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Nonlinear Trapped-Electron Mode and Anomalous Heat Transport in Tokamaks

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Abstract

We take the phenomenological point of view that the anomalous electron thermal conductivity produced by the nonlinear trapped electron mode should also influence the stability properties of the mode itself. Using a model equation, we show that this effect makes the mode self-stabilizing. A simple expression for the anomalous thermal conductivity is derived and its scaling properties discussed.

It is well-known that electrons in a tokamak plasma lose heat at an anomalously rapid rate.[1] This loss is usually interpreted in terms of an anomalous perpendicular electron thermal conductivity coefficient $\chi_{e\perp}$. An explanation of the magnitude of $\chi_{e\perp}$ and its scaling with various physical parameters is still an unsolved problem of tokamak physics.

In this note we wish to explore consequences of the following <u>ansatz</u>: The anomalous thermal conductivity $\chi_{e\perp}$ is felt, not only by the gross thermal dynamics of the discharge, but also by the microscopic fluctuations associated with microinstabilities. We shall <u>assume</u> that $\chi_{e\perp}$ is independent of scale-size and is isotropic in the two dimensions $\perp$ to the B_0 field. In particular we shall examine the influence of $\chi_{e\perp}$ on the drift instability driven by trapped electron dissipation.[2] This instability is of special interest because most tokamak discharges have a substantial part of the plasma in the "banana" regime ($\nu_{eff} < \omega_{be}$), where this instability should be operative. We demonstrate below (using a model equation) that a sufficiently large $\chi_{e\perp}$ can lead to stabilization of the trapped electron instability. The following scenario may now be put forward: Typically, most tokamak discharges are in the parameter space where a substantial part of the plasma is unstable to <u>linear</u> trapped electron instability. As the fluctuation level comes up, it leads to an anomalous perpendicular thermal conductivity $\chi_{e\perp}$ of electrons. $\chi_{e\perp}$ not only produces the observed anomalous electron heat transport, but it also shuts off the instability. We thus have a self-consistent mechanism determining the instability saturation and the anomalous electron heat transport.

Let us make some qualitative estimates to orient our thinking. The time scale for diffusing a T_e perturbation associated with the microscopic mode is $(\chi_{e\perp}\nabla^2)^{-1}$. Taking $\chi_{e\perp} \sim a^2/\tau_{Ee}$ (a is the plasma radius and τ_{Ee}, the electron energy confinement time) and $|\nabla| \sim k_\theta \sim \rho_i^{-1}$ (ρ_i is the ion Larmor radius), we find

$$\chi_{e\perp}\nabla^2 \approx (a^2/\rho_i^2)\tau_{E\ell}^{-1} \approx 10^6 - 10^7 \ \mathrm{sec}^{-1} .$$

Thus we find that $|\omega_{be}| > |\chi_{e\perp}\nabla^2| > \nu_{eff}, \omega$. Thus if the observed $\chi_{e\perp}$ is also operative on microscales ($\sim \rho_i$) , it produces a nonlinear dissipative mechanism which can replace the linear dissipative mechanism (ν_{eff}) associated with detrapping of trapped electrons. Since $|\chi_{e\perp}\nabla^2| > \omega \sim \omega_* \cong (cT_e/eB)(d\ln n_e/dr)$, the mode is nonlinearly driven into the dissipative regime. Note that $\chi_{e\perp}$ primarily operates on trapped electrons since they have an associated temperature fluctuation; the untrapped electrons basically follow a Boltzmann distribution and have $\tilde{T}_e \approx 0$.

Our starting equations are the quasi-neutrality condition

$$\tilde{n}_{eu} + \tilde{n}_{et} = \tilde{n}_i , \tag{1}$$

the Boltzmann response for untrapped electrons (we neglect resonant electron effects since their contribution to growth rate is typically smaller)

$$\tilde{n}_{eu}/n_{ou} = e\phi/T_e , \tag{2}$$

the usual fluid response for cold ions

$$\tilde{n}_i/n_o = \frac{e\phi}{T_e}\left[\frac{\omega_{*e}}{\omega} + \frac{k_\parallel^2 c_s^2}{\omega^2}\right] \tag{3}$$

($c_s = \sqrt{T_e/m_i}$ is the ion-acoustic speed), and a model equation for the trapped-electron distribution function, averaged over the "bounce" motion viz.

$$\frac{\partial}{\partial t} f_T + v_{\underline{E}\times B} \cdot \nabla f_{oT} - \chi_{e\perp} \nabla_\perp^2 (f_T - \sqrt{\varepsilon}\, f_u) = 0. \tag{4}$$

