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MASS SPLITTING BETWEEN B^+ AND B^0 MESONS *

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ABSTRACT

Using the mass differences $K^+ - K^0$ and $D^+ - D^0$ as constraints, the mass splitting between B^+ and B^0 mesons has been calculated as -1.3 ± 1.22 MeV in the nonrelativistic quark model and -1.6 ± 0.72 MeV in the chiral symmetry breaking theory.

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The recent observation of the upsilon resonances [1] seems to establish the existence of a new flavor of the quark, the bottom flavor. These resonances are interpreted in the quarkonium model as the bound states of the $\bar{b}b$ quark system where b is the bottom-flavored quark.

One of the interesting problems concerning the bottom flavor may be the mass splitting between the pseudoscalar $B^+(u\bar{b})$ and $B^0(d\bar{b})$ mesons, belonging to the same isospin doublet. Though the direct observation of these particles is as yet difficult, we expect that they will be found in the near future. Once their mass splitting $\Delta B = B^+ - B^0$ be measured, it will provide an independent test of the theories which have been given for the explanation of the mass splittings $\Delta D = D^+ - D^0$ and $\Delta K = K^+ - K^0$. This is possible because any theory concerning the isospin nonconservation should explain ΔB as well as ΔD and ΔK in a consistent way, since all of them are of the same origin.

We wish to calculate in this letter the mass splitting ΔB within the theories which have been applied to the calculations of ΔD. In this way, we are keeping the consistency and can provide a meaningful test of those theories when B^+ and B^0 are observed in the future. If the future measurement of ΔB would show an agreement with our result, those theories may survive. On the other hand if there is a considerable discrepancy between them, we expect that those theories should be modified.

Since the experimental data [2]

$$\Delta D = 5.0 \pm 0.8 \text{ MeV}$$
$$\Delta K = -4.01 \pm 0.13 \text{ MeV} \qquad (1)$$

enter as constraints in our calculation, the number of free parameters will be reduced in our case than in the previous calculations where ΔD has been an unknown.

In the literature, we can find many ideas [3,4,5] on the isospin breaking which have been applied to the calculation of ΔD. Among others, ΔD has been estimated 6.7 MeV by Lane and Weinberg [4] in the nonrelativistic quark model and the Dashen's theorem [6], while ΔD was given between 2.5 and 4.5 MeV in the chiral symmetry breaking theory [5]. We consider that these two estimates are most compatible with the experiments, eq.(1). Therefore, we will take the above two theories in our calculation of ΔB.

Notice first that the mass splitting of any isospin multiplet can be divided into two parts: one is the difference in the Coulomb interactions between the constituent quarks and the other is the difference in the masses of the constituent quarks. Thus, we may write down the mass splitting as

$$\Delta\mu = (\Delta\mu)_{Coul} + (\Delta\mu)_{quark} , \qquad (2)$$

where μ = K, D or B and $(\Delta\mu)_{quark} = (m_u - m_d)_\mu$ is essentially the mass difference between the up and the down quark in μ.

In the nonrelativistic quark model, the Coulomb part may be parametrized with a constant $\langle 1/r \rangle$ times the difference of the products of the quark charges. We have $(\Delta K)_{Coul} = (\alpha/3)\langle 1/r_K \rangle$ and $(\Delta D)_{Coul} = (2\alpha/3)\langle 1/r_D \rangle$. As Lane and Weinberg have calculated, neglecting the higher-order couplings, $(\Delta K)_{Coul}$ is given as 2/3 of the total photon exchange contributions to ΔK :

$$(\Delta K)_{Coul} = (2/3)(\Delta K)_\gamma . \qquad (3)$$

Applying the same procedure for the d mesons, one would obtain that $(\Delta D)_{Coul} = (4/3)(\Delta D)_\gamma$. If $\langle 1/r_\mu \rangle$ is given equal for both the K and the D mesons, then $(\Delta K)_\gamma = (\Delta D)_\gamma$. Moreover, as long as $\langle 1/r_\mu \rangle$ remains constant we can easily see that $(\Delta\mu)_\gamma$ is equal for all μ.

