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QCD SUM RULES OF THE LAPLACE TRANSFORM TYPE FOR THE GLUON COMPONENT OF THE U(1)$_A$ MESON MASS *

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ABSTRACT

We get upper bound of the gluon component of the U(1)$_A$ meson mass using QCD sum rules of the Laplace transform type to the two-point functions associated to the divergence of the U(1)$_A$ current in the chiral limit. For $\Lambda \simeq 70 \sim 210$ MeV and $f_{\eta'} \simeq (0.5 \sim 0.7)\sqrt{3}\, f_\pi$, we obtain

$$M_G \lesssim (0.6 \sim 0.85) \text{ GeV} \quad ,$$

which indicates an important gluon contribution to the η' mass.

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progress in extending the applicability domain of Quantum Chromodynamics (QCD) to obtain predictions on low-energy parameters (hadron masses and coupling constants). The approach is based on sum rules obeyed by the spectral functions of a specific two-point function of current operators, as a consequence of general analyticity properties. There exist a variety of QCD sum rules in the literature [1-3] depending on how these analyticity and positivity properties are exploited. Of particular interest for low-energy phenomenology are the sum rules of the Laplace transform type

$$\mathcal{F}(M^2) = \frac{1}{\pi}\int_0^\infty dt\, e^{-t/M^2}\, \mathrm{Im}\,\Pi(t) \tag{1}$$

proposed by SVZ [1a] for the light quark system and recently revised by NR [1b]. Here $\frac{1}{\pi}\mathrm{Im}\,\Pi(t)$ denotes a specific spectral function (e.g. the hadronic vacuum polarization measured in the $e^+e^- \to$ hadrons); $\mathcal{F}(M^2)$ is a quantity, which in principle can be computed asymptotically in QCD provided that M^2 is larger than the QCD scale Λ which we shall take to be $\Lambda \simeq 70 \sim 210$ MeV in the dimensional renormalization $\overline{\mathrm{MS}}$ scheme $^{1)}$ as a result issued from the QCD sum rule analysis of the isovector of the $e^+e^- \to$ hadrons total cross-section [5]. Clearly, the sum rule in Eq.(1) is more selective on the low-energy behaviour of the spectral function for small enough M than the right-hand side of the usual dispersion relation

$$\Pi(Q^2 \equiv -q^2 > 0) = \frac{1}{\pi}\int_0^\infty \frac{dt}{t+Q^2}\, \mathrm{Im}\,\Pi(t) \quad + \text{"subtraction"} \quad . \tag{2}$$

In this letter we propose to extend the applicability of the QCD sum rules to the two-point functions:

$$\psi_G(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\,\partial_\mu J_5^\mu(x)\left(\partial_\mu J_5^\mu(0)\right)^\dagger|0\rangle \quad , \tag{3}$$

associated to the divergence of the U(1)$_A$ current which reads in the chiral limit and for n flavours:

$$\partial_\mu J_5^\mu(x) = \left(\frac{g^2}{4\pi^2}\right)\frac{n}{8}\,\epsilon_{\mu\nu\rho\sigma}\,F^{\mu\nu}_{(a)}(x)\,F^{\rho\sigma}_{(a)}(x) \quad . \tag{4}$$

To lowest order of QCD and including non-perturbative effects, the expression of $\psi_G(q^2)$ in the $\overline{\mathrm{MS}}$ renormalization scheme is [6]

$$\Psi_G(q^2=-Q^2) = \left(\frac{\;}{4\pi}\right)(Q\;\;\frac{\;}{\pi^2}\left(\;\;\frac{\;}{\sqrt{2}}\;\frac{\;}{(Q^2)^2}\right.$$

$$-\frac{4\pi^2}{(Q^2)^3}\,f^{abc}\langle F^3_{abc}\rangle + O\left(\frac{1}{Q^8}\right)\Big\} \tag{5}$$

