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DIRAC'S LARGE NUMBER HYPOTHESIS AND THE RED SHIFTS OF DISTANT GALAXIES *

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ABSTRACT

Dirac has postulated that all large dimensionless numbers occurring in Nature must be inter-related. On this basis he suggested that the gravitational coupling decreases linearly with time (as measured by atomic clocks) while the number of nucleons in the observable Universe increases quadratically with time. It has already been demonstrated that one of the three possible modes for Dirac's theory to work, the "multiplicative creation" mode, is not compatible with the observations of pulsars. Here we demonstrate that the other mode of creation of matter also faces severe problems, and we suggest that the comparison of red shifted galaxies could provide a comparatively "clean" test of Dirac's theory.

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Dirac has argued that all large dimensionless numbers should be related to the age of the Universe 1)-4). The reasoning is that the age of the Universe, measured in units of the time taken to traverse the classical radius of the electron is a large number $\sim 7 \times 10^{39}$. Assuming that all the other dimensionless constants were related to each other by geometrical factors (like 2π etc.) in the early Universe, any large number must come from the variation of time. Though the significance of the classical radius of the electron for these purposes is not clear, the idea is certainly very attractive. However, it seems virtually impossible to develop a testable theory from such a general principle. It is a tribute to Dirac's inventiveness that he was able to build such a theory. He pointed out that the ratio of gravitational to electromagnetic forces between a proton and an electron is $\sim 2 \times 10^{39}$. Thus, the universal gravitational "constant", G, should be related to the age of the Universe, t, by

$$G \sim t^{-1} \quad . \tag{1}$$

The operational statement is that the gravitational force will be seen to decrease if measured by atomic (as opposed to gravitational dynamical) methods. He also noticed that the estimated number of nucleons in the observable Universe, n, is $\sim 10^{78}$-10^{79}, which is roughly the square of the age of the Universe, measured in the appropriate units, i.e.

$$n \sim t^2 \quad . \tag{2}$$

Thus, either matter is being created in the form of nucleons, or more and more matter is coming into the range of observation. Using the facts contained in Eqs.(1) and (2), Dirac develops a testable theory having three possible modes.

The three possible modes just mentioned are the means whereby Eq.(2) can be satisfied. One method would be to have continuous creation of matter throughout a finite Universe, being more where more matter is present. Dirac calls this mode "multiplicative creation," and we shall denote it by "(a)". Another method would be to have matter created uniformly throughout the finite Universe. This is Dirac's "additive creation" which we shall denote by "(b)". Yet another possibility is that the Universe is infinite and matter is not being created, but more and more is coming into the range of our observation. This effective increase of the number of nucleons can only be obtained by putting a limit on the distance to which our "range of observation" extends. Dirac puts this limit at the distance where the speed of recession is c/2, c being the speed of light. He obtains the increase by supposing that the rate of expansion of the Universe is reducing. Thus the distance to which we can see goes on increasing. We shall call this mode the "slowing expansion" mode and denote it by (c).

Dirac has shown [3),4)] that the radii of planetary orbits increase with time in (a) and decrease with time in (b) and (c). Unfortunately, these tests are not decisive in any way. The problem with them is the high accuracy required to measure effects $\sim 10^{-10}$ per year, and the impossibility of demonstrating that the observation of such effects, if made, is not due to other (perturbational, relativistic, etc.) effects. A "clean" test of the theory, which gives an unequivocal answer in favour of, or against, the theory is required. It has been demonstrated that the mode (a) is inconsistent with the neutron star model of pulsars,[5)] and can be safely discounted therefore. It should be mentioned that Dirac believed this mode to be the least likely.

The method used in considering mode (a) was to look for a "critical situation", which was provided by the high density and radiation rate of neutron stars. This method is inapplicable in the other two cases as there is no local mass increase in either case. The critical situation could only occur near the initial big bang, but as Dirac explicitly avoids any statement about that epoch, no test of his theory can be devised by the above-mentioned method. Thus, for a "clean" test, we must look for cumulative effects. Hence we are forced to look at the Universe as it was a long time ago. Luckily, we have an excellent "time machine" available due to the finite speed of light. Namely, if we look sufficiently far away, we see the Universe as it was a long time ago. To test Dirac's theory we must look at distant galaxies. This idea of looking far so as to look into the past is certainly not new, but its application to Dirac's theory presents some novel problems. As the main method for measuring distances is by red shifts, any attempt to make predictions of the change of red shift with distance according to Dirac's theory will be untestable in practice. To avoid making the error of using a different concept of distance from that which is actually used, we define "distance" operationally in terms of the red shifts by using Hubble's formula

$$r = v/H \quad , \tag{3}$$

where r is the distance, v the recession speed and H is Hubble's parameter. In terms of a convenient red shift parameter $y = \nu'/\nu$, we have

$$r(y) = (c/H)\,(1-y^2)/(1+y^2) \quad . \tag{4}$$

Notice that $y = (1+z)^{-1}$ in terms of the usual red shift z, and thus $0 < y \leq 1$, where $y = 1$ corresponds to our position, $r = 0$.

Let us consider mode (b) first. What becomes of the created matter? Does it form into galaxies, fill up intergalactic space, or some combination of the two? Let us consider each possibility separately. In the first case we would expect matter to go on accreting into the previous galaxies, effectively like mode (a) on the galactic scale. Thus distant galaxies would be less massive than closer ones. The increase of mass of a typical galaxy with age, from t_1 to t_2, would be given by

$$M(t_1)/M(t_2) = (t_1/t_2)^2 \quad . \tag{5}$$

Taking the epoch of galaxy formation at 10^6-10^7 years, there should be a variation of galaxy mass with distance (from us) of 10^6-10^8 times. It can be argued that, due to the much higher value of G earlier, stars forming at 10^7 years after the big bang could be 10^3 times less massive than the stars which we observe today. However, there would still be 10^3 times fewer stars in the older galaxies. As this variation is not observed, we conclude that matter could not go entirely into galaxy formation.

