like $f(M) \approx g^2(M)$) and then evolving it up to the value K_0 that will make $f(K_0)$ vanish. The solution will have the form

$$K_0 = M \exp\left(-\frac{16\pi^2}{g^2(M)} F(M)\right) \qquad (13)$$

The scalars that remain massless, when the symmetry is broken at the scale M, will acquire a mass of order K_0, as the remaining symmetry is broken still further. In this case the hierarchy will be given by Eq.(13). In the SU(5) model this calculation was accomplished[11] and the result indicated the necessity of coupling constant adjustments for the scheme to work out. We show in our simple model, that in addition to simply adjusting the couplings, we are still left with the same constraint stated above. To do this we write down the effective potential at one loop level (neglecting the scalar contributions)

$$\begin{aligned} V_1(\eta,\chi) = {} & \frac{1}{4} f_1 \chi^4 + \frac{1}{4} f_2 \eta^4 + \frac{1}{2} f_3 \chi^2 \eta^2 + \frac{1}{2} f_4 (\chi\cdot\eta)^2 \\ & + \frac{1}{32\pi^2}\left\{ \frac{3}{2}(N-2)\left[g^4 \eta^4 \ln\frac{\eta^2}{M^2} + g^4 \chi^4 \ln\frac{\chi^2}{M^2} - \frac{25}{6} g^4 (\chi^4 + \eta^4)\right]\right. \\ & \left. + \frac{3}{2} g^4 \left[(\eta^2+\chi^2)^2 \ln\frac{\chi^2+\eta^2}{M^2} - \frac{25}{6}(\chi^2+\eta^2)^2 + \frac{10}{3}\chi^2\eta^2 \right] \right\} \end{aligned} \qquad (14$$

We assume that $\eta^2 \ll \chi^2$ (at least of $O(\alpha)$) and expand the logarithmic terms in powers of η^2/χ^2 (and take N = 3), to derive the stationary conditions

$$\eta^2\left(f_2 + 3\alpha^2 \ln\frac{\eta^2}{M^2} - 22\alpha^2 + 3\alpha^2 \ln\frac{\chi^2}{M^2} + 3\alpha^2\frac{\eta^2}{\chi^2}\right) + \chi^2\left(f_3 + 3\alpha^2\ln\frac{\chi^2}{M^2} + 3\alpha^2\frac{\eta^2}{\chi^2} - 6\alpha^2\right) = 0 \tag{15a}$$

$$\chi^2\left(f_1 + 6\alpha^2\ln\frac{\chi^2}{M^2} - 22\alpha^2 + 3\alpha^2\frac{\eta^2}{\chi^2}\right) + \eta^2\left(f_3 + 3\alpha^2\ln\frac{\chi^2}{M^2} + 3\alpha^2\frac{\eta^2}{\chi^2} - 6\alpha^2\right) = 0 \tag{15b}$$

We can obtain an approximate solution, in the limit when we have hierarchy, by taking $\eta^2 = 0$. Then $\chi^2 = M^2 \exp\left(\frac{22 - f_1/\alpha^2}{6}\right)$ and it will imply that the masses associated to the fields χ and η are

$$m_\chi^2 = 12\alpha^2 M^2 \exp\left(\frac{22 - \frac{f_1}{\alpha^2}}{6}\right) \tag{16a}$$

$$m_\eta^2 = \left(f_3 + 5\alpha^2 - \frac{f_1}{2}\right) M^2 \exp\left(\frac{22 - f_1/\alpha^2}{6}\right) \tag{16b}$$

The η field has a mass of the same order as the λ field, unless $\left(f_3 + 5\alpha^2 - \frac{f_1}{2}\right)$ is very small. But here again the smallness of m_η^2 is bounded by the next order in loop expansion (even when the renormalization group tells us that one of the couplings goes to zero), so we have

$$\left.\frac{m_\lambda}{m_\eta}\right|_{1\,\mathrm{Loop}} < \alpha^{-1/2}$$ ([illegible])

That is to say, m_η^2 can receive contributions of $O(\alpha^3)$ and they must be taken into account, and the same with contributions of $O(\alpha^4)$ and so on. Therefore we are faced with the same problem stated in the item before this.

Sachrajda[6] has recently proposed a scheme to impose gauge hierarchy based on an improved renormalization group. Suposing the existence of hierarchy he starts in search of solutions of the renormalization group equations compatible with that hierarchy. He shows, that with consistent redefinitions of coupling constants, hierarchy is possible. In our model, we can read off similar initial conditions as those obtained in ref.6. From eq.(15a) and eq.(15.b), with

$\eta^2/\chi^2 \ll 1$ we have

$$f_3 + 3\alpha^2 \ln \frac{\chi^2}{M^2} - 6\alpha^2 \approx 0 \qquad (18\,a)$$

$$f_1 + 6\alpha^2 \ln \frac{\chi^2}{M^2} - 22\alpha^2 \approx 0 \qquad (18\,b)$$

Following Sachrajda[6], redefining the coupling constants allow us to find solutions to these equations, but in the same way, as we discussed before, the solutions will not warrant hierarchy up to next order.

Concluding these comments we stress that the unnaturalness of gauge hierarchy, is not only a problem of a simple adjustment of the coupling constants at the tree level, but it is rather a problem of . lculating contributions of a very large number of loops to the effective potential in order to perform a fine tuning of the coupling constants. This only reflects the fact that the communication between different scalars, occurs by the interchange of gauge bosons (also scalars) at all orders. No perturbative method could account for this problem. In this way we believe that a dynamical mechanism[12] for breaking the gauge symmetry is a better mechanism of avoiding the hierarchy problem.

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References

1. P.Langacker,"Grand Unified Theories and Proton Decay", SLAC-PUB-2544/80
2. E.Gildener, Phys.Rev. D14,1667(1976)
3. O.K.Kalashnikov and V.V.Klimov, Phys.Lett. 80B,75(1978); K.T.Mahanthappa and D.G.Unger, Phys.Lett.78B,604(1978); K.T.Mahanthappa, M.A.Sher and D.G.Unger, Phys.Lett. 84B, 113(1979).
4. R.N.Mohapatra and G.Senjanovic, Hadron.Journ.1,903(1978).
5. S.Weinberg, Phys.Lett. 82B,387(1979).
6. C.T.Sachrajda, Phys.Lett. 98B,261(1981).
7. T.N.Sherry, Phys.Lett. 88B,76(1979); Journ.of Phys. A13, 2205(1980).
8. E.Gildener, Phys.Lett. 92B,111(1980).
9. S.Coleman and E.Weinberg, Phys.Rev. D7,1888(1973); S.Weinberg, Phys.Rev. D7,2887(1973); R.Jackiw, Phys.Rev. D9,1686(1974).
10. This amounts to assume $f_i \ll g^2$. We have taken separately the scalar contributions wich apart from extra computational work, do not modify our conclusions.
11. J.Ellis, M.K.Gaillard, A.Peterman and C.Sachrajda, Nucl. Phys. B164,253(1980).
12. S.Weinberg, Phys.Rev. D13,974(1976); Phys.Rev.D19,1277 (1979); L.Susskind, Phys.Rev.D20,2619(1979).