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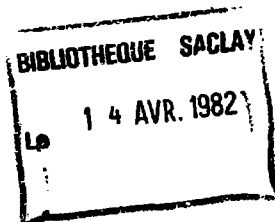
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LEPTON AND QUARK GENERATIONS IN THE GEOMETRICAL RISHON MODEL

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Abstract :

We propose a concrete representation of leptons and quarks in different generations in the geometrical approach to the rishon model where rishons behave as the fundamental representations of the  $SU(3)_C \otimes SU(3)_H$  group. The model allows a unified description of both hadronic and leptonic decays of elementary particles.



## 1. INTRODUCTION

The rishon model was introduced <sup>1</sup> as a simple way of explaining the spectroscopy of quarks and leptons. However, in the absence of a dynamical scheme the model raised more questions than it answered. For instance, it was very difficult to understand the non-existence of spin 3/2 leptons and quarks and/or composites of two rishons and one antirishon, the statistics obeyed by rishons, etc. Recently, in a series of papers <sup>2</sup>, Harari and Seiberg have supplemented the original proposal with a dynamical theory which explains most of the questions listed above. They start with a local gauge theory based on the group  $SU(3)_C \otimes SU(3)_H$ , which is assumed to be an exact symmetry of nature at all levels. One assumes also that the scale parameters obey  $\Lambda_C \ll \Lambda_H$ . All rishons are massless, spin 1/2 objects, obeying ordinary Fermi statistics. The rishons T and V belong to the (3,3) and  $(\bar{3},3)$  representations of  $SU(3)_C \otimes SU(3)_H$ . It is further assumed that all non-singlets, whether in color or hypercolor become confined. Only hypercolor singlets survive at energies below  $\Lambda_H$  and only color singlets survive at energies below  $\Lambda_C$ , where they will all be approximately massless compared to the corresponding scales. Any other multiplets are supposed to acquire masses of the order  $\Lambda$ .

One central question that must be answered by any model which reproduces the observed lepton and quark spectroscopy is the question of generations. Apart from the rather trivial statement that higher generations are excitations of the first one, there are only two attempts to explain the existence of generations in the rishon model.

Harari and Seiberg <sup>3</sup>, based on the identification of a broken global axial symmetry, propose that higher generations are different combinations of seven-rishon states: the 3 rishons of the first generation plus a  $T\bar{T}V\bar{V}$  composite. Different generations are distinguished by the value of an axial charge X.

A different approach was recently considered by Gelmini <sup>4</sup>, who assumes that the distinction between generations is given by the different color structure of the original 3 rishons: the physical states (leptons and quarks) are obtained by combining rishon-antirishon pairs or gluons with the 3 rishons in order to obtain color singlets (leptons) or triplets (quarks). Both these approaches are too general yet to explain the processes in which the generation number is not conserved.

A different development of the rishon model has been advanced by one of us <sup>5</sup>. One constructs rishons as vectors in a color space and then one proceeds to construct objects which are either singlets (leptons) or triplets (quarks) in color and which are composed out of 3 rishons. Using the graphical techniques of vector algebra a comprehensive study of the first few generations has been made and the decay modes of quarks

and leptons have been investigated. One drawback, however, was that in the absence or hypercolor, this model suffered from the same limitations as the original Harari-Shupe conjecture.

In the present paper we propose a new, improved version of the geometrical approach that takes into account both color and hypercolor. It gives an elegant and unified description of both hadronic and leptonic decays of elementary particles.

The rishons are defined by their transformation properties under  $SU(3)_C \otimes SU(3)_H$ . We have the following assignments :  $T \in (3,3)$ ,  $\bar{T} \in (\bar{3},\bar{3})$ ,  $V \in (\bar{3},3)$  and  $\bar{V} \in (3,\bar{3})$ . Since all rishons belong to the fundamental representation of a  $SU(3)$  we can interpret the rishons as vectors (rank-one tensors) in the color space and vectors in the hypercolor space. Graphically we distinguish the representation 3 from  $\bar{3}$  by attaching to each line a direction (arrow). We denote a vector in color space by a straight line and a vector in hypercolor space by a dotted line. Our conventions are shown in the graphical representations of the rishons in Fig.1 :



Fig. 1

It will be convenient to work directly on the graphical representations of the rishons in constructing leptons, quarks and hadrons. We therefore present a short review of the graphical techniques of spin algebras.

