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NONLINEAR SATURATION OF NON-RESONANT INTERNAL INSTABILITIES IN A STRAIGHT SPHEROMAK

By

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Nonlinear Saturation of Non-Resonant Internal Instabilities in a Straight Spheromak

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Abstract

An initial value numerical solution of the time dependent nonlinear ideal magnetohydrodynamic equations demonstrates that spheromak equilibria which are linearly unstable to nonresonant helical internal perturbations saturate at low amplitude without developing singularities. These "instabilities" thus represent the transition from an axisymmetric to a non-axisymmetric equilibrium state, caused by a peaking of the current density.

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If the toroidal current becomes too concentrated at the magnetic axis of an axisymmetric spheromak, the configuration will become unstable to an ideal magnetohydrodynamic instability, even at zero plasma pressure. Since the temperature dependence of the plasma conductivity causes this current peaking in ohmically heated spheromaks, the nonlinear behavior of these instabilities is of considerable interest.

Recently, a study has appeared[1] in which the linear stability boundaries of the spheromak configuration have been mapped out. One of the results of this study is that the internal mode stability of a toroidal spheromak can be well approximated by considering the circular cylindrical spheromak, an idealized configuration in which the torus has been straightened out while maintaining its periodic boundary conditions. This study also found nonresonant ($nq - m \neq 0$ anywhere in the plasma) $m = 1$ internal modes to be the dominant internal ideal instabilities in a spheromak, where n is the toroidal mode number and m is the poloidal mode number. In the present paper, we study the nonlinear evolution of these <u>nonresonant</u> modes in a circular cylindrical spheromak. This is compared to the results of Ref. 2 where the <u>resonant</u> $m = 1$ *internal kink mode* in the tokamak was found to develop nonlinearly into a saturated helical state with a singularity in the current density.

Using standard (r,θ,z) cylindrical coordinates, we define a periodicity length in the z-direction of $L = 2\pi k$, where k is the analog of the reciprocal major radius in a toroidal device, $k = 1/R$. The equilibrium safety factor profile is taken to be of the form

$$q(r) = rk\ B_z(r)/B_\theta(r) = q_o(1 - r^2) \quad . \tag{1}$$

Force free equilibrium fields for this system are obtained by performing the integral

$$B_\theta(r) = rB_{\theta o} \exp \left(- \int_o^r \frac{2r + (d/dr)(q^2/2k^2)}{(q^2/k^2 + r^2)} dr \right) \quad . \tag{2}$$

In addition to the overall scale factor $B_{\theta o}$, the equilibrium profiles are seen to depend only on the pitch parameter q_o/k.

The linear stability properties of this class of configurations are illustrated in Fig. 1 (taken from Ref. 1) where we plot the unstable region in $(nk,\ nq_o)$ space. A given equilibrium with a fixed value of q_o and k is unstable if the line passing through the origin with slope $(q_o/k)^{-1}$ intersects the shaded (unstable) region and a point corresponding to a discrete mode with n an integer lies within the shaded region.

We have studied the nonlinear evolution of unstable equilibria of the form given by Eqs. (1) and (2). This can be treated as a two-dimensional problem since helical symmetry is preserved in the present system. The plasma is treated as incompressible for convenience in the numerical calculation. The final saturated equilibrium obtained assuming incompressibility will also be an equilibrium state for a compressible plasma with finite pressure. The dynamic equations used are the exact equations of incompressible magnetohydrodynamics with fixed helicity, first used in Ref. 2. The velocity and magnetic fields are represented by a stream function U and the helical flux A:

$$\vec{V} = g\nabla U \times \hat{e} + V_e\hat{e} \quad , \tag{3}$$

$$\vec{B} = g\nabla A \times \hat{e} + B_e\hat{e} \ . \tag{4}$$

Here, $g \equiv [1 + \alpha^2(r/a)^2]^{-1/2}$, $\phi = \theta + \alpha z$ is a helical coordinate with helicity $\alpha = nk$, and $\hat{e} = \hat{r} \times \hat{\phi}$ is the unit vector in the symmetry direction. All the scalar components on the unit basis vectors $(\hat{r},\hat{\phi},\hat{e})$ are functions of r and ϕ only. The fixed helicity incompressible magnetohydrodynamic equations are (in rationalized emu units with minor radius $a = 1$ and homogeneous density $\rho_o = 1$)

