HE

IC/80/112

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

THE BJORKEN-PASCHOS RELATION IN THE UNIFIED GAUGE THEORY

Sun Kun Oh

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1980 MIRAMARE-TRIESTE

IC/80/112

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

THE BJORKEN-PASCHOS RELATION IN THE UNIFIED GAUGE THEORY *

Sun Kun Oh **

International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We have examined in the unified gauge theory with the integrally-charged quark model the Bjorken-Paschos relation within the impulse approximation of the parton picture. We find that the relation is valid for the forward scattering region and provides a reliable way to test the charge assignment of the quarks and the gluons.

MIRAMARE - TRIESTE
July 1980

* To be submitted for publication.

** On leave of absence from Department of Basic Science, Korea Advanced Institute of Science, Seoul, Korea.

The idea that hadrons are composed of the fundamental constituents called partons become one of the basic ingredients to the particle physics since it has been introduced about a decade ago. Historically, there have been two ways of developments in the parton idea: One is the field-theoretic justification provided by Drell, Levy and Yan[1], and the other is the impulse approximation, conjectured by Feynman[2], where the probing particle interacts with the parton inside the target hadron instantaneously and incoherently from other partons.

It was within the impulse approximation that Bjorken and Paschos[3] investigated the relationship between the structure of the proton and the inelastic Compton and electron-proton scattering. They showed that the inelastic Compton scattering as well as the inelastic electron scattering off the proton may reveal the structure of the proton, noticing that the coupling strengths for these processes are proportional to the charge of the partons.

Meanwhile, they have obtained a relation concerning about the differential cross sections between the inelastic Compton scattering and the inelastic electron-proton scattering within the Born approximation. This is because the electron-proton scattering within the one-photon exchange approximation can be regarded as a Compton scattering of a virtual photon off the parton inside the proton.

Explicitly, the relation reads

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\gamma p} = \frac{\nu^2}{kk'}\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{ep} \left\langle \sum_i Q_i^4 \right\rangle \Big/ \left\langle \sum_i Q_i^2 \right\rangle , \qquad (1)$$

where $\langle \sum_i Q_i^4 \rangle$ is the average of the $(\text{charge})^4$ of the partons weighted by their probability distributions.

In this note, we are going to consider eq.(1), which we will call afterwards the Bjorken-Paschos relation, mainly for two reasons. Since nowadays it is generally accepted that the hadrons are composed of quarks and gluons, we want to examine the Bjorken-Paschos relation for the system of the spin-$\frac{1}{2}$ and spin-1 partons. The other is, as suggested by themselves, that this relation may distinguish clearly the integrally-charged quark model from the fractionally-charged quark model.

We find that the precise measurement of the cross section of the inelastic Compton scattering may tell not only about the parton picture but also about the charge assignment of the partons definitely, through the Bjorken-Paschos relation.

We may take the quantum chromodynamics as a gauge theory combined with the fractionally-charged quark model; and we will take as an integrally-charged quark model the unified gauge theory with the Han-Nambu model[4], proposed by Pati and Salam[5]. The group structure of this theory coincides with the Weinberg-Salam model for the weak and electromagnetic interactions and the quarks remain fractionally charged, prior to the color symmetry breaking. When the color symmetry is broken spontaneously, we see that the gauge fields for the strong interactions and those for the weak and electromagnetic interactions get mixed. After diagonalization, we find that the half of the gluon octet become

charged and the quarks acquire additional color charges to be intagrally charged. Since we are going to deal with the electromagnetic couplings, we expect that the color contributions may arise from the color charges due to the spontaneous breaking of the color symmetry. In the remainder of this note, we will consider within the unified gauge theory and will notice the color contributions when necessary, as they can be easily traced.

Consider first the electroproduction process $e + p \longrightarrow e + X$. Within the Born approximation, the mediating gauge bosons are the virtual photon and its color partner U^0. The differential cross section for this process is related to the proton structure functions according to