where $\langle F^2\rangle \equiv \langle 0|F^{\mu\nu}_{(a)}F^{(a)}_{\mu\nu}|0\rangle$, $\langle F^3\rangle \equiv \langle 0|F^{\mu\nu}F_{\alpha\beta}F^{\beta\mu}|0\rangle$;

the charmonium analysis fixes $\langle\alpha_S F^2\rangle$ to be of the order of 0.04 GeV^4 [7]. The behaviour of the two-point function at $q = 0$ is known from the QCD multicolour analysis to be [8] :

$$\psi_G(0) \sim M_P^2 f_P^2 \left(\frac{N_c}{3}\right)^2 \tag{6}$$

where N_C is the number of colours, M_P is the gluon component of the $U(1)_A$ meson mass and f_P its decay amplitude normalized as:

$$\langle 0|J^\mu_5(x)|P\rangle = \sqrt{2}\, f_P\, q^\mu , \tag{7}$$

where q^μ is the momentum of the pseudoscalar meson $|p\rangle$. Eq.(7) is analogous to the chiral symmetry relation from PCAC analysis

$$\psi_5(0) \equiv -(m_u+m_d)\langle 0|\bar\psi_u\psi_u + \bar\psi_d\psi_d|0\rangle \simeq 2 f_\pi^2 m_\pi^2 , \tag{8}$$

where $\psi_5(q)$ is the two-point function associated to the divergence of the $SU(n)_L \times SU(n)_R$ axial vector current. As we learned from QCD sum rules of $\psi_5(q^2)$ [3] , care must be taken when working with $\psi_5(q^2)$ because the value of $\psi_5(0)$ can cancel the pole contribution to the sum rule and so the analysis becomes misleading, ignoring this term $\psi_5(0)$. As $\psi_5(0)$ is constant, one can escape this difficulty working with various derivatives of $\psi_5(q^2)$. On the contrary, various derivatives of a quantity like $\psi_5(q^2)/q^2$ appears to depend highly on $\psi_5(0)$. 2) Similar care must be taken for the case of $\psi_G(q^2)$ in the $U(1)_A$ sector. So, to our knowledge, the results in Ref.6

to the fact that the authors ignore the effect of $\psi_G(0)$. In fact, one can either use the result in Eq.(6) or extend PCAC to the η' meson to get at a first approximation

$$\psi_G(0) \simeq 2 f_P^2 M_P^2 . \tag{9}$$

Now, work for example,as in Ref.6, with the second derivative of $\psi_G(q^2)/q^2$ which is superficially convergent

$$\frac{\partial^2}{(\partial q^2)^2}\left\{\psi_G(q^2)/q^2\right\} = \frac{2}{\pi}\int_0^\infty \frac{dt}{(t+Q^2)^3}\,\frac{1}{t}\,\mathrm{Im}\,\psi_G(t). \tag{10}$$

Take into account the non-perturbative effect of Eq.(9) into Eq.(10), saturate the spectral function $\mathrm{Im}\,\psi_G(t)$ by the η' . Using the positivity of the continuum and Eq.(5), we get for large Q^2 the useless inequality 3)

$$\frac{1}{Q^2}\left(\frac{3\bar\alpha_s}{4\pi}\right)^2\frac{2}{\pi^2} \geq 0 , \tag{11}$$

where we have introduced the running coupling constant

$$\frac{\bar\alpha_s}{\pi}(Q^2) = \frac{1}{-\beta_1 \log Q/\Lambda} \quad \text{with} \quad \beta_1 = -\frac{1}{2}\left(11+\frac{2n}{3}\right) \text{ for } SU(3)_c\times SU(n)_F .$$

