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QCD SUM RULES FOR THE LIGHT QUARKS

VACUUM CONDENSATE

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International Atomic Energy Agency and United Nations Educational Scientific and Cultural Organization INTERNATIONAL CESTEE FOR THEORETICAL PHYSICS

> QCD SUM RULES FOR THE LIGHT QUARKS VACUUM CONDENSATE[®]

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ABSTRACT

Vfe re-evaluate the quark condensate renormalization group invariant term $(m^{\prime}_{ij} + m^{\prime}_{ij}) < 0$ $\psi_{ij} \psi_{ij} + \psi_{ij} \psi_{ij}$ o $\psi_{ij} \psi_{ij}$ i $i \in \{1, 3, \ldots, m\}$ and QCD sum rules of the Laplace transform type to two-loops. It is shown that the PCAC result is respected for the u,d quarks but is violated by a factor 30 to 44% for the u,s quarks. Using the recent estimates of the invariant quark masses \mathbb{m}_{α} + \mathbb{m}_{α} and \mathbb{m}_{α} + \mathbb{m}_{α} , we extract bounds on the spontaneous masses \mathbb{m}_{α} defined to One loop as:

$$
< 0 \quad \overline{\psi}_1 \quad \psi_1 \, \big(0 \, > \, (Q^2) \, = \, (-\widehat{\psi}_1)^3 \, , \, (\log \, Q/\hbar)^{12/(33-2n)} \, \text{ for } \, \text{SU(3)}_C \, \times \, \text{SU(n)}_F.
$$

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Recently, various QCD sum rules have been proposed in the literature in order to understand the low energy physics (aadron masses and couplings) via the information of the short distance behaviour of the two-point functions associated to various types of currents $\{1,2,3\}$. For a particular interest for light mesons physics is the sum rule of the Laplace transform type, originally proposed by SVZ and collaborators [l] and recently revised by S.R.[U]. Within the framework of this later sum rule, it has been shovn for example, that the p-meson coupling to the photon is predicted with a good accuracy and that the sum of invariant masses $\widehat{m}_{1} + \widehat{m}_{3}$, $\widehat{m}_{1} + \widehat{m}_{5}$ differs by a factor 2 to 3 to the well known current algebra Leutwyler masses [5]. Within such a sum rule programme, it is also a time to revise the PCAC estimate of the quark-condensate programme, it is also a time to revise the PCAC estimate of the quark-condensate operators \mathcal{M}^{1D} , if \mathcal{M}^{1D} , if In fact, the well known PCAC results are

$$
(\frac{m_{u}+m_{d}}{c})\langle 0|\bar{f}_{u}f_{u}+\bar{f}_{d}'f_{d}/c\rangle=-2\bar{f}_{f}^{2}m_{f}^{2}, \qquad (1a)
$$

and

$$
(m_{u}+m_{g})<0/\bar{F_{u}}F_{u}+\bar{F_{g}}F_{g}/0>=-e\int_{0}^{2\pi}M_{k}^{2}
$$
 (1b)

where the quantities in the LHS are renormalisation group invariant and have no anomalous dimension, $r_{\pi} \ge 93.28$ MeV and $r_{K} \ge 1.16f_{\pi}$ are the pion and kaon decay amplitudes which control the $\pi + \mu\nu$ and $K + \mu\nu$ decays. To our knowledge, an appropriate sum rule which is very sensitive to $O_M^1 = -(m_u + m_i)^*$ • < 0| $\bar{\psi}_1$, $\psi_1 + \bar{\psi}_i$, ψ_3 |0 > , i \cdot d, s is the sum rule related to the longitudinal part of the axial-vector two-point function $D_5(q^2)^{1}$ studied in BNRY [3]. In fact $D_c(q^2)$ is a renormalization scheme dependent to lowest order of QCD, so it is convenient to work with the superficially convergent function:

$$
\phi(q^t) = \frac{Q}{2q^2} Q_{\overline{q}}(q^t) , \qquad (2)
$$

which obeys the unsubstracted dispersion relation:

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$$
\phi(q^2) = \frac{1}{r} \int_{t_0}^{t} \frac{dt}{(t+Q^2)^2} \frac{1}{t} Im \int_{S}^{L}(t)_{y=0}^{t-1} \frac{q^2}{z} \phi^2 D_{(3)}
$$

where $\psi_{\varsigma}(t)$ is the two-point function associated to the pseudoscalar current which is related to $D_{\rm g}(q^2)$ by the Ward identity:

$$
g^2 D_s (94) = \frac{11}{15} (94) - \frac{11}{15} (9).
$$
 (4)

la the Nambu-Goldstone realization of chiral symmetry, the non-perturbative effect to $\psi_{\mathsf{C}}(0)$ is known to be:

$$
\frac{1}{2}(0) = - (m_{u} + m_{i}) \langle 0 / \sqrt{\frac{1}{n}} \frac{u}{u} + \frac{\overline{y}}{t} \frac{u}{t} / 0 \rangle, i \in d_{1} \delta_{2} \tag{5}
$$

where the RHS is given by eq. (1) if PCAC is used. In the following we shall derive an alternative estimate of $\psi_5(0)$ using the dispersion relation and the QCD sum rules obeyed by $D_{\varsigma}(q^{\varsigma})$. For the sake of simplicity, let's first use the lowest order QCD expression of $\psi_5(q^2)$ within the WES dimensional
renormalization scheme², Using eqs. 2 to 4, saturating the spectral function $Im\psi_{\epsilon}(t)$ by the K-meson $(K^{+}$ channel) -sad using the positivity of the "continuum" contribution to the spectral function, we get the naive inequality:

$$
\frac{16}{4}(\frac{6}{4})
$$
 $\frac{12}{4} \frac{16}{4} \frac{1}{16} \frac{1}{4} - \frac{3}{8} \left(\frac{1}{2} \frac{1}{16} + \frac{1}{16} \right)^2$ $\frac{1}{4} \frac{1}{16} \frac{1}{4} \frac{1}{16} \frac{$

with $d = \gamma_1/\beta_1 = 12/(33 - 2n)$ for $SU(3)_C$ x $SU(n)_R$ ($\gamma_1 = 2$ and $\beta_1 = - \frac{11}{2} + \frac{n}{2}$ are respectively the first coefficients of the mass - anomalous dimension and of the β -function).

However, the naive constraint in eq. (6) can be improved, using the QCD sum rules techniques $[1,2,3]$. A particular interest for us is the Laplace transform sum rule discussed in ref. $[3,4]$, which is obtained by applying to both sides of eq. (3) the operator:

$$
\sum_{n=0}^{\infty} = \lim_{\substack{q' \to \infty \\ n \to \infty}} \frac{q^2}{n^2} m^2 \frac{(-1)^n}{n!} (\partial^2)^n \frac{\partial^n}{(\partial \partial^2)^n} . \tag{7}
$$

-3-

Then, the Laplace transform of eq. (3) reads:

$$
F(m) = \frac{1}{r} \int_{0}^{a} dt e^{-\frac{t}{m}} = \frac{1}{t} L_{m} \frac{1}{s}(t) .
$$
 (8)

The information on the RHS of eq. (8) can be improved by adding to the K-meson contribution that of the continuum vhich is:

$$
F_c(m^t) = e^{-\frac{t}{k}/m^t} \int\limits_0^a dt \, e^{-\frac{t}{k}/m^t} \int\limits_{t+t_c} Jm \, \frac{\mu}{k}(t+t_e), \quad (9)
$$

where $\sqrt{t}_c \neq M_K + 2m_\pi$ is the continuum threshold. Using the QCD model for
the continuum, one can analytically evaluate $F_c(M^2)$, e.g. expanding $\overrightarrow{(\text{tt}_c)}$ Im $\psi_5(\text{tt}_c)$ in the form of Taylor series.

From a practical point of view, we need the Laplace transform:

$$
\hat{L} \frac{d^n}{(dz)^n} \left\{ \frac{1}{z^{\alpha+1}} \left(\frac{1}{\log z} \right)^{\beta+1} \right\}, \tag{10}
$$

where $x = \frac{f}{h^2}$. This can be done in a suitable way, once we use the master formula given in ref $[4]$. After some algebra:

$$
\frac{2}{(\pi + \frac{t_{e}}{h})^{a+1}} \frac{1}{(\frac{t_{0}}{h})(\pi + \frac{t_{e}}{h})^{\beta+1}} = e^{-\frac{t_{e}}{h}(h^{2} + \frac{1}{h})}
$$

$$
(\frac{1}{h_{0}y})^{a+1} \{1 - (\beta + 1) \}^{a+1} (\frac{1}{h_{0}y} + \frac{1}{h_{0}z_{y}}) \}^{(11)}
$$

with: $\frac{1}{f(x)} = \frac{1}{f(x)} \frac{dF(z)}{dz}$ and $y = \frac{M^{2}}{h^{2}}$.