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{ep} = \frac{4\alpha^2 E'^2}{Q^4} \cos^2\frac{\theta}{2} \left[W_2(\nu,Q^2) + 2W_1(\nu,Q^2)\tan^2\frac{\theta}{2} \right], \quad (2)$$

where $Q^2 = 4EE'\sin^2\frac{\theta}{2}$, $\nu = E - E'$ and [6,7]

$$\nu W_2(\nu,Q^2) = x\,\delta\left(x - \frac{Q^2}{2M\nu}\right) \left[\frac{4}{3}(u+\bar{u}) + \frac{1}{3}(d+\bar{d}) + \frac{1}{3}(s+\bar{s}) + \frac{4}{3}(c+\bar{c}) \right]$$
$$+ \Delta^2(\xi)\,\rho(Q^2)\; x\,\delta\left(x - \frac{Q^2}{2M\nu}\right) \left[\frac{2}{3}\sum_q (q+\bar{q}) + \left(4 + \frac{4}{3}\xi + \frac{1}{3}\xi^2\right) g \right], \quad (3)$$

$$M W_1(\nu,Q^2) = \frac{1}{2}\,\delta\left(x - \frac{Q^2}{2M\nu}\right) \left[\frac{4}{3}(u+\bar{u}) + \frac{1}{3}(d+\bar{d}) + \frac{1}{3}(s+\bar{s}) + \frac{4}{3}(c+\bar{c}) \right]$$

$$+\Delta^2(\xi)\,\rho(Q^2)\,\frac{1}{2}\,\delta\!\left(x-\frac{Q^2}{2M\nu}\right)\left[\frac{2}{3}\sum\left(q+\bar{q}\right)+\frac{8}{3}(4+\xi)\,g\right] \qquad (4)$$

with $\Delta^2(\xi) = (1+\xi)^{-2}$, $\xi = Q^2/m_g^2$, m_g being the gluon mass. The function $\rho(Q^2)$ is a threshold factor for the exhibition of the color contributions. The quark and the gluon distribution functions are denoted by $q(x)$ and $g(x)$. We note that the terms multiplied by the factor are the color contributions.

It is well-known that the total transverse and longitudinal cross sections are conveniently defined as[8]

$$W_1 = \frac{K}{4\pi^2\alpha}\,\sigma_T \qquad \text{and} \qquad W_2 = \frac{K}{4\pi^2\alpha}\cdot\frac{Q^2}{Q^2+\nu^2}\left(\sigma_T+\sigma_L\right) \quad , \qquad (5)$$

where $K = \nu - Q^2/2M$. Using the ratio $R = \sigma_L/\sigma_T$, we may rewrite eq.(2) as

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{ep} = \frac{4\alpha^2E'^2}{Q^4}\,W_2(\nu,Q^2)\left[1+\frac{1}{1+R}\,\frac{\nu^2}{2EE'}\right] . \qquad (6)$$

It can be easily seen that $R \to \infty$ for the spin-1 parton and $R \to 0$ for the spin-$\frac{1}{2}$ parton in the deep-inelastic limit of ν and $Q^2 \to \infty$ while their ratio fixed. We should remark that in the deep-inelastic limit of this process, the effective charges for the quarks and the gluons become equal to the charge assignment of the fractionally-charged quark model:

$$Q_{quark} = Q_{flavor} + \Delta^2(\xi)\,\rho(Q^2)\,Q_{color} \to Q_{flavor}$$

$$Q_{gluon} = \Delta^2(\xi)\,\rho(Q^2)\,Q_{color} \to 0 \quad .$$

Next, let us consider the inelastic Compton scattering, $\gamma + p \to \gamma + X$. The impulse approximation enables us to describe this process as the incoherent point interaction between the incident photon and the parton. Bjorken and Paschos have calculated that the differential cross section for the inelastic Compton scattering off the spin-$\frac{1}{2}$ parton is

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)^{\frac{1}{2}}_{\gamma p} = \frac{4\alpha^2E'^2}{Q^4}\cdot\frac{\nu}{kk'}\left(1+\frac{\nu^2}{2kk'}\right)x f_{\frac{1}{2}}(x) \quad , \qquad (7)$$

where $f_{\frac{1}{2}}(x)$ is the probability distribution of the spin-$\frac{1}{2}$ parton having the momentum fraction x of the proton momentum. On the other hand, the inelastic Compton scattering off the spin-1 parton has been calculated recently.[9] We have carefully recalculated for the differential cross section and we find that the result is

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)^{1}_{\gamma p} = \frac{4\alpha^2E'^2}{Q^4}\cdot\frac{\nu}{kk'}\left[\frac{2}{3}+\frac{2}{3}\,\frac{\nu^2}{kk'}+\frac{1}{6}\left(\frac{\nu^2}{kk'}\right)^2\right]x f_1(x) \ . \qquad (8)$$

Comparing eqs.(6)-(8) with eq.(1), with the identification[3]

$$k, k' \leftrightarrow E, E' \qquad \text{and} \qquad -t \leftrightarrow Q^2$$

we can immediately conclude that the unified gauge theory does not satisfy the Bjorken-Paschos relation. This is because of the charged gluon contributions. The participation of the charged gluons in the inelastic Compton scattering yield the spin-dependent expressions for eqs. (6)-(8).