$$dA/dt = 0 \quad , \tag{5}$$

$$(d/dt)(gB_e) = \vec{B}\cdot\nabla(gV_e) + 2\alpha g^4\vec{V}\cdot\nabla A \quad , \tag{6}$$

$$(d/dt)(\Delta^*U) = -\vec{B}\cdot\nabla(gJ_e) + \alpha^2 g^2\left[\partial(V_e^2 - B_e^2)/\partial\phi\right] - 2\alpha\vec{V}\cdot\nabla(g^3V_e) \quad , \tag{7}$$

$$(d/dt)(V_e/g) = \vec{B}\cdot\nabla(B_e/g) \ . \tag{8}$$

Here, $\nabla f = (\partial f/\partial r)\hat{r} + (1/rg)(\partial f/\partial\phi)\hat{\phi}$, $\Delta^* f \equiv \nabla\cdot(g^2\nabla f)$
$= (1/r)[(\partial/\partial r)(rg^2\partial f/\partial r)] + (1/r^2)(\partial^2 f/\partial\phi^2)$,
$J_e = -(\Delta^*A/g + 2\alpha g^2 B_e)$, $\vec{V}\cdot\nabla f = (\nabla f \times g\nabla U)\cdot\hat{e}$, $\vec{B}\cdot\nabla f = (\nabla f \times g\nabla A)\cdot\hat{e}$
and $d/dt = \partial/\partial t + V\cdot\nabla$. (Note that the usual reduced tokamak equations[3] can be readily obtained from the above equations by taking only the lowest order terms in α.[2] In this lowest order, however, the modes we study are not present.)

An axisymmetric equilibrium is given an initial perturbation of the given helicity and the time advancement is followed numerically. A damping term is added in the nonlinear phase to provide a mechanism for energy dissipation, allowing the plasma to settle into a neighboring lower energy equilibrium if one should exist. The numerical scheme used is similar to the one used in

Ref. 2.

In Fig. 2 we plot the linear growth rates and the nonlinear saturation amplitude of the unstable modes for various $(nk)^2$ values with the equilibrium denoted by the dashed line $q_o/k = 0.3$ in Fig. 1. The saturation amplitude is characterized by ξ_a, the displacement of the magnetic axis from the equilibrium state.

In Fig. 3 we show the flux contours A in the initial unperturbed (unstable) axisymmetric state (3a) and in the nonlinearly saturated (stable) helical state for the mode with nk = 2. Similarly, in Fig. 4 we show the contours of constant current density in the symmetry direction, J_e, for the axisymmetric and helically saturated states.

From Figs. 3 and 4 we see that the saturated helical states for these modes do not possess singular current densities. This is in marked contrast to the singular saturated states for the internal kink mode in tokamaks studied in Ref. 2. The critical difference is that the spheromak unstable internal modes of the present study do not have singular surfaces in the plasma. The absence of singular current densities in the saturated state implies that resistive reconnection does not play a role as it does in the nonlinear evolution of the internal kink mode in the tokamak.

Figures 1 and 2 enable us to put together a picture of the evolution of a spheromak discharge. If the spheromak is initially formed with a relatively broad current distribution it will have $q_o/k > 2/3$ and will thus be stable to all the current driven internal modes in Fig. 1. As the center of the plasma ohmically heats and the temperature peaks, the current will concentrate on axis causing q_o/k to decrease below $q_o/k \lesssim 2/3$ and thus cause the spheromak to become unstable to one or more internal modes. These modes will grow to a finite but small amplitude, resulting in a smooth transition to a stable non-axisymmetric new equilibrium configuration, without causing the phenomenon of "sawtoothing"[4] associated with the nonlinear development of internal modes in tokamaks. It should be remembered, however, that the present study assumes helical symmetry, and thus an extension to three dimensions may still reveal a different character such as the development of current singularities due to the coupling to modes of different helicities.