## 2. GRAPHICAL TECHNIQUES <sup>6</sup>

We use a graphical technique analogous to the one currently employed in the analysis of angular momentum algebra <sup>6</sup> and which is just a visualization of explicit analytic calculations. There exists in fact a bijection between the graphical method and the analytic one. For instance, the summation over repeated indices rule is graphically translated as the rule in Fig.2 :

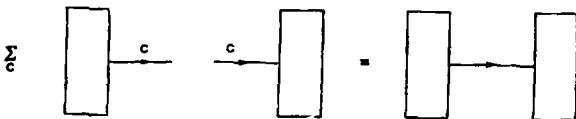


Fig. 2

Thus, any diagram without free lines (i.e. all lines start and end in a vertex) will be an invariant in the corresponding space (i.e. will belong to the identity representation). Similarly, when one takes the tensor product of two representations  $3$  and  $\bar{3}$  in the color or hypercolor space, the singlet in  $3 \otimes \bar{3} = 1 + 8$  is represented by a closed diagram (see Fig.3) :

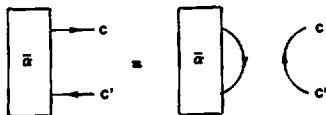


Fig. 3

The notation  $\bar{3}$  implies that the only dependence of the considered expression on color indices is given by the free lines representing a vector and a covector (which transform respectively as  $3$  and  $\bar{3}$ ). The closed diagram corresponds to the color singlet while the remaining line is just  $\delta_{cc'}$ .

The third fundamental rule of the graphical method refers to the decomposition of the product of two identical representations :  $3 \otimes 3 = \bar{3} + 6$  (see Fig.4) :

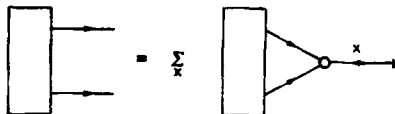


Fig. 4

The lines representing tensors of rank 1,  $X$  can be a tensor of rank 0, 1 or 2. Since we are interested only in the state belonging to  $\bar{3}$  (i.e. we work only with vectors) and not in the one corresponding to 6 ( $X = 0$  and  $X = 2$ ) we shall omit the sum over  $X$  and always consider that  $X = 1$ .

We note that the 3-vertex diagram represents just the Clebsch-Gordan coefficient in the tensor decomposition <sup>7</sup> and, since all our tensors have rank 1 ,

$$\begin{array}{c} \diagup \\ \text{---} i \\ \diagdown \\ \text{---} i \end{array} \begin{array}{c} \text{---} k \\ \text{---} \\ \text{---} \end{array} = \frac{1}{\sqrt{2}} \epsilon_{ijk}$$

As a consequence of the preceding rules one obtains without difficulty the results shown in Fig.5 :

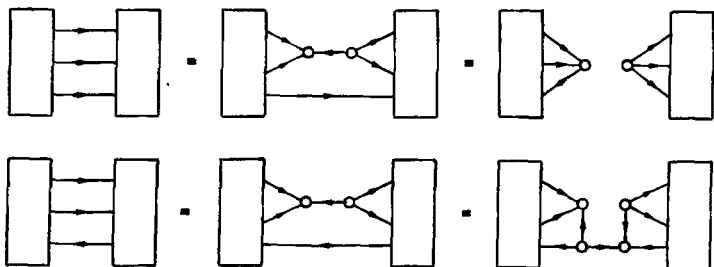


Fig. 5

We shall refer to the results in Fig.5 as the pinching rules.

### 3. CONSTRUCTION OF LEPTONS, QUARKS AND HADRONS

Our model, by construction, will contain only singlets or triplets in either color or hypercolor. Following Harari and Seiberg, we assume that only hypercolor singlets have masses in the range accessible to present experiments and therefore we shall ignore hypercolor triplets. We shall also avoid the problem of bosons made up of rishons which are not quark - antiquark combinations.