The one way to remedy the difficulty of the spin-dependence is to consider the Bjorken-Paschos relation in a kinematically restricted region, that is, in a region where

$$\frac{\nu^2}{kk'} \ll 1 \quad . \tag{9}$$

We note that this is a region of the forward scattering.

Then, we can neglect all the spin-dependent terms in eqs.(6)-(8) and can restore the original form of the relation. However, it should be noticed in eq.(8) that the factor of $2/3$ must be multiplied to the probability distribution of the spin-1 parton. Thus, it can be written as

$$\frac{1}{\langle \sum_i Q_i^4 \rangle}\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\gamma p} = \frac{1}{\langle \sum_i Q_i^2 \rangle}\cdot\frac{\nu^2}{kk'}\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{ep} \quad , \tag{10}$$

where

$$\langle \sum_i Q_i^4 \rangle = \int_0^1 x\,dx \left[\sum Q_{quark}^4 \{q(x)+\bar{q}(x)\} + Q_{gluon}^4 \cdot \frac{2}{3} g(x) \right] \tag{11}$$

and

$$\langle \sum_i Q_i^2 \rangle = \int_0^1 x\,dx \left[\sum Q_{quark}^2 \{q(x)+\bar{q}(x)\} + Q_{gluon}^2 \cdot g(x) \right] \tag{12}$$

in the forward scattering region.

Having established the Bjorken-Paschos relation in the unified gauge theory, what we are going to show next is that this relation may clearly distinguish the integrally-charged quark model from the fractionally-charged quark model.

Let us first remark that Q_{quark} and Q_{gluon} in eqs.(11) and (12) are the effective charges so that they may be different in the different processes. Really, it is the case since there occurs no cancellation in the inelastic Compton scattering while there occurs the cancellation due to the U^0 exchange in the inelastic electron-proton scattering, as described by the factor $\Delta^2(\xi)$. Therefore, we have to calculate $\langle \sum_i Q_i^4 \rangle$ and $\langle \sum_i Q_i^2 \rangle$ separately for the two processes. As mentioned earlier, the effective charges of the quarks and the gluons become equal to the charge assignment of the fractionally-charged model, so that we have in the deep-inelastic limit

$$\frac{\langle \sum_i Q_i^2 \rangle^{INT}}{\langle \sum_i Q_i^2 \rangle^{FRA}} \to 1 \quad . \tag{13}$$

On the other hand, the ratio of the factor of $(\text{charge})^4$ is calculated as

$$\langle \sum_i Q_i^4 \rangle^{INT} / \langle \sum_i Q_i^4 \rangle^{FRA}$$

$$= \frac{\int_0^1 x\,dx \left[2(u+\bar{u}) + (d+\bar{d}) + (s+\bar{s}) + 2(c+\bar{c}) \right] + \int_0^1 x\,dx \cdot 4 \left[\frac{2}{3} g(x) \right]}{\int_0^1 x\,dx \left\{ \left(\frac{2}{3}\right)^4 (u+\bar{u}) + \left(\frac{1}{3}\right)^4 (d+\bar{d}) + \left(\frac{1}{3}\right)^4 (s+\bar{s}) + \left(\frac{2}{3}\right)^4 (c+\bar{c}) \right\} \cdot 3}$$

$$= 4.0 + 2.8 \quad ,$$

where the numerical value is evaluated from the parametrization of the parton distribution functions of Pakvasa et al.[10] The gluon distribution is required to satisfy the sum rule for the momentum conservation,

$$\sum \int_0^1 x\,dx \left\{ q(x) + \bar{q}(x) \right\} + \int_0^1 x\,dx \left[8\,g(x) \right] = 1 \quad .$$

Consequently, we see that the Bjoken-Paschos relation predicts the cross section of the inelastic Compton scattering in the unified gauge theory about 6.8 times larger than in the fractionally-charged quark model. Although we may as well predict the cross section of the inelastic Compton scattering by directly using eqs.(7) and (8), but it is reliable and consistent to predict the cross section via the Bjoken-Paschos relation. This is because the prediction using eqs.(7) and (8) has two basic assumptions; the idea of the parton picture and the integrally-charged quark model. While the latter assumption can be tested straight-forward by the experiments on the inelastic Compton scattering, the former one must be consistently justified. The Bjorken-Paschos relation may justify the impulse approximation of the parton picture by comparing the two processes, as remarked by themselves.[3]

We have some closing remarks.