However, it seems to be reasonable that $\langle 1/r_\mu \rangle$ is not a constant but a variable in μ, since an increasing behavior of $\langle 1/r_\mu \rangle$ may be exhibited in the nonrelativistic linear potential model [7] as the reduced mass of the constituent quarks increases. According to this approach, the ratios among $\langle 1/r_\mu \rangle$ are calculated as

$$\langle 1/r_B \rangle = 1.15 \langle 1/r_K \rangle \quad \text{and} \quad \langle 1/r_D \rangle = 1.1 \langle 1/r_K \rangle , \qquad (4)$$

where we take $m_u = m_d$ = 340 MeV, m_s = 540 MeV, m_c = 1500 MeV and m_b = 4700 MeV. It immediately follows that

$$(\Delta B)_{Coul} = 1.15(\Delta K)_{Coul} \quad \text{and} \quad (\Delta D)_{Coul} = 2.2(\Delta K)_{Coul} . \qquad (5)$$

Moreover, regarding $\langle 1/r_\mu \rangle$ as a typical constant related to the size of the hadron, it would be interesting to notice that the increasing behavior is also expected in the bag model [8,9], where the inverse of the bag radius increases in response to the reduced pressure as one of the constituent quarks becomes heavier. Therefore, following these arguments, we would take the idea of the variable $\langle 1/r_\mu \rangle$ as given by eq.(4).

Another assumption we wish to make is that $(\Delta\mu)_{quark} = (m_u - m_d)_\mu$ may differ between different isospin multiplets. This assumption might be justified in the bag model where the predictions of the electromagnetic mass differences of the mesons with fixed Δ_{quark} [10] have been no better than those with varying Δ_{quark} [9]. If we set Δ_{quark} to be fixed, then by subtracting $(\Delta K)_{quark} = (\Delta D)_{quark}$ from eq.(1) we find that

$$(\Delta K)_\gamma = 0.47 \text{ MeV}$$

where we have retained the relations eq.(5).

However, this value reveals a remarkable disagreement with Dashen's theorem [6] on the photon exchange contributions to the squared-mass splittings. There we have

$$(m_{K^+}^2 - m_{K^0}^2)_\gamma = (m_{\pi^+}^2 - m_{\pi^0}^2)_\gamma .$$

Since the pion mass splitting is solely due to the photon exchange, it follows that $(m_{K^+}^2 - m_{K^0}^2)_\gamma = m_{\pi^+}^2 - m_{\pi^0}^2$, or [11]

$$(\Delta K)_\gamma = (m_{K^+} - m_{K^0})_\gamma = 1.27 \text{ MeV}. \quad (6)$$

Hence, as far as we believe in the correctness of Dashen's theorem, we would have to let Δ_{quark} vary.

Adopting Dashen's value, eq.(6), we apply it into eq.(3) with eq.(5) to calculate $(\Delta K)_{Coul}$ and $(\Delta D)_{Coul}$. The results are sustituted into eq.(2). Then the experimental data in eq.(1) would yield

$$(\Delta K)_{quark} = -4.86 \pm 0.13 \text{ MeV}$$
$$(\Delta D)_{quark} = 3.14 \pm 0.8 \text{ MeV}.$$

The physical intuition suggests that $|(\Delta B)_{quark}|$ may well be less than 3.14 MeV, since there exists a tendency of decreasing $(\Delta\mu)_{quark}$ as the mass of the constituent quark increases. In accordance with the bag model [9], we parametrize this tendency as

$$(\Delta\mu)_{quark} = A \langle 1/r_\mu \rangle + B$$

identifying r_μ with a typical length proportional to the bag radius. Since we have the ratios among $\langle 1/r_\mu \rangle$ from eq.(4), we need not evaluate the coefficients A and B . Rather, we can find easily after some arithmetics that

$$(\Delta B)_{quark} = -(\Delta D)_{quark} - 0.5 \left\{ (\Delta D)_{quark} + (\Delta K)_{quark} \right\}$$

which can be rewritten in terms of $(\Delta K)_\gamma$ as

$$(\Delta B)_{quark} = -1.5 \left\{ \Delta D - 2.2(2/3)(\Delta K)_\gamma \right\} - 0.5 \left\{ \Delta K - (2/3)(\Delta K)_\gamma \right\} , \quad (7)$$

where the sign is chosen so as to give a negative value for $(\Delta B)_{quark}$. On the other hand, $(\Delta B)_{Coul}$ can be calculated from eq.(5) as

$$(\Delta B)_{Coul} = 1.15(2/3)(\Delta K)_\gamma . \quad (8)$$

Adding eqs.(7) and (8) together, we obtain an expression for ΔB in terms of $(\Delta K)_\gamma$:

$$\Delta B = -(1.5\Delta D + 0.5\Delta K) + 4.95(2/3)(\Delta K)_\gamma . \quad (9)$$

We plot eq.(9) versus $(\Delta K)_\gamma$ in Fig.1. The Dashen's value gives

$$\Delta B = -1.3 \pm 1.22 \text{ MeV} , \quad (10)$$

where the uncertainty is propagated from the experimental errors.

We note that the numerical coefficients in eq.(9) depend primarily on the ratios in eq.(4). However, the above result would not change within the uncertainty bound though the ratios be slightly altered.