The result in Eq.(11) shows that it is more interesting working with quantities independent of $\psi_G(0)$. That can be achieved by taking the Laplace transform of $\psi_G(q^2)$ obtained from the application of the operator

$$\hat{L} \equiv \frac{(-1)^P}{(P-1)!}\,\frac{\partial^P}{(\partial Q^2)^P} \tag{12}$$

to both sides of the dispersion relation of the type in Eq.(2) obeyed by $\psi_G(q^2)$. So, we get the sum rule

$$\frac{1}{M^6}\int_0^\infty dt\, e^{-t/M^2}\,\frac{1}{\pi}\,\mathrm{Im}\,\psi_G(t) \simeq \frac{4\pi^2}{9}\left(\frac{\bar\alpha_s}{\pi}\right)\left\{\left(\frac{\bar\alpha_s}{\pi}\right) + O\left(\frac{\bar\alpha_s}{M^4}\,\alpha_s\langle F^2\rangle\right)\right\}. \tag{13}$$

The reader can notice that the non-perturbative terms $\alpha_S \langle F^2 \rangle 1/M^4$ have disappeared in the sum rule. So, the so-called low-energy theorem of Ref.6 will now concern the $\langle F^3 \rangle$ term of Eq.(13). Now, let us optimize Eq.(13), taking $M \simeq 1$ GeV from our experience on light meson sum rules [1] . [4)]

Let us use the experimental data on f_P from the radiative decay of the ψ meson ($\psi \to \eta' \gamma$) [10]

$$f_{\eta'} \simeq (0.5 \sim 0.7) \sqrt{3} f_\pi \ . \qquad (13)$$

Saturating $\mathrm{Im}\, \psi_G(t)$ by the P meson, and using the positivity of the continuum contribution to the spectral function, we deduce

$$M_P \lesssim e^{M_P^2/4M^2} \left\{ \frac{3}{2\pi} \frac{1}{\sqrt{2}} \left(\frac{\bar{\alpha}_s}{\pi}\right) \frac{M^3}{f_P} \right\}^{1/2} , \qquad (14)$$

which for $M \simeq 1$ GeV

$$M_P \lesssim \begin{matrix} (0.85 \sim 0.72) \\ (0.71 \sim 0.6) \end{matrix} \text{ GeV for } \Lambda \simeq \begin{matrix} 210 \\ 70 \end{matrix} \text{ MeV} . \qquad (15)$$

Such a result of the gluon component of the $U(1)_A$ meson mass is interesting for the understanding of the classical so-called "$U(1)_A$ problem" [5)]. We expect to have an important gluonic contribution to the mass of η' (0.96 GeV). The agreement between theory and experiment can be improved taking into account the quark contribution to the η' mass [6)]. We are investigating within QCD sum rules an analysis of the quark contribution as well as on the contribution of the possible interference term between quark and gluons.

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FOOTNOTES

1) For a recent review on the dimensional renormalization scheme, see e.g. Ref.4.

2) As the non-perturbative part of $\psi_5(0)$ is renormalization group invariant, it has the same value at various values of the subtraction scale ν . So, as $\psi_5(q)_{\text{renormalized}} = \psi_5|_{\text{bare}} - \psi_5(\nu)$, automatically contributes $\psi_5(\nu)$ to various derivatives of $\psi_5(q^2)/(q^2)^n$.

3) This observation makes a great reservation on the results obtained in Ref.6 (so-called low-energy theorem and estimate of the η' mass).

4) Note that for such value of M, the $1/M^6$ contribution to the sum rule is less than 5% if one adopts the following factorization of Ref.7b" $\langle 0 | g^3 f_{abc} F^3_{abc} | 0 \rangle \simeq \frac{48\pi}{5} \rho_c^{-2} \langle \alpha_S | F^2 \rangle \simeq 4.55\ 10^{-2}$ GeV^6, where $\rho_c \simeq (200 \text{ MeV})^{-1}$ is the confinement radius.

5) For a review of the present status of the $U(1)_A$ problem, see e.g. Ref.11.

6) Attempts to understand the quark contribution of the $U(1)_A$ meson mass are, e.g. in Ref.12 and references therein. I thank N. Fuchs for informing me of his result.

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