(i) We first remark that the kinematical region satisfying eq.(9) has been considered before. It has been observed by Brodsky and Roy[11] that in the deep-inelastic processes involving two photons the impulse approximation of the parton picture is consistent with the field-theoretical method of Drell, Levy and Yan for a limited region of kinematics. It is the region satisfying eq.(9). Therefore, the justification of the Bjorken-Paschos relation is provided by both of the two approaches of the parton picture in the forward scattering region.

(ii) The test of the unified gauge theory via the Bjorken-Paschos relation requires the simultaneous measurements of the inelastic Compton scattering and the inelastic electron-proton scattering at the same incident energy. Also, the experiments should be prepared to measure the forward scattering cross section. Since the momentum transfer is expressed as $Q^2 = 4EE' \sin^2 \frac{\theta}{2}$ where we restrict the angle $\theta \ll 1$, we should have very high energies for the incident electron and photon to make Q^2 large enough to satisfy eq.(13). Roughly, Q^2 should be as large as 4 or 5 times of the gluon mass to have $\Delta^2(\xi)$ less than 5 %. Moreover, independently of the gluon mass, we should have large Q^2 to be sure that the parton idea is applicable.

(iii) The Q^2-dependent higher-order corrections are seen to make only a small change to the ratio of $\langle \sum_i Q_i^4 \rangle^{INT} / \langle \sum_i Q_i^4 \rangle^{FRA}$, since the range of the value of Q^2 is restricted to satisfy the forward scattering condition. In an experiment using the 100 GeV incident beam, Q^2 cannot be larger than 30 GeV^2 if $\theta < 4°$. As the momentum proportion carried by each kind of partons does not change significantly for Q^2 up to 30 GeV^2, we expect that the factor of 6.8 remains unchanged.

(iv) One assumption we have made during the calculations is that the parton distributions are effectively the same for both of the quark model. This kind of assumption cannot be justified directly. However, there seems to be no reason that the parton distributions should be definitely different according to the charges of the quarks. We only note that the ratio in eq.(14) suffers only a minor change within 5 % when we use for $\langle \sum_i Q_i^4 \rangle^{INT}$ and $\langle \sum_i Q_i^4 \rangle^{FRA}$ the different parametrizations.[12]

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

References

1 S. D. Drell, D. J. Levy and T.-M. Yan, Phys. Rev. 187(1969), 2159; D1(1970), 1035; D1(1970), 1617; T.-M. Yan and S. D. Drell, Phys. Rev. D1(1970), 2402.

2 R. P. Feynman, Phys. Rev. Lett. 23(1969),1415.

3 J. D. Bjorken and F. A. Paschos, Phys. Rev. 185(1969),1975.

4 Y. Nambu and M. Y. Han, Phys. Rev. D10(1974),674.

5 J. Pati and A. Salam, Phys. Rev. D8(1973),1240; D10(1974),275.

6 G. Rajasekaran and P. Roy, Pramana 5(1975), 303.

7 J. Pati and A. Salam, Phys. Rev. Lett. 36(1976),11; 37(1976),E1312; G. Rajasekaran and P. Roy, Phys. Rev. Lett. 36(1976),355.

8 L. Hand, Phys. Rev. 129(1963),1834; see for a review M. Perl, in High Energy Hadron Physics(1974, Wiley, New York).

9 H. K. Lee and J. K. Kim, Phys. Rev. D18(1978),3985; D21(1980), E 312.

10 J. Pakvasa, D. Parashar and S. F. Tuan, Phys. Rev. D10(1974),2124.

11 S. J. Brodsky and P. Roy, Phys. Rev. D3(1971), 2914.

12 J. K.-Andre and F. E. Paige, Phys. Rev. D19(1979),221; F. E. Paige, T. L. Trueman and T. N. Tudron, Phys. Rev. D19(1979),935.