Now, let us calculate ΔB in the chiral symmetry breaking theory for the sake of comparison. Within the chiral symmetry breaking schemes proposed by Gell-Mann et al., [12] the square-mass splittings are explained as the isospin symmetry breaking terms plus the photon exchange contributions. Thus, the subtraction of the photon exchange contributions would yield a linear relationship between the square-mass differences and the isospin breaking. This may be expressed as

$$\Delta u_3 = 2 m_\mu \frac{f_\mu}{\sqrt{Z_\mu}} \left\{ \Delta\mu - (\Delta\mu)_\gamma \right\} , \quad (11)$$

where m_μ is the averaged mass of the μ-th isospin doublet and Δu_3 represents the isospin breaking due to the third scalar density of the chiral symmetry. Since u_3 is the only source of the isospin breaking in this scheme, it follows that Δu_3 is equal for all μ. Assuming $\sqrt{Z}_\mu$ = constant [11], we have

$$2 f_B m_B \left\{ (\Delta B)_\gamma - \Delta B \right\} = 2 f_D m_D \left\{ \Delta D - (\Delta D)_\gamma \right\} . \qquad (12)$$

In order to estimate ΔB, we have to know the decay constants f_D and f_B. This can be done by observing that the current quark masses are related with the mass spectra of the pseudoscalar mesons. The relation reads [13]

$$m_b / m_c = (2 f_B m_B^2 - f_\pi m_\pi^2)/(2 f_D m_D^2 - f_\pi m_\pi^2) . \qquad (13)$$

Neglecting the small pion mass, we substitute eq.(13) into eq.(12) to obtain

$$(\Delta B)_\gamma - \Delta B = (m_c/m_b)(m_B/m_D)\left\{ \Delta D - (\Delta D)_\gamma \right\} . \qquad (14)$$

Again, we can rewrite eq.(14) in terms of $(\Delta K)_\gamma$ as

$$\Delta B = -\frac{m_c}{m_b}\frac{m_B}{m_D}\Delta D + \left\{ 2.2\frac{m_c}{m_b}\frac{m_B}{m_D} + 1.15 \right\}(\Delta K)_\gamma . \qquad (15)$$

Taking m_c and m_b as 1.5 GeV and 4.7 GeV respectively, and m_B as 5.27 GeV [14], we plot eq.(15) versus $(\Delta K)_\gamma$ in Fig.1. The Dashen's value gives in this case

$$\Delta B = -1.6 \pm 0.72 \text{ MeV} . \qquad (16)$$

We consider that this numerical result is quite compatible with the

estimate from the nonrelativistic quark model, eq.(10).

We note that if we attempt to evaluate ΔB from ΔK rather than ΔD as done above, we would have a different result. Using m_s = 150 MeV for the current mass of the strange quark, we find ΔK gives ΔB = +0.6 MeV, a positive value. However, it seems to be reasonable to take ΔD rather than ΔK for the evaluation of ΔB, since the approximation of neglecting the pion mass is not adequate for the K mesons [4].

Anyhow, it is remarkable that ΔB is predicted much smaller than K both in the nonrelativistic quark model and in the chiral symmetry breaking theory.

Up to now, we have estimated the mass difference between B^+ and B^0 using ΔK and ΔD as constraints. We have investigated how ΔB is predicted in those theories concerning the isospin nonconservation we have found that the mass splitting ΔB is quite small. We may conclude that $|\Delta B| \lesssim$ 2.0 MeV in the nonrelativistic quark model and in the chiral symmetry breaking theory. We expect that the future measurement of ΔB may give certain criteria for those theories.

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References.

[1] S. W. Herb et al., Phys. Rev. Lett. 39(1977) 252;
T. Boehringer et al., Phys. Rev. Lett. 44(1980) 1111;
D. Andrews et al., Phys. Rev. Lett. 45(1980) 219.

[2] Particle Data Group, Phys. Lett. 75B(1978) 1.

[3] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. Lett. 37(1976) 255;
S. Ono, Phys. Rev. Lett. 37(1976) 655;
W. Celmaster, Phys. Rev. Lett. 37(1976) 1042;
N. G. Deshpande, D. A. Dicus, K. Johnson and V. L. Teplitz, refs. 9 and 10 .

[4] K. Lane and S. Weinberg, Phys. Rev. Lett. 37(1976) 717.

[5] R. Dutt and S. N. Sinha, Phys. Lett. 70B(1977) 103.

[6] R. Dashen, Phys. Rev. 183(1969) 1245.

[7] H. J. Schnitzer, Phys. Rev. D13(1976) 74.

[8] E. Golowich, Phys. Lett. 91B(1980) 271;
J. F. Donoghue and K. Johnson, Phys. Rev. D21(1980)1975.