Using the QCD expression of $F(M^2)$ and $F_C(M^2)$ to two loops and including the leading non-perturbative effects, the improved Laplace transform version of eq. (6) reads:³⁾

-k-

$$
\frac{V_{s}(0)}{M^{2}} \geq a_{1}^{2}k^{2} \frac{M_{s}^{2}}{M^{2}}e^{-\frac{M_{s}^{2}/M^{2}}{M_{s}}}-\frac{3}{\theta\pi^{2}}(\frac{M_{u}+M_{s}}{M_{s}})^{2}\frac{1}{(\frac{1}{\theta_{0}}M_{h})^{201}/\beta_{1}}
$$

\n
$$
-(1-e^{-2\frac{1}{2}(M_{s})})\left\{1+\frac{2}{\pi}(\frac{M_{s}}{M_{s}})\frac{M_{s}}{M_{s}}+2\frac{1}{\theta_{2}}(\frac{M_{s}}{M_{s}})^{2}\right\}
$$

\n+ $\frac{2\frac{1}{2}\beta_{2}}{\beta_{2}^{2}}\frac{1}{\theta_{0}}\frac{M_{s}}{M_{s}}\frac{M_{s}}{M_{s}}-\frac{m_{u}}{M_{s}}\frac{1}{\theta_{0}}\frac{M_{s}}{\overline{m}_{s}^{2}}$
\n- $\frac{1}{3}\pi^{2}\frac{1}{M_{s}}(\frac{1}{\theta_{0}}-\frac{m_{u}}{R})<\frac{\overline{u}_{1}}{M_{s}}\frac{1}{\theta_{0}}+\frac{m_{u}-m_{s}}{R_{s}})\frac{\overline{v}_{1}}{M_{s}}\frac{(12)}{M_{s}}\frac{1}{\theta_{0}}\frac{1}{\theta_{$

= $\frac{1}{\sqrt{2\pi}} \frac{1}{\log M/\Lambda}$, $\frac{1}{\pi}$ and $\frac{1}{\pi}$ are respectively the invariant and running juark masses in the MS scheme it the two-loop level, $\tau_{\rm g}$ = 0.5772 is the Euler constant, $\theta_{\gamma} = -\frac{21}{4} + \frac{19}{12}$ n and $\gamma_{\gamma} = \frac{101}{12} - \frac{5}{18}$ n fo $SU(3)_C$ x $SU(n)_{\overline{F}}$. The leading non-perturbative effects are controlled by the vacuum expectation values $\langle 0|\bar{\psi}_i|\psi_i|0\rangle$ and $\alpha_{\rm g} \langle 0|\bm{G}^{\rm HV}\bm{G}_{\rm inv}^{(\bm{a})}|0\rangle$. The

recent estimate [7] via charmonium sum rules gives:

The quark condensate contribution is left as a free parameter, but we assume for convenience $'\colon \, \, \check{\textbf{w}}_1 \not\vdash_u \, \check{\textbf{w}}_2 \, \times \, \overline{\textbf{w}}_5 \, \, \vdash_v, \, \, \textbf{s}$ o that we rewrite to a first approximation:

$$
(m_{\varphi}-\frac{m_{\mu}}{2})<\overline{Y_{\mu}}Y_{\mu}>\qquad(m_{\mu}-\frac{m_{\mu}}{2})<\overline{Y_{\varphi}}Y_{\varphi}>\simeq-\frac{1}{2}\gamma_{\varphi}^{\mu}(0),
$$
 (13)

The above term will however improve the estimate of $\psi_6(0)$ by a correction practically negligible. We use as input in eq. (12) the invariant mass \hat{m} .

obtained in ref [4] and the values of A^{π}_{MS} \rightarrow 70 v 210 MeV obtained in ref [8] from the uses of the QCD sum rules to the isovector part of the e e" • Hadrons data (note that small values of $\Lambda_{\overline{MS}}$ correspond to higher values of \widehat{m}_i as shown in ref [4]). The shape of $\psi_{\varsigma}(0)$ against M^2 for various \hat{m} $\left(\widehat{m} \equiv \widehat{m}_{1} + \widehat{m}_{\alpha}\right)$ and the consistent values of $A_{\overline{MS}}$ is given in Fig. 1 for two flavours. We optimize the estimate of $\psi_{\varsigma}(0)$ by demanding that the continuum contribution to the spectral function is less than 10% in order to justify the K-meson dominance. We ask also that the QCD corrections to the lowest order expression is less than 40% . Such conditions are obtained for $M\geq 2M_v$, i.e. at the value of M around the \blacklozenge -meson mass. Then, we deduce our optimal estimate:*^*