The simplest fermion, built out of 3 rishons, which belongs to the (1,1) representation will involve a mixed product in both color and hypercolor. That is, one takes the vector product of two rishons and then the scalar product of the dirishon with the third rishon. The graphical representation is shown in Fig.6 . It is obvious that the method of construction of this object is just the projection on 1 in the product  $3 \otimes 3 \otimes 3$  (or  $\bar{3} \otimes \bar{3} \otimes \bar{3}$ ). We stress that in every 3-vertex, whether in color or in hypercolor, the three arrows must all point the same way with respect to the vertex. By definition, one can only take vector products of vectors belonging to the same representation and scalar products of vectors belonging to conjugate representations. The object just constructed is identified to a lepton. All the arguments presented in Ref.2 about the spin, mass, etc. and in Ref.5 about internal quantum numbers apply equally well here.

Depending on the directions of the arrows, we identify the following four leptons :  
 $e^+ = (T T T)$  ,  $e^- = (\bar{T} \bar{T} \bar{T})$  ,  $\nu_e = (V V V)$  and  $\bar{\nu}_e = (\bar{V} \bar{V} \bar{V})$  . As an example,  
 Fig.6 shows the  $e^+$  and  $\bar{\nu}_e$  .

Note that an isospin-doublet corresponds to identical color line-orientation and to different hypercolor line-orientation :

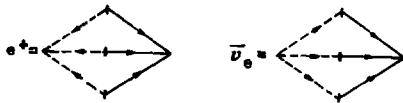


Fig. 6

We introduce next the graphical representation of vector bosons. Since we shall assume that all decays can be obtained by applying the pinching rules, we construct vector bosons by joining two leptons and applying the reverse of the pinching rule. A few examples are given in Fig.7, where we used the reverse of the pinching rule on the color lines, and they correspond respectively to  $\gamma (Z^0)$  ,  $W^-$  and  $W^+$  :



Fig. 7

First we note that it is not possible to construct in this way an object which separates, when applying the pinching rule, into  $e^- \nu_e$  or  $e^+ \bar{\nu}_e$  . In other words, interactions between leptons mediated by vector bosons will conserve the lepton quantum numbers. Second, our use of the label " vector - boson " is not entirely justified. Indeed, these diagrams determine only the internal quantum numbers of the bosons, not their spin. As we shall see later, certain scalar objects will have the same diagrammatical representation.

Out of three fishons one can construct only one other kind of fermion, which will be singlet in hypercolor and triplet in color. This object, to be identified as a quark, is constructed by taking a mixed product in hypercolor and a double vector product in color. Two examples are shown in Fig.8 :

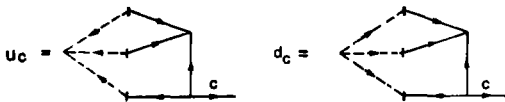


Fig. 8

The free color line indicates that  $u$  and  $d$  are triplets in color while  $\bar{u}$  and  $\bar{d}$  will be antitriplets.

Hadrons built up of two quarks will be constructed by taking the scalar product (in color space) of the two quarks. Since one can only take scalar products of a triplet with an antitriplet, one sees that all such hadrons will be made of a quark and an anti-quark. As an example we present the graphical representation of a  $M_2^+ = \pi^+ (\rho^+)$  in Fig. 9 :

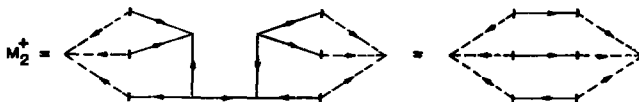


Fig. 9

An important observation that can be made at this point is that the charge of the vector bosons represented in Fig. 7, does not depend on the orientation of the color lines. This fact will enable us to obtain the coupling of the vector mesons to quarks. First, we make the assumption that all structures of the kind shown in Fig. 7, which differ only by the orientation of one or more color lines, represent the same object. This is a natural consequence of the fact that internal quantum numbers are not changed when we change the arrow on one color line. For instance, all four diagrams in Fig. 10 represent a  $W^-$  or a  $\rho^-$  :

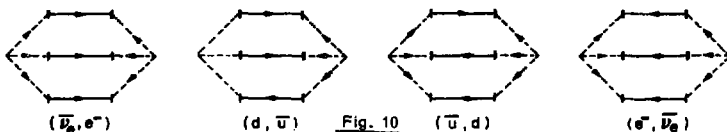


Fig. 10

In the second and third diagrams, applying twice the pinching rule on two lines, as shown in Fig.5, we easily identify the  $\bar{u}$  and  $d$  quarks.

