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CATASTROPHE THEORY AND DISORDERS OF THE FAMILY SYSTEM *

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ABSTRACT

A brief synopsis for the family therapist of catastrophe theory with a view to its possible use in modelling disorders of the family system.

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PREAMBLE

This short article is intended to be a very brief review of the basic ideas behind catastrophe theory and to discuss how it might be applied to the family system. However,it is written by a theoretical physicists who has had no previous experience either with catastrophe theory as such or, what is more pertinent, with family therapy or the analysis of the family system. One might therefore ask why has he written this article. The answer lies in an idea of John Byng-Hall in utilizing interdisciplinary communication in order to try out successful concepts and models in other fields, in particular, through the use of mathematical models. The catastrophe theory of René Thom, as demonstrated by his many works [1] and those of E.C. Zeeman [2] and others [3], is an excellent example. In physics, a subject divided into many branches or even sub-branches (cosmology, astrophysics, plasma, solid state, atomic, nuclear, subnuclear and elementary particle physics, to mention a few), one frequently finds that ideas and concepts from different branches can be taken over in a completely new context. In my own field, elementary particle physics, there are many examples of this, in which models based on analogy with the atom are constructed or techniques and concepts arising in solid state physics, such as the high temperature expansion or super-conductivity, are used. In all these cases the basic physical entities are in no way related however, the laws, which govern them, can be modelled with (or described in terms of) the same mathematics.

Catastrophe theory has been applied to a very wide range of phenomenon spanning the instabilities in mechanical systems to brain activity. Some applications may be speculative in the extreme, however where no well organized mathematical structures exist, a model based on catastrophe theory is always something one can try if certain prerequisites or patterns are apparent. The latter become evident when we note what in fact catastrophe theory is about, which is the subject of the next paragraph.

I. RUDIMENTS OF CATASTROPHE THEORY

To state catastrophe theory in its entirety is a job for a mathematician and would require a considerable volume. However in simple terms one can follow E.C. Zeeman [4] and consider the following mathematical system, consisting of a function f in some multi-dimensional space (x,y,z,...) which depends on a set of parameters a,b,c,... (later to be identified with the control variables of a system). An example of f would be $f = a\,x^2 + b\,y^2 + c\,z^2$, the level surfaces of which (i.e. f = constant) are the conics (sphere, ellipsoid and hyperboloid). The particular type of conic is determined by the values of a, b and c or the relationship between them. (For example, a sphere corresponds to all the parameters being positive and equal e.g. $a = 1$, so that $f = R^2 = x^2 + y^2 + z^2$.) If one views the situation with the conics in the parameter space (a,b,c) then there will be special values or lines, which mark the transition point from one type of conic to another. This division of the function f into its dependence on the space of variables $(x,y,z,\ldots)$ and the parameter space $(a,b,c,\ldots)$ is an essential ingredient in catastrophe theory.

Let us now consider the set of stationary points in the space $(x,y,z,\ldots)$ defined by the vanishing derivatives of f i.e. $\partial f/\partial x = \partial f/\partial y = \partial f/\partial z = \cdots = 0$.*) If we denote this set of stationary values of f by S, then S is a smooth surface in the space $(x,y,z,\ldots,a,b,c,\ldots)$ with the only singularities (i.e. points of abrupt change or multi-valuedness) of the projection of the surface S onto the parameter space $(a,b,c,\ldots)$ (e.g. plane (a,b) in the case of only two parameters) being certain kinds of folded curves, cusps and the like. These are called the elementary catastrophes as classified by René Thom. This is provided the function f itself is a smooth generic function, which for our purposes means f is essentially a smooth continuous function of its variables $(x,y,z,\ldots)$ over the domain, in which it is defined. We shall refer to the work of Thom and Zeeman for a more precise and complete definition. We shall also leave to the excellent literature on the subject [1]-[3] a complete discussion of Thom's monumental theorem, only briefly summarizing the main gist here, which we shall illustrate with some simple and standard examples.