Acknowledgment

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Figure Captions

Fig. 1 Stability boundaries for internal modes in the pressureless cylindrical spheromak with parabolic q-profile. A given equilibrium corresponds to a line emanating from the origin with slope $[q_0/k]^{-1}$.

Fig. 2 Linear growth rate (γ) and shift of the magnetic axis in the saturated state (ξ_a) for equilibrium with $q_0/k = 0.3$. Note that in a physical device with $k \equiv 1/R$, nk can only take on discrete values.

Fig. 3 Helical flux contours, A, for an unstable axisymmetric equilibrium with $q_0/k = 0.3$, nk = 2 (a), and for the nonlinearly saturated state of the same equilibrium (b).

Fig. 4 Helical current contours, J_e, for the axisymmetric (a) and helical saturated state (b) of Fig. 3. Note that the current density remains smooth in the saturated state.

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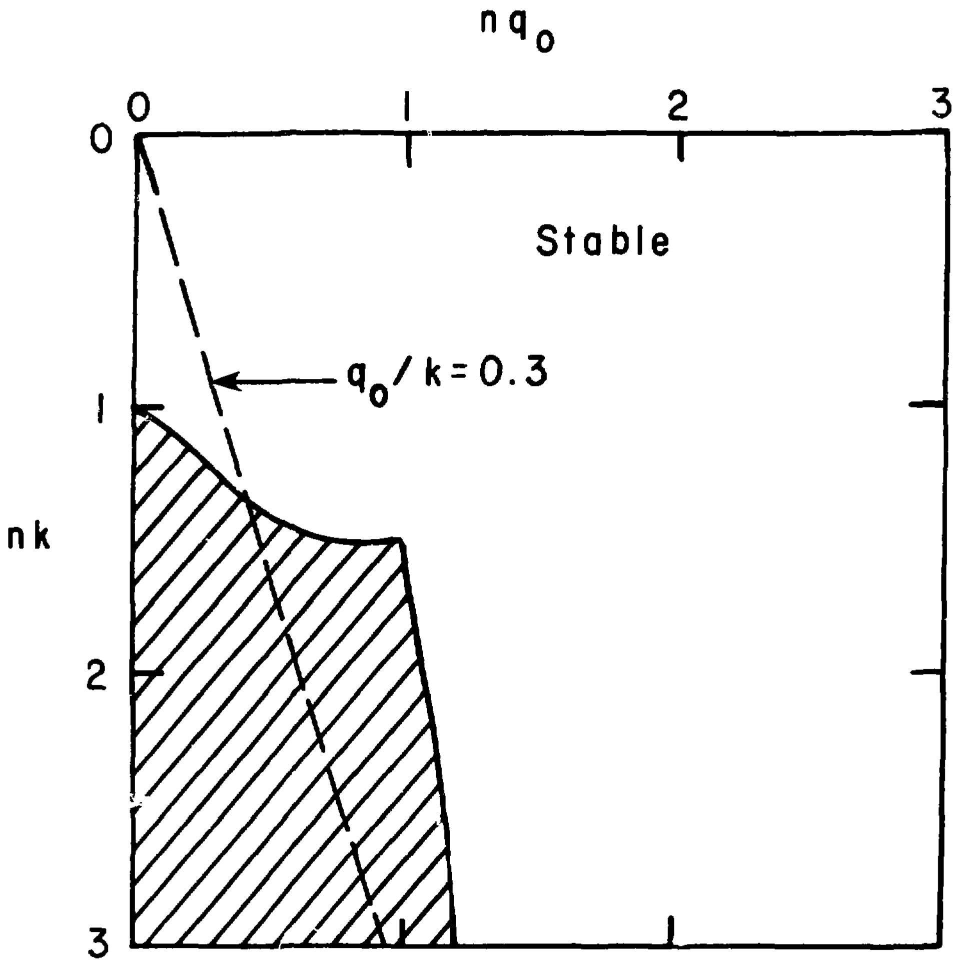