In general, in any rishon model, if we couple  $W^-$  at one end to leptons ( $\bar{\tau}\bar{\tau}\bar{\tau} - \bar{\nu}\bar{\nu}\bar{\nu}$ ) and at the other end to quarks ( $\bar{\tau}\bar{\tau}\bar{\nu} - \bar{\nu}\bar{\nu}\bar{\tau}$ ), a rearrangement of the rishons is necessary in between the two couplings. In our model this is obtained by simply changing the arrow on one color line. In the absence of a dynamical theory, which would allow calculating the effect of flipping the arrow on a color line, one could roughly associate a factor  $\approx m^2/f$  with each flip. In addition to vector meson dominance, which here follows from the identity of the diagrams describing the photon,  $\rho$ ,  $\omega$ , etc, we predict also that  $W^+$  ( $W^-$ ) will be dominated by  $\rho^+$  ( $\rho^-$ ) at  $\rho_W^2 = m_p^2$ . We stress again that spin is not incorporated in our approach and the diagrams in Fig.10 can equally well describe objects of spin 0.

We go next to the construction of higher generations of leptons and quarks. We assume that higher generations are described by more complicated color structures. In principle, the hypercolor structure could also be more complicated, however, since hypercolor forces are restricted to a much smaller range, the different hypercolor configurations will fluctuate rapidly into each other, destroying the separate identity of each configuration. This point has been emphasized also in Ref.4.

The simplest way to increase the complexity of a color structure would be to introduce insertions like the one in Fig.11 on color lines. But, it is easily seen applying the pinching rule on two lines, that one bubble is equivalent to an arbitrary number of bubbles:

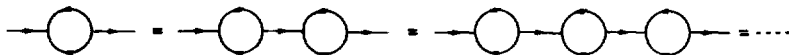


Fig. 11

We will therefore assume that the kind of insertions shown in Fig.11 can be made to appear or disappear on any color line, without changing the character of the object to which the line belongs. In other words, we will consider only structures which are "one-line irreducible". The simplest one is the "Mercedes star" shown in Fig.12.



We identify the four leptons, corresponding to the different possibilities of arrow orientations on color and hypercolor lines as  $\mu^+$ ,  $\mu^-$ ,  $\nu_\mu$  and  $\bar{\nu}_\mu$ . The four quarks belonging to the second generation are shown in the same figure.

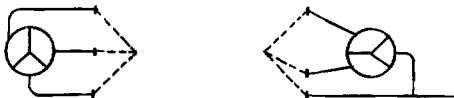


Fig. 12

The insertion of the Mercedes star bubble could have been done also on the lower vertex of the diagrams in Fig.8. One can either assume that these two possibilities are identical, and use either of them as convenient, or take a linear combination of the two when defining states belonging to the second generation. In this paper, we shall assume the former to be true. None of the results to be derived here will depend on this choice.

Finally, we give in Fig.13 the structure of the leptons and quarks belonging to the third generation.

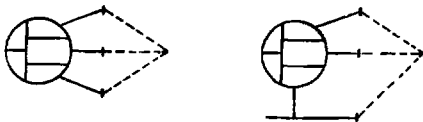


Fig. 13

One can continue and construct higher generations. In our model there is no limit on the number of generations. It is however conceivable that a dynamical calculation will show that some color configurations might be highly unstable and thus limit the number of generations.

## 4. DECAYS OF ELEMENTARY PARTICLES

We are at this point ready to consider the decays of leptons and hadrons. But before doing that we have to discuss certain constraints on the way we use the pinching rule, which are imposed by the physical fact that the vector bosons couple to lepton - antilepton (or quark - antiquark) pairs of the same generation.