The kind of catastrophes that can occur (i.e. sudden jumps in S as a function of $a,b,c,\ldots$) does not depend on the dimensions of the space $(x,y,z,\ldots)$, which we shall for now think of as the response space of variables corresponding to the control variables $a,b,c,\ldots$ In fact the catastrophe can be described and classified in a general way in terms of only the variation and number of control parameters. It is in this latter property that the utility of the theory, as a mathematical model of complex (many variable) systems, lies. This is provided that the complexity lies in the dimension of the response space and not in the control space. An example of such a system, which has been discussed in terms of catastrophe theory for example by Zeeman, is the human brain. The latter is thought of as a very large number of coupled biochemical subsystems (cells) responding to a relatively few external stimuli.

*) If f was the energy function of a mechanical system, then this condition defines the stable points about which the system can oscillate. Hence the terminology structural stability in Thom's work.

Let us complete this brief discussion by a few simple mathematical examples, which can be thought of as standard reference models of Thom's elementary catastrophes. Consider the following (one-dimensional) function:

$$f = x^6/6 - ax - bx^2/2 - cx^3/3 - dx^4/4 \quad .$$

This is an example of a smooth generic function of the variable x, depending on four control parameters a, b, c and d, which is used as the reference of the so-called butterfly catastrophe. The latter embodies the lower order catastrophes such as the cusp and swallow-tail, when thought of as functions of only the parameters (a,b) and (a,b,c) respectively. (These names arise from their graphical shape.)

The stationary surface S defined by $df/dx = 0$ has the form

$$x^5 - a - bx - cx^2 - dx^3 = 0 \quad . \qquad (1)$$

The projection of this surface onto the space (a,b,c,d) will not be single-valued and to illustrate how one can picture this projection, we start with a single parameter a and the quadratic form $x^2 - a = 0$. As a function of the parameter a this has the folded graph shown in Fig.1(a), the upper and lower branches of which correspond respectively to $x = +\sqrt{a}$ and $x = -\sqrt{a}$. If we look at the situation with the cubic form $x^3 - a - bx = 0$ and draw the surface S above the (a,b) plane, we find a folded sheet like that indicated in Fig.1(b). The projection of the fold onto the (a,b) plane is a cusp with two branches B_1 and B_2, respectively, which mark the boundaries of the region of multi-valuedness of x as a function of a and b. In the case where only the upper and lower sheets of S are accessible, one calls the region between B_1 and B_2 the bifurcation set and its size depends on the parameter b, which is accordingly called the splitting parameter. The butterfly catastrophe has the two additional parameters c and d, the possible role of which in a specific model will be discussed later. Returning to the standard example (1), one finds that the variation of the parameter c has the effect of moving the surface S up or down and the cusp left or right in the (a,b) plane as illustrated in Fig.1(c). For this reason it is thought of as the bias factor. Finally, the parameter d (called the butterfly factor) creates a new structure when it becomes positive. The projection of this latter structure on the (a,b) plane is shown in Fig.1(d). The situation as regards the surface S is shown in Fig.2, in which we see the new structure corresponds to a pocket in the upper surface leading to a response level between the two in the bifurcation set of the cusp case.

The whole situation discussed above is referred to as the butterfly catastrophe and although we have described it in terms of a one-dimensional response space (corresponding to the variable x), the considerations go through independently of the dimension of the latter space. This means if we consider the response of a complex multi-dimensional system like the human brain, to two or three external stimuli, we could monitor simply the behaviour of the whole system through the level of a certain chemical or the behaviour of the person involved, as regards determining the regions of sudden change.

One could go on to describe in a similar way other kinds of catastrophe in Thom's general classification theorem [5]. However this I feel is better left to the literature provided by the main proponents of catastrophe theory, some of which are mentioned in the bibliography. I will instead consider a toy model describing one of the most obvious family syndromes, namely its response to work and sleep, in order to provide an illustration of how one might make use of catastrophe theory in the context of the family system.