In writing Eqs. (1) - (3) we have used the usual local WKB approximation. The second moment of Eq. (4) shows that the temperature fluctuation in trapped electron fluid diffuses at a rate $|\chi_{e\perp}\nabla_\perp^2|$ as desired. The density moment of (4) indicates that the model term relaxes the trapped electron density perturbation $\tilde{n}_{eT}$ towards the equilibrium Boltzmann value $\sqrt{\varepsilon}\,\tilde{n}_{eu} = \sqrt{\varepsilon}(e\phi/T_e)n_{ou}$. Proceeding in the standard manner with (4), we get

$$n_{eT} = n_o \sqrt{\varepsilon}\, \frac{e\phi}{T_e} \left[1 - \left\langle \frac{\omega - \omega_{*T}}{\omega + ik_\perp^2 \chi_{e\perp}} f_{oT} \right\rangle\right] \quad , \tag{5}$$

where

$$\omega_{*T} = \omega_*\left[1 - \frac{d\ell n T_e}{d\ell n\, n_e}\left(\frac{3}{2} - \bar{v}^2\right)\right].$$

It is well-known that if $\chi_{e\perp}$ is independent of velocity and f_{oT} is a Maxwellian, the temperature gradient term drops out of (5). We believe that the temperature gradient destabilization is important and therefore assume an appropriate velocity dependence; the result will be written in terms of an unknown coefficient $\alpha \sim O(1)$. We may now write a new dispersion relation for the trapped electron mode, viz. $\omega \approx \omega_r + i\gamma$, where

$$\omega_r = \omega_*/[1 + k_\perp^2 \rho_s^2] \tag{6a}$$

$$\gamma \simeq \alpha\, \varepsilon^{1/2} \frac{\omega_*^2}{k_\perp^2 \chi_{e\perp}} [1 + k_\perp^2 \rho_s^2]^{-2} \left[\frac{3}{2} \frac{d\ell n\, T_e}{d\ell n\, n_e} + \frac{k_\perp^2 \rho_s^2}{1 + k_\perp^2 \rho_s^2}\right] . \tag{6b}$$

Note that the above "dissipative" trapped electron regime is relevant, because $k_\perp^2\chi_{e\perp} > \omega_*$ is expected. Note also that the anomalous $\chi_{e\perp}$ replaces ν_{eff} as the basic dissipative mechanism responsible for instability. However, since $\chi_{e\perp}$ is large, the growth rate is reduced (below the linear value due to ν_{eff}) and becomes smaller as $\chi_{e\perp}$ increases. It is also interesting to point out that in this "dissipative" regime, the effects due to curvature-drift resonances, etc., are not too critical.

We now assume that we can use the slab model for sheared magnetic fields.[3] One could argue with some justification that, in the presence of substantial fluctuations, the toroidal nature of the linear problem is irrelevant. The mode instability now requires that the local growth rate γ given by (6b) exceed a shear damping rate (for n = 0 radial eigenmode)

$$\gamma_d \simeq \omega_*(L_n/L_s) \quad , \tag{6c}$$

where $L_n^{-1} \equiv (\nabla n/n)$, $L_s^{-1} = (r/R)(q'/q^2)$, $q' \equiv (dq/dr)$ and $q \equiv B_T r/B_p(r)R$ is the well-known safety factor. Typically, $\gamma_d/\omega_* \simeq 0.1$. An additional damping mechanism could be the nonlinear Landau damping with broadened ion resonances. The magnitude of this effect is

$$\gamma_{ion}/\omega_* \simeq \frac{(k_\perp^2 D)}{*}\frac{k_\parallel^2 v_{ith}^2}{\omega^2} < \left(\frac{k_\perp^2\chi_{e\perp}}{\omega_*}\right)\left(\frac{v_{ith}^2}{q^2R^2\omega^2}\right) \sim 10^{-1} - 10^{-2}$$

and appears to be weaker than the linear shear damping. The mode is thus self-stabilized when $\chi_{e\perp}$ becomes large enough to make $\gamma \simeq \gamma_d$. Equating (6b,c) we find

$$\chi_{e\perp} = \frac{\alpha\sqrt{\varepsilon}}{k_\perp^2}\,\frac{\omega_*}{[1+k_\perp^2\rho_s^2]^2}\,\frac{d\ln T_e}{(d\ln q)}\,\frac{q}{\varepsilon} \tag{7}$$

where $\epsilon \approx r/R$, and we have assumed that the temperature gradient term dominates in (6b), as usual, and the (3/2) has been absorbed in α. The basic ansatz of this paper is that the $\chi_{e\perp}$ [given by (7)] which shuts off the trapped electron mode is identical to the $\chi_{e\perp}$ measured by the gross thermal dynamics of the discharge. We now explore further consequences of this identification.