Fig. 1

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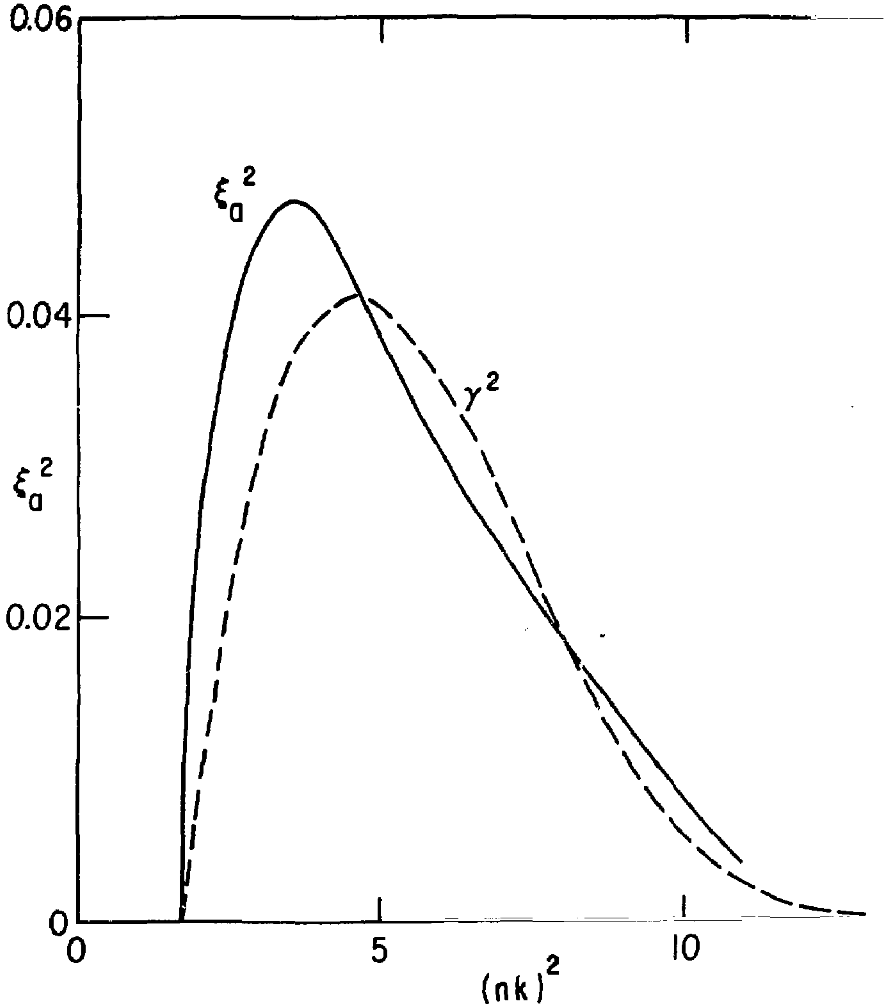

Fig. 2

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(a)

(b)