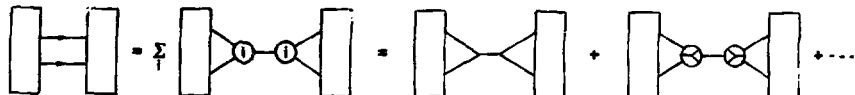


Fig. 14

Diagrammatically, the constraint is shown in Fig. 14, where  $i$  denotes the structure associated with the leptons of the  $i$ -th generation. By requiring that the two vertices must belong to the same generation we guarantee that the vector bosons do not couple to two leptons (quarks) of different generation. Indeed, using the pinching rules in Fig. 14 we get :

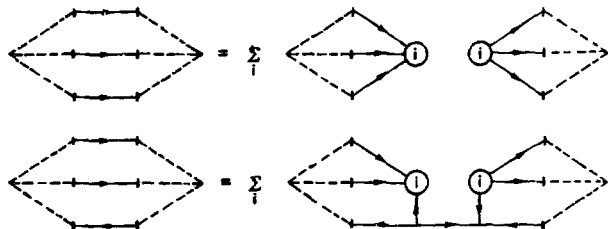


Fig. 15

The sum over  $i$  (number of generations) will be naturally cut-off by energy-momentum  $\delta$ -functions.

We proceed now to discuss the decays of known leptons and hadrons. We consider first decays in which the generation number is conserved. Every color line can be interrupted by a hypercolor line without affecting the quantum numbers : this is equivalent to creating (or annihilating, if we reverse this process)  $T\bar{T}$  or  $V\bar{V}$  pairs in a manner which makes them obviously irrelevant in counting the quantum numbers. Furthermore, all decays will be obtained via the process shown in Fig.16 , i.e. by pinching on the hypercolor lines :

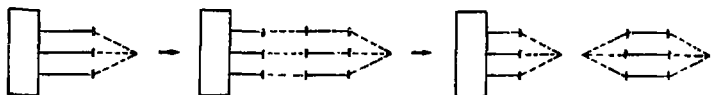


Fig. 16

$\mu$  decay :

Fig.17 exemplifies the above rule in the case of  $\mu$  decay :

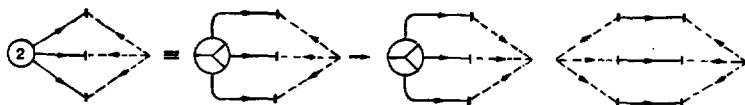


Fig. 17

The figure shows that  $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$ . The decay  $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e \gamma$  can be obtained by applying once again the pinching rule. Fig.16 shows that in general, whenever we have a charged particle in the final state we also have the same final state plus a photon (or an  $e^+ e^-$  pair, if the photon is off mass-shell).

$\tau$  decays :

Fig.16 implies  $\tau^- \rightarrow \nu_\tau W^-$  and  $W^- \rightarrow e^- \bar{\nu}_e$  or  $W^- \rightarrow \mu^- \bar{\nu}_\mu$ . Reversing the arrow on one color line in the  $W^-$  we obtain the structure corresponding to  $\bar{u} d$ . Thus, we get  $\tau^- \rightarrow \nu_\tau \rho^-$  and  $\tau^- \rightarrow \nu_\tau \pi^-$ . Applying the pinching rule on the  $\bar{u} d$  structure one gets the other decay modes :  $\tau^- \rightarrow \pi^- \rho^0 \nu_\tau$ ,  $\tau^- \rightarrow \rho^- \pi^0 \nu_\tau$ ,  $\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu_\tau$ , etc.

$\pi$  decays :

Fig.18 shows the decay  $\pi^+ \longrightarrow \pi^0 e^+ \nu_e$  :



Fig- 18

Using the reverse of the pinching rule and flipping the arrow on the color line as in Fig.19 one obtains the decays  $\pi \longrightarrow \mu \nu, e \nu, \mu \nu \gamma$  and  $e \nu \gamma$ .



Fig. 19

The ratio of the  $\pi \longrightarrow \mu \nu$  versus the  $\pi \longrightarrow e \nu$  decay rates is well understood as a mass effect. A standard calculation<sup>8</sup> gives :

$$R = \frac{\Gamma(\pi \longrightarrow e \nu)}{\Gamma(\pi \longrightarrow \mu \nu)} = \left(\frac{m_e}{m_\mu}\right)^2 \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} = 1.28 \cdot 10^{-4}$$

The diagram in Fig.19 describes also  $\pi^0 \longrightarrow \gamma \gamma, \gamma e^+ e^-,$  etc. Note that the decay of  $\pi^0$  into two photons rather than one is implied by charge conjugation, which is not contained in our diagrams. In general, all conservation laws related to external symmetries (C, P, T) will have to be considered separately, since they are not included in the diagrams.