Unfortunately, Eq. (7) has the undesirable feature that $\chi_{e\perp}$ depends on $k_\perp$. For $k_\perp\rho_s \lesssim 1$, the dependence is $k_\perp^{-1}$. We now postulate that $\chi_{e\perp}$ has to be large enough to shut off the most unstable mode, which typically occurs in linear theory when $k_\perp\rho_s \approx 1$. Using this equation to eliminate $k_\perp$ we find

$$\chi_{e\perp} = \frac{\alpha}{4} \sqrt{\epsilon} \left(\frac{q}{\epsilon}\right)(c_s\rho_s^2)(d\ell nT_e)\left(\frac{d\ell n\ n}{d\ell n\ q}\right) . \tag{8}$$

Equation (8) is the desired expression for the anomalous electron thermal conductivity associated with the nonlinearly saturated trapped electron instability.

Equation (8) has some very interesting features:

(1) For a typical Ohmic heated discharge, we may take $\epsilon \approx r/R \approx 1/6$, $q \approx 1.5$, $T_e \approx 1$ kev, B = 25 K Gauss, $(d\ell n\ T_e)(d\ell n\ n)/(d\ell nq) \sim r^{-1} \sim (20\ cm)^{-1}$. This gives

$$\chi_{e\perp} \approx \alpha\ (1.4)\ 10^4\ cm^2\ sec^{-1} ,$$

which is close to the measured experimental values for $\alpha \sim 0(1)$. Thus the $\chi_{e\perp}$ in Eq. (8) has the right order of magnitude.

(2) Equation (8) predicts no direct density scaling in contradiction to Alcator scaling $\chi_e \sim 5.10^{17}/n_{cm^{-3}}$. However, there is some experimental

evidence that the density profiles are fla er at higher densities.[4] Thus a density scaling could enter through $(\nabla n/n)$. Equation (8) has a direct scaling with q. Higher currents (lower q) give stronger shear and hence reduced $\chi_{e\perp}$. This is consistent with recent experiments on neutral beam heated discharges.[4]

(3) The temperature scaling of Eq. (8) is quite complicated. There is a direct $(\nabla T_e)T_e^{1/2}$ dependence. Furthermore, there is an implicit dependence through q(r). If the plasma resistivity is classical and $J_z(r)/J_z(0) = [T_e(r)/T_e(0)]^{3/2}$, we have the simple relationship

$$r\,\frac{d\ln q}{dr} = 2\left[1 - \left(\frac{T_e(r)}{T_e(o)}\right)^{3/2} \frac{q(r)}{q(o)}\right] . \tag{9}$$

(4) The most interesting feature of Eq. (8) is the dependence of $\chi_{e\perp}$ on the magnetic shear and the temperature gradient. Equation (8) suggests that the plasma is trying to maximize heat transport across magnetic surfaces by weakening the shear. However, to keep the anomalous transport going, the instability must be present, and a minimum temperature gradient is needed. Thus the temperature profile and the shear profile are locked in to a "nonlinear marginally stable" form. This argument is reminiscent of the "marginal stability" theory of trapped electron mode put forward by Manheimer et al.[5] There is an important distinction, however. Our "marginally stable" state is nonlinear in that the linear shear damping is being balanced by a nonlinearly reduced growth term. The basic objection[6] against the Manheimer theory that experimentally observed temperature profiles are linearly unstable and not linearly marginally stable, is thus inapplicable to our arguments. Recent arguments by Coppi[7] and Coppi and Mazzucato[7] in favor of a "principle of profile consistency" for electron temperature profiles in tokamaks is also consistent with our point of view, viz. that the electron temperature profile

is held close to a critical form by the nonlinearly saturated state of a temperature gradient driven instability.

(5) Our model Eq. (4) is basically a model for renormalization of the trapped electron fluid response. (Sin e resonant electron effects have been assumed small, the usual shear-induced resonance broadening[8] for untrapped particles did not enter our theory.) A detailed nonlinear theory should provide an expression for $\chi_{e\perp}$ in terms of the fluctuation amplitudes. If electrostatic theories are used, one may estimate

$$\chi_{e\perp}^{NL} \sim \left(\frac{cE}{B}\right)^2 \frac{1}{\omega_*} \sim \left(\frac{kcE}{B\omega_*}\right)^2 \frac{\omega_*}{k^2} .$$

Typical saturation amplitudes $\tilde{n}/n_o \sim 10^{-2}$ give $kcE/B\omega_* \sim 1$ and $k^2\chi_{e\perp}^{NL} \sim \omega_*$. Thus $\chi_{e\perp}^{NL}$ is too small to account for the measured $\chi_{e\perp}$. Alternatively, a fluctuating $\tilde{b}$ model may be invoked to estimate $\chi_{e\perp}$. Magnetic field fluctuations associated with the trapped electron mode may be significant. However, their effect on renormalization of the <u>trapped</u> electron fluid is complicated and may depend on the explicit time dependence of $\tilde{b}$. We emphasize once again our point of view, viz. that $\chi_{e\perp}$ will manage to get large enough to saturate the instability nonlinearly.

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