3.2 10²
$$
\leq
$$
 0² \leq 4.2 10³ 6eV⁴ 3 $\frac{1}{2}$ 0.97 \leq 1 $\frac{1}{10}$ \leq 0.21 6eV $\Big|_{(14)}$

vhieh is significantly smaller than the PCAC estimate:

$$
\left.\frac{\partial}{\partial n}\right|_{PCAC} \simeq 5.7 \; \text{A}^{-3} \text{GeV}^{\frac{1}{2}}. \tag{15}
$$

In the π – channel, the natural scale of optimization is $M_{\rm g}$ (see e.g. ref 4). At this value of M, the PCAC estimate of $\mathcal{O}_{\rm M}^{\rm u}\cong 2 f_\pi^c$ m is obtained from eq. (12) with a 5% of accuracy due to the smallness of the π and u, d masses. Let's introduce the renormalization group invariant operators $\hat{\mu}_i$ of BURY, which to the one-loop of QCD is defined by

$$
\langle o | \overline{\psi_i} \psi_i | o \rangle = \left(-\hat{\mu}_i \right)^3 \left(\log \vartheta / \sqrt{\frac{\pi}{33 - 2}} n \right) \tag{16}
$$

and let's use the recent estimates of the sum of invariant quark mass values \widehat{m}_{1} [4]; we can deduce from eqs. (14) and (15), assuming $\widehat{\mu}_{11} \sim \widehat{\mu}_{d}$ (SU(2)_p symmetric vacuum):

$$
\hat{p}_{\mu} \approx \hat{p}_{\mu} \leq \frac{180 \text{ MeV}}{260 \text{ MeV}} \text{ for } \hat{m}_{\mu} + \hat{m}_{\mu} \geq \frac{30 \text{ MeV}}{10 \text{ MeV}} \tag{17}
$$

and

$$
\widehat{\mu}_{0} \leq \frac{170 \text{ MeV}}{220 \text{ MeV}} \qquad \text{for} \quad \widehat{m}_{u} + \widehat{m}_{0} \geq \frac{300 \text{ MeV}}{150 \text{ MeV}} \qquad (18)
$$

The above results indicate that the chiral $SU(3)_{\overline{p}}$ symmetry relation $\hat{\psi}_n \leq \hat{\psi}_n \leq \hat{\psi}_n$ could be realized though the FCAC relation is highly broken for the case of the s-quark. An extension of the above discussion to the case of heavy quark could easily be done. However, we still need more precise information on the heavy pseudoscalar meson decay amplitude analogous to f^{\prime}_{π} .

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The author vould like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He would also like to thank N. Paver, E. de Rafael and F. J. Yndurain for discussions. (6)

FOOTNOTES

- 1) In this paper, we shall follow the notation and normalization of BNRY in ref [3].
- 2) For a recent review of dimensional renormallzation applied to QCD, see e.g. ref [6] and references therein.
- 3) We shall still conserve the inequality sign due to the positivity contribution of the radial excitation of the K-states to the spectral function.
- 4) The reader must note that the contribution of the $\langle \bar{\psi} | \psi \rangle$ term to the RHS of the sum rule is small at the value of M^2 which we choose for our optimization, so that our assumption $\langle \psi_u \psi_u \rangle \geq \langle \psi_g \psi_g \rangle$ does not affect in a crucial way our estimate of $\psi_5(0)$.
- 5) Note that in our conditions of optimization, the inequality sign in Eq. 12 can be considered as an estimate. In fact, there is no experimental evidence of low mass orbital excitations of the K-meson which could notably increase the value of 0_M^8 .
- 6) In preparing this paper, we have been informed by N.S. Craigie of the recent work in ref [9] where the authors discuss the ≤ 0 [$\bar{\psi}_i \psi_i$] o > term $i \equiv u$, d within the framework of QCD sum rules applied to the vector and tensor currents. However at their choice $M^2 \simeq M^2$, the contribution of the next non-leading logarithms and higher twist operators to their estimate could be important.

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FIGURE CAPTION

<u>Fig. 1</u> Behaviour of $\psi_{\bf q}(0) \equiv -({\bf m}_{_{11}} +$ for various values of $\hat{m} = \hat{m}_{11} + \hat{m}_{22}$ and A in the MS scheme. versus M renormalization

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