K decays :

The diagram in Fig.18 applies when one vertex is of second generation and shows that the only decay possible is  $K^0 \rightarrow K^+ e^- \bar{\nu}_e$ . Due to the small phase space available, this process is very rare. One expects a branching ratio similar to the one for  $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ , i.e. about  $10^{-9}$ .<sup>8</sup>

These examples should suffice to convince the reader that all generation number conserving decays can be obtained via the mechanism shown in Fig.16.

Let us now consider decays in which the generation number is not conserved. The mechanism through which a jump is made from one generation to another is the following : a pinching on two hypercolor lines is made in a configuration in which the pinching rule does not normally apply. An example will explain better what this means. Consider the decay  $\mu \rightarrow e \gamma$ . The diagrams in Fig.20 show how  $\mu$  becomes an off-shell  $e$  :

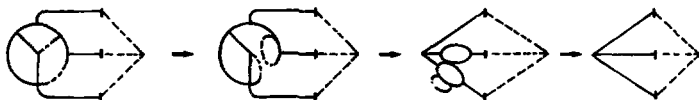


Fig. 20

The electron which is now off mass shell ( $p_e^2 = m_\mu^2$ ) will decay into an electron and a photon through the mechanism in Fig.16. We see that the pinching rule was applied on two lines which do not obey the conditions under which the rule is usually applied : the two lines do not separate two blocks, at least one of which is closed.

A possible explanation of the smallness of generation number breaking decays compared to generation number conserving decays can be given in the context of a naive interpretation of the color and hypercolor lines as strings of different length. The hypercolor lines are to be viewed as very short strings (of the order of  $\Lambda_H^{-1}$ ) while the color lines are much longer (of the order of  $\Lambda_C^{-1}$ ). The pinching rule can be applied to two or three hypercolor lines when they are within a distance  $\Lambda_H^{-1}$  of each other. It is obvious that the probability of this happening is much higher when the hypercolor strings are near the ends of the color strings, since these are already kept in a small

region by the existing hypercolor strings (see Fig.16). On the other hand, the two color strings on which the hypercolor strings appear in Fig.20 can be found in a much larger region and the probability of them being within  $\Lambda_H^{-1}$  of each other is very small. In the absence of a dynamical theory which could lend support to the above picture, we present it as a rough guide to guesstimates of relative branching ratios.

Continuing our discussion of the  $\mu$  decay we see that Fig.20 gives, by applying the pinching rule on the off-shell electron, the decays  $\mu \longrightarrow e\gamma$ ,  $\mu \longrightarrow e\bar{e}e$  and  $\mu \longrightarrow e\gamma\gamma$ . The suppression of the  $\mu \longrightarrow e\gamma$  decay with respect to the other two can be understood as a spin effect: it is the only decay which requires a spin flip.

$\tau$  decays:

Depending on which lines we choose to make the two-line pinching on, the  $\tau$  becomes either an off-shell  $\mu$  or an off-shell  $e$ . Either then decays into an on-mass-shell lepton and a photon (or  $e^+e^-$ ).

K decays:

Consider the decay of the  $K^+$ : when the second generation vertex inside the  $\bar{s}$  quark becomes a first generation vertex, the structure obtained is that of a  $u\bar{d}$  pair, having a mass equal to the  $K$  mass. This object decays, via the mechanism in Fig.16 into  $\pi^+\pi^0$  or  $\pi^+\pi^+\pi^-$ ,  $\mu^+\nu_\mu$ ,  $e^+\nu_e$ ,  $\pi^0\mu^+\nu_\mu$ ,  $\pi^0e^+\nu_e$ , etc. All the decay modes can be obtained by choosing all possible orientations for the arrows on the color and hypercolor lines.

D decays:

There are three generation number conserving decays: first  $D^+ \longrightarrow D^0 e^+ \nu_e$  and  $D^+ \longrightarrow \bar{K}^0 X^+$  where  $X^+ = \mu^+ \nu_\mu$ ,  $e^+ \nu_e$ ,  $p^+$ ,  $\pi^+$ ,  $\pi^+\pi^0$ , etc. There is enough phase space also for  $D^+ \longrightarrow \bar{K}^0 \mu^+ X^+$ . The third decay is obtained by applying the pinching rule simultaneously at both ends of the  $D^+$  structure: one obtains  $D^+ \longrightarrow K^- X^+ X^+$  where  $X^+$  is as above. Finally, all the other hadronic and semi-leptonic and leptonic decays can be accounted for using the rule in Fig.20.