[9] N. Deshpande, D. A. Dicus, K. Johnson and V. L. Teplitz, Phys. Rev. D15(1977) 1885.

[10] N. Deshpande, D. A. Dicus, K. Johnson and V. L. Teplitz, Phys. Rev. Lett. 37(1976) 1305.

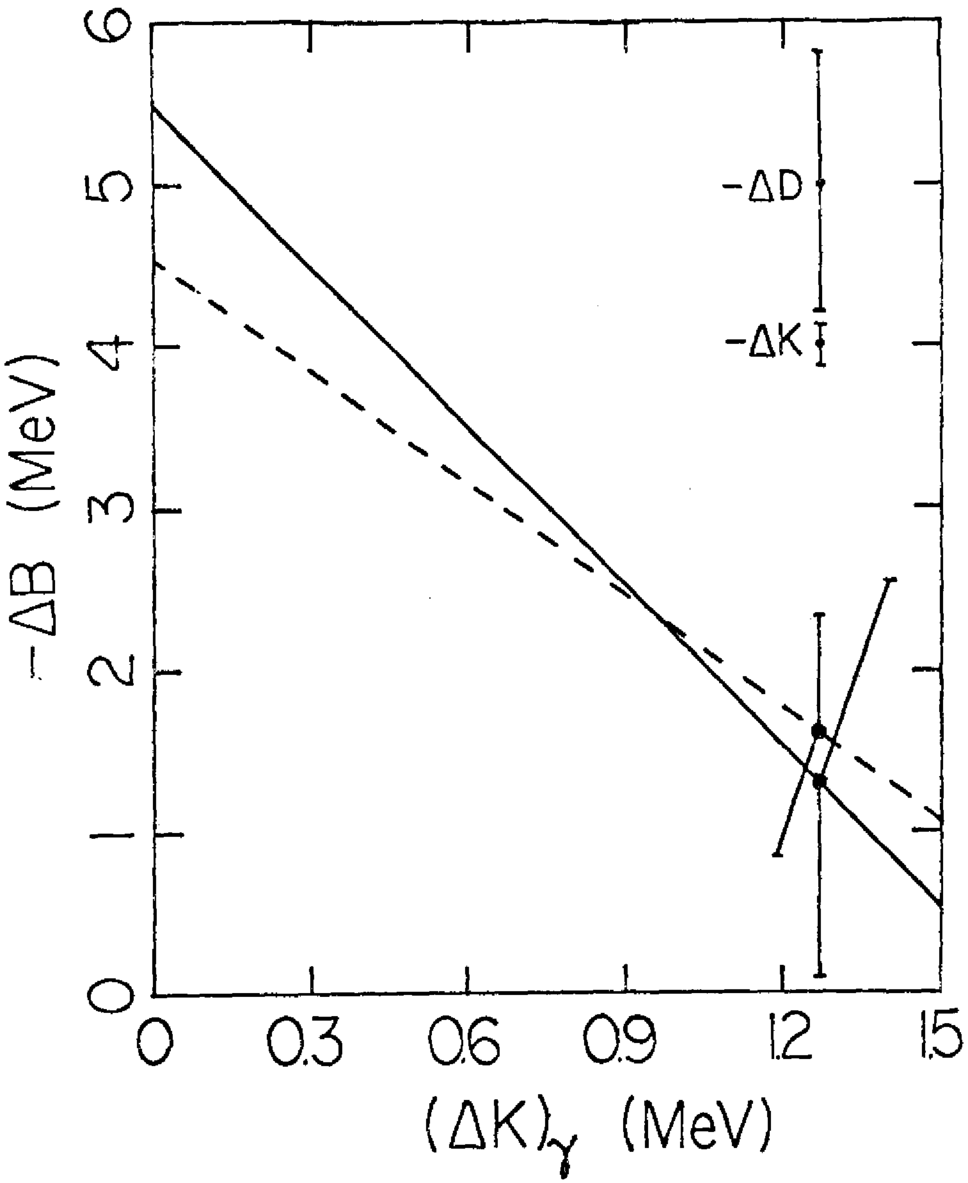
[11] N. G. Deshpande and D. Soper, Phys. Rev. Lett. 41(1978) 735.

[12] M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. 175(1968) 2195.

[13] S. K. Oh, to be submitted in Phys. Lett. B.

[14] G. Finocchiaro et al., Phys. Rev. Lett. 45(1980) 222.

Fig.1. ΔB versus $(\Delta K)_\gamma$ in the nonrelativistic quark model (solid line) and in the chiral symmetry breaking theory (dashed line). For the sake of comparison, ΔK and ΔD are also represented at the Dashen value of $(\Delta K)_\gamma$.



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