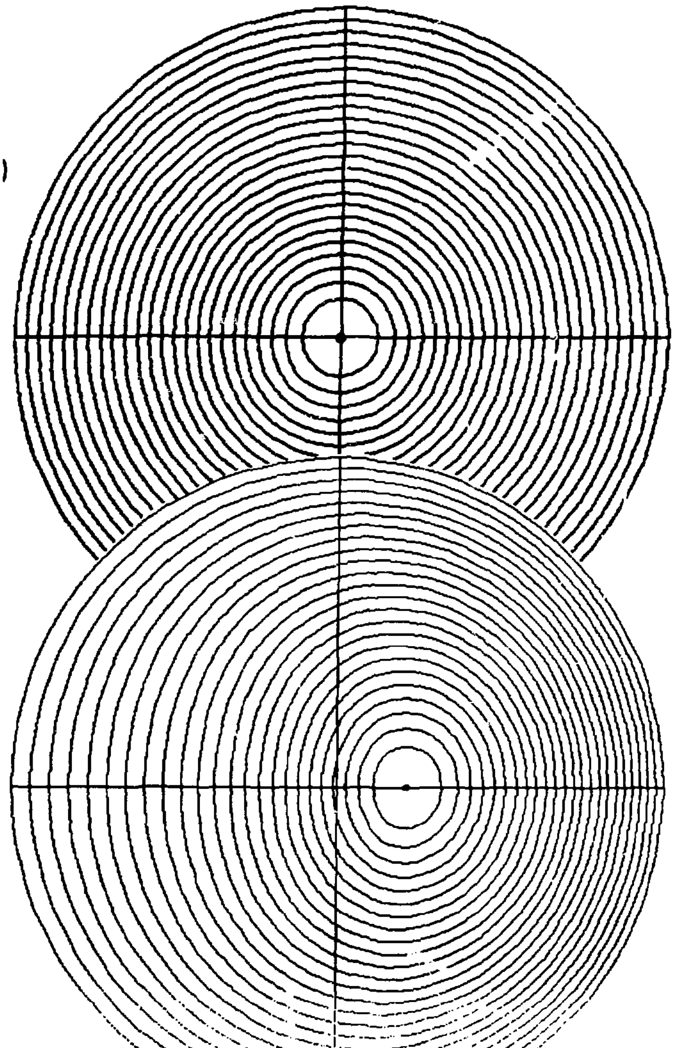

Fig. 3

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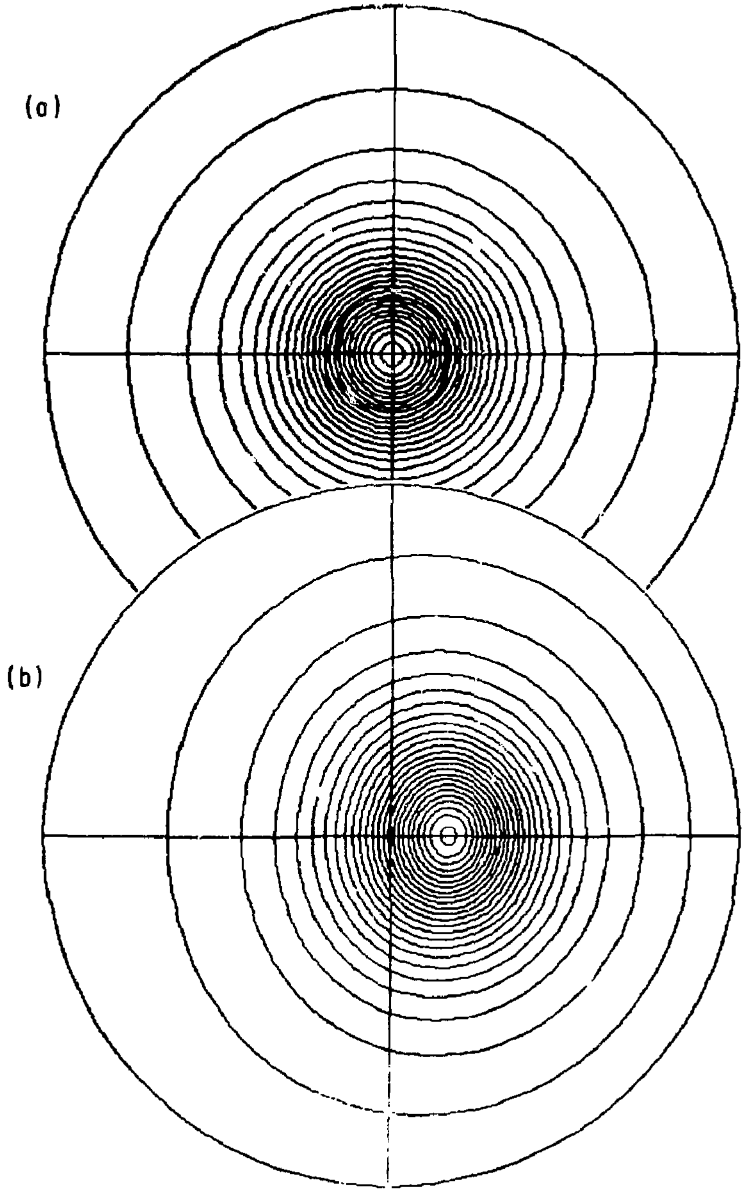

Fig. 4