If we suppose that after the first 10^7 years the created matter has gone entirely into intergalactic space, by Eq.(2) there must be a million times as much dark matter in the Universe as visible matter in the galaxy. This would imply that the density of the dark matter in the Universe is greater than the density of visible matter in the galaxies! Apart from the unacceptability of this requirement, the number of nucleons in the Universe would no longer be $\sim 10^{78}$, but $\sim 10^{84}$, and so the large number hypothesis would be violated. Hence we must discard this mode also.

For mode (b) we are forced to suppose that galaxy formation continued till some time, after which matter only went into intergalactic space. The time for the end of galaxy formation that causes least problems is $\sim 10^9$ years. In this case the galaxy size variations, and the ratio of dark to visible matter, go down to ~ 100 each, which is not too unacceptable. However we get a problem with the extra reduction of luminosities of galaxies with distance. From Eq.(4) the usual reduction of luminosities should be

$$(L_2/L_1)_u = [(1-y_1^2)(1+y_2^2)]^2/[(1+y_1^2)(1-y_2^2)]^2 \quad . \tag{6}$$

However, the extra-galactic matter will further reduce the luminosity roughly proportionally to the distance. Thus, by Dirac's theory with mode (b)

$$(L_2/L_1)_D = [(1-y_1^2)(1+y_2^2)]^3/[(1+y_1^2)(1-y_2^2)]^3 \quad . \tag{7}$$

These two formulae will be measurably different as

$$(L_2/L_1)_D/(L_2/L_1)_u = [(1-y_1^2)(1+y_2^2)]/[(1+y_1^2)(1-y_2^2)] \quad . \tag{8}$$

In terms of the recession speeds v_1, v_2 we get

$$(L_2/L_1)_D/(L_2/L_1)_u = v_1/v_2 \quad . \tag{9}$$

As no such discrepancy has been noted so far, we must conclude that mode (b) is inconsistent with observation. It should be pointed out here that Dirac has also given up the idea of creation of matter 4) and is concentrating on mode (c). We now proceed to consider this mode.

There appears to be a serious problem of principle in this third mode, (c), because of the arbitrariness of the definition of the horizon within which the number of nucleons is the required amount. It is clear that nothing past the red shift horizon can come into the range of observation. Thus there must be an infinite amount of matter within the red shift horizon, which must have an infinite amount of matter within it to be able to continue to supply the ever-increasing matter within the "observable Universe". Thus the arbitrary choice of c/2 as the limit of the speed of recession is important to be able to claim that the number of nucleons increases as the square of time, as we could have arbitrarily many nucleons in our "observable Universe" by choosing a sufficiently large fraction of c. However, assuming that some solution to this problem may be found, let us address ourselves to the task of obtaining some testable predictions for this mode, (c), of Dirac's theory.

Dirac has shown 4) that the density of the Universe decreases linearly with time in (c),

$$\rho \sim t^{-1} \quad . \tag{10}$$

Now, all the matter in the Universe must be divided into visible and dark matter, in the same ratio throughout all space, as no matter is being created in this mode. Thus the number of galaxies at a given distance must be reduced by this factor, coming from Eq.(10). Taking the ratio of the number of galaxies at two given distances corresponding to two different times, t_1 and t_2, as given by Dirac's theory, $k_D = [n(t_1)/n(t_2)]_D$, and comparing it with the corresponding ratio in the standard cosmology, k_S, we get

$$k_D/k_S = t_2/t_1 \quad . \tag{11}$$

In terms of the distance, r, since we are looking back along the light cone, we have

$$t_2/t_1 = (1 - r_2/ct_p)/(1 - r_1/ct_p) \quad , \tag{12}$$

where t_p is the present age of the Universe, H^{-1}. Thus, using Eq.(4) along with Eqs.(11) and (12), we obtain

$$k_D(y_1,y_2)/k_S(y_1,y_2) = (1+y_2^{-2})/(1+y_1^{-2}) \quad . \tag{13}$$

Now, the relationship between the red shift parameter y, and the recession speed v, gives us

$$1 + y^{-2} = 2/(1 - v/c) \quad . \tag{14}$$

Using Eq.(14) in Eq.(13) we finally obtain

$$k_D\{v_1,v_2)/k_S(v_1,v_2) = (1-v_1/c)/(1-v_2/c). \tag{15}$$

Thus, comparing the number of distant galaxies with the closer ones, we see that we expect many more in Dirac's theory than in the usual cosmology. Doing a statistical analysis of the number of galaxies showing given red shifts should provide us with a "clean" test of mode (c) of Dirac's theory.

We have seen that the additive creation mode, (b), of Dirac's theory can be ruled out along with mode (a). Dirac himself does not believe in these modes. The "slowing expansion" mode, (c), of Dirac's theory is the most important. This mode appears to involve a serious problem of principle regarding the definition of the "observable Universe". Disregarding that problem, we suggest that an analysis of the red shifts of distant galaxies may give a "clean" test of Dirac's large number hypothesis. Admittedly, the "cleanliness" is messed up by the statistical analysis required and by the difficulty of obtaining reliable data for large recession speeds.

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