II. A TOY MODEL FOR THE FAMILY SYSTEM

We end this glimpse of catastrophe theory by considering a toy model of the disorders we suppose can occur to the family, when viewed as a system responding to control factors such as work and sleep. By the word toy we simply mean that the model we describe is in no way supposed to be taken seriously in its own right, but instead used to illustrate the type of strategy or thought that might go into using the theory to model family disorders. For a far more serious example of a somewhat parallel nature and involving apparently considerable research, we refer to the discussion of the anorexia disorder by J. Hevesi and E.C. Zeeman [6].

Consider a family, consisting of say, a father, mother and a child and further consider its daily routine of work and sleep. In a crude way we could monitor the family relationships in response to the average number of hours of work and sleep, by the amount and nature of contact among the members of the family (with some suitable weighting). We could give the set of behaviours S middle values for non-aggressive contacts (talk, play, etc.); lower values for aggressive contacts (arguments, tantrums, etc.) and very high values for little or not contact (i.e. separation). We note that the precise values assigned to the set of behaviours S is, by the basic theorem of Thom, immaterial. The disorders we have in mind in the family system are at the one extreme fights and tantrums, while at the other, some kind of separation or running away.

The control variables a and b of the catastrophe model would be respectively the average number of hours of sleep (or rest) per member of the family each day and the corresponding average number of hours work (or other tiring activity). An important factor in defining such variables will be the weight given to each member of the family, which could vary from family to family and depend on things like the age of the child. The variable a will therefore be expected to be of the form:

$$a = w_f a_f + w_m a_m + w_c a_c \quad \text{with} \quad w_f + w_m + w_c = 1 ,$$

where the weights corresponding to each member of the family (w_f, w_m, w_c) will take into account their individual effect relative to the other members. (Here the subscripted a's are defined for the individual members of the family.) As a function of the control variables a and b the normal rhythm of the family might look like the cycle C_1 on the response surface depicted in Fig.3(a). However in our model we shall think of work as a splitting factor, because it can cause stress, resulting in aggression and the breakdown of the normal order. For example, an over-worked family may well have regular arguments and serious upsets, this would be depicted in the model by the cycle C_2 in Fig.3(a). In this latter cycle the family never gets adequate sleep, which is further aggravated by the upsets. However the situation is still stable, the family copes and self-control is never completely lost.

In our model there was another path an over-worked family could have gone along, namely the amount of contact steadily decreases, until the family is effectively separated. This situation corresponds to the upper portion of the folded surface in Fig.3(a). The whole picture of the family reponse to these control variables is being represented by Thom's cusp catastrophe. If we go further and add to the model the possibility that under the stress cycle (or spiral) of behaviour C_2, one (or more) member of the family looses self control, then we can represent this by a shift of the cusp to the left as shown in Fig.3(b). This can be represented by a change in a new variable, namely the bias factor c discussed in Sec.I. In our model this variable would be a quantitative way of describing the ability of the family to cope with the stress situation discussed above. It would depend on the history and nature of the family (i.e. a psychological factor). When cycle C_2 crosses the cusp boundary B_2, then a catastrophe occurs, that is there is a sudden change in the behaviour of the family, for example, it breaks up. This might take the form of the husband and wife separating for a time or even the child running away. The advantage of a model based on catastrophe theory is that one does not have to worry about the precise definition of the response variables, provided the family system can be described in terms of the model, the same causes (i.e. changes in a,b,c) can be attributed to both the catastrophic response mentioned above.

This toy model can be pushed one stage further in analogy to the anorexia example of Zeeman and Hevesi mentioned earlier, by supposing there exists a fourth control variable d, to which the family can respond. If this variable is of the form of the butterfly factor described above, then its variation can cause a pocket to form in the upper surface, giving rise to three reponse levels and the three-surface situation depicted in Fig.2. In the anorexia example the factor d corresponds to reassurance given to the patient during hypnosis, the latter state itself corresponds to being on the middle surface created by the pocket. In our toy,example d could be linked to an outside influence such as some kind of family therapy. The idea one might persue is that if the sudden jumps between the lower surface S_L in Fig.2 and the upper response level in the model (surface S_U in Fig.2) can be transformed into jumps to the middle surface S_P in Fig.2, created by the therapy iself, then one could model progress in the therapy by a new route opened up by the structure of the butterfly catastrophe. Such a route is depicted by the arrowed path in Fig.2.