Next, we mention the process of pinching directly on color lines, without hypercolor. The structures thus created are reabsorbed at another vertex. An example is given in Fig.21. These processes are for instance responsible for  $K^0 - \bar{K}^0$  mixing,  $D^0 \longrightarrow K^+ K^-$ , etc.

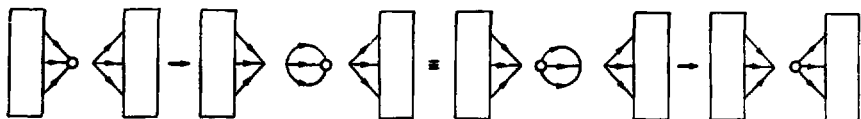


Fig. 21

Finally, if we construct baryons by taking a mixed product in color space of three quarks, we obtain the structure in Fig. 22 :

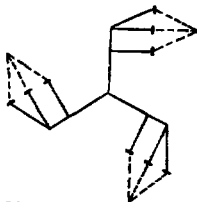


Fig. 22

Using the previous rules, one can easily account for all known decay modes, hadronic and semi-leptonic of the baryons.

## 5. CONCLUSIONS

We note first that, since spin and all external quantum numbers are not accounted for in this approach, it is not obvious from the diagrams whether a  $u\bar{d}$  structure represents a  $\pi^+$  or a  $\rho^+$ , or whether  $\omega$  decays into  $2\pi$ 's or  $3\pi$ 's. All the constraints coming from conservation laws based on external symmetries must be added by hand to decide on the spin-parity-charge conjugation properties of the decay products. This situation is very similar to the one existing when quark diagrams were introduced. Indeed, quark diagrams, supplemented by the OZI rule, allowed the description of all hadronic decay modes as far as the internal quantum numbers of the products was concerned, but spin had to be put in by hand. Our scheme is entirely analogous to quark diagrams. It allows an understanding of which decays are possible and which are not and a rough qualitative estimate of branching ratios. The main advantage with respect to quark diagrams is that it allows us to find not only the hadronic decays but all the semi-leptonic and leptonic ones as well. We present it as further evidence that the rishon model can do more than just provide a mnemonics for the spectroscopy of quarks and leptons.

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Figure Captions

Fig. 1 : The graphical representations of the rishons  $T$  and  $V$ . The rishon  $\bar{T}(\bar{V})$  is obtained graphically by changing the direction of arrows on the lines representing the rishon  $T(V)$ .

Fig. 2 : Graphical rule for summation over repeated indices.

Fig. 3 : Graphical representation of the tensor product of the representations  $3$  and  $\bar{3}$  in a closed diagram.

Fig. 4 : Graphical representation of the tensor product of the representation  $3$  in a closed diagram.

Fig. 5 : Graphical representations for  $3 \otimes 3 \otimes 3$  and  $3 \otimes 3 \otimes \bar{3}$ .

Fig. 6 : Graphical representation of the leptons  $e^+$  and  $\bar{\nu}_e$ .

Fig. 7 : Graphical representation of the  $\tau(Z^0)$ ,  $W^-$  and  $W^+$ .

Fig. 8 : Graphical representation of  $u$  and  $d$ .

Fig. 9 : Graphical representation of  $\pi^+$  ( $\rho^+$ ).

Fig. 10 : Equivalent graphical representations of  $W^-$ .

Fig. 11 : One-line reducible insertions on color lines.

Fig. 12 : Simplest irreducible diagram characterising the leptons and quarks of the second generation.

Fig. 13 : Graphical representations of the leptons and quarks of the third generation.

Fig. 14 : Graphical representation of the constraints imposed on the pinching rule by generation number conservation.

Fig. 15 : Generation number conserving pinching rules for vector bosons.

Fig. 16 : General rule for generation number conserving decays.

Fig. 17 :  $\mu$  decay.

Fig. 18 :  $\pi$  decay.

Fig. 19 :  $\pi$  decay.

Fig. 20 : Generation number breaking mechanism for the decay  $\mu \rightarrow e\gamma$ .

Fig. 21 : Example of pinching directly on color lines.

Fig. 22 : Graphical representation of a baryon.