At this point I feel I have pushed the toy example far enough. However I hope it has illustrated, in relation to catastrophe theory, how one might go about organizing ones thoughts with a view to modelling disorders of the family system, if they are of a sudden or abrupt nature and can be linked to recognizable control factors. Whether or not such models say anything about reality will in general be an exceedingly difficult (probably impossible) task to prove on theoretical grounds (i.e. from the laws of chemistry and biology). Just as in the anorexia example, it would rather have to be established on empirical grounds. Even the problem of identifying and quantifying the control variables of the system will be a matter of much experimentation and experience.

Let me end this article with the following observation. It often happens that the use of a model provides no deep truth in itself. However the method in applying it opens up new doors, which lead to the deeper understanding we are seeking.

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[6] Described in Ref.2, pp.33-51.

FIGURE CAPTIONS

Fig.1 a) One-dimensional example of a double-valued function;

b) Two-dimensional example of the folded curve represented by Thom's cusp catastrophe;

c) Effect of control parameter c on the cusp projection represented by branches B_1 and B_2;

d) Projection of butterfly catastrophe on the a-b plane, showing four branches or boundaries of abrupt change.

Fig.2 Depicts the response surface corresponding to the butterfly catastrophe. The surface is folded like Fig.1(b), moreover it has an additional pocket starting from the upper surface and extending beneath it to a point above the lower surface.

Fig.3 a) A model for the family system based on the cusp catastrophe;

b) Shift of the response cycle into a region of abrupt change.

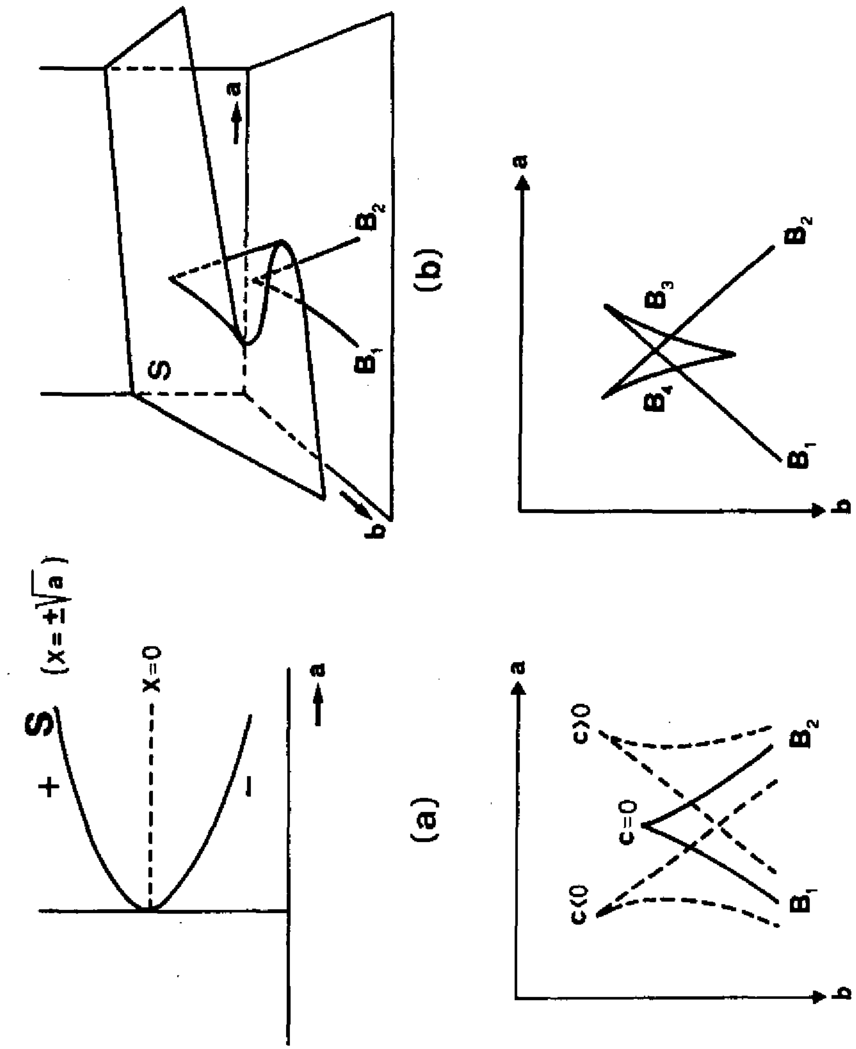

Fig 1

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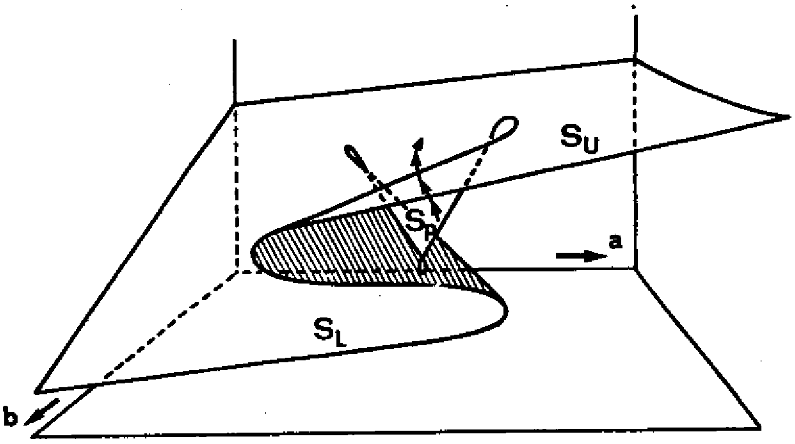

Fig 2

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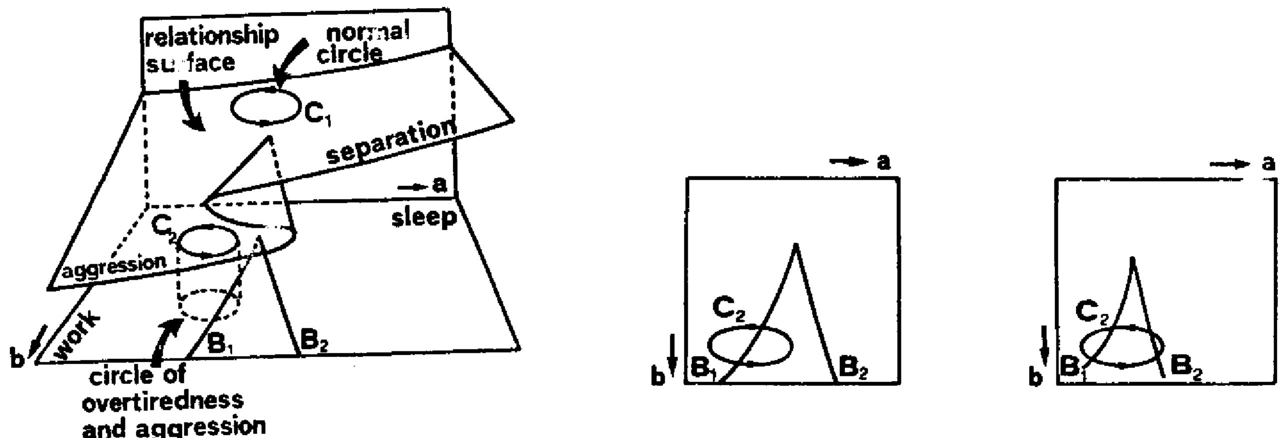

(a) (b)

FIG 3

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