CURRENT ICTP PREPRINTS AND INTERNAL REPORTS

IC/79/163	G. GELMINI: The s-wave $Qc\bar{Q}\bar{c}$ states.
IC/79/165	S. MARCULESCU and L. MEZINCESCU: Phase factors and point-splitting in supersymmetry.
IC/79/166	A. FASOLINO, G. SANTORO and E. TOSATTI: Instability, distortion and dynamics of the W(100) surface.
IC/79/168	T.D. PALEEV: The trace formulae for realizations of Lie algebras with A-operators. Analysis of the Fock representations.
IC/80/1	W. KRÓLIKOWSKI: Towards a dynamical preon model.
IC/80/5 INT.REP.*	RIAZUDDIN: Two-body D-meson decays in non-relativistic quark model.
IC/80/6 INT.REP.*	G. ALBERI, M. BLESZYNSKI, T. JAROSZEWICZ and S. SANTOS: Deuteron D-wave and the non-eikonal effects in tensor asymmetries in elastic proton-deuteron scattering.
IC/80/7 INT.REP.*	A.M. KURBATOV and D.P. SANKOVICH: On the one variational principle in quantum statistical mechanics.
IC/80/8 INT.REP.*	G. STRATAN: On the alpha decay branching ratios of nuclei around A = 110.
IC/80/9	N.Y. AYOUB: Effect of high lying states on the ground and few low lying excited 0^+ energy levels of some closed-shell nuclei.
IC/80/10	L. FONDA, N. MANKOČ-BORŠTNIK and M. ROSINA: The decay of coherent rotational states subject to random quantum measurements.
IC/80/11	G.C. GHIRARDI, V. GORINI and G. PARRAVICINI: Spatial localization of quantum states and physical meaning of the marix elements of the resolvent operator.
IC/80/12 INT.REP.*	M.V. MIHAILOVIČ and M.A. NAGARAJAN: A proposal for calculating the importance of exchange effects in rearrangement collisions.
IC/80/13	A. BULGAC, F. CARSTOIU and O. DUMITRESCU: Double folded Yukawa interaction potential between two heavy ions.
IC/80/14 INT.REP.*	W. KRÓLIKOWSKI: Lepton and quark families as quantum-dynamical systems.

* Internal Reports: Limited distribution
THESE PREPRINTS ARE AVAILABLE FROM THE PUBLICATIONS OFFICE, ICTP, P.O. BOX 586, I-34100 TRIESTE, ITALY.

IC/80/16 AHMED OSMAN: Four-body problem for four bound alpha particles in ^{16}O.

IC/80/17 INT.REP.* NAMIK K. PAK: Introduction to instantons in Yang-Mills theory (Part I).

IC/80/18 INT.REP.* RIAZUDDIN: Neutral current weak interaction without electro-weak unification.

IC/80/19 N.S. CRAIGIE, S. NARISON and RIAZUDDIN: An apparent inconsistency between the Dyson and renormalization group equations in QCD.

IC/80/20 INT.REP.* W.I. FURMAN and G. STRATAN: On alpha decay of some isomeric states in Po-Bi region.

IC/80/23 P. ROZMEJ, J. DUDEK and W. NAZAREWICZ: Possible interplay between non-axial and hexadecapole degrees of freedom - An explanation for "enormously" large Q_4?

IC/80/24 REVISED.

IC/80/25 INT.REP.* G. MAIELLA: Path-integral measure and the Fermion-Bose equivalence in the Schwinger model.

IC/80/26 N.S. CRAIGIE, S. NARISON and RIAZUDDIN: A critical analysis of the electromagnetic mass shift problem in QCD.

IC/80/27 INT.REP.* D.K. CHATURVEDI, U. MARINI BETTOLO MARCONI and M.P. TOSI: Mode-coupling theory of charge fluctuation spectrum in a binary ionic liquid.

IC/80/28 ABDUS SALAM: Gauge unification of fundamental forces (Nobel lecture).

IC/80/30 R. NOJAROV, E. NADJAKOV and V. ANTONOVA: High spin structure in a coupled bands model.

IC/80/31 AHMED OSMAN: Rearrangement collisions between four identical particles as a four-body problem.

IC/80/34 INT.REP.* I.Zh. PETKOV and M.V. STOITSOV: On a generalization of the Thomas-Fermi method to finite Fermi systems.

IC/80/37 A.N. ANTONOV, V.A. NIKOLAEV and I.Zh. PETKOV: Nucleon momentum and density distributions of nuclei.

IC/80/40 INT.REP.* W. KRÓLIKOWSKI: An integral transform of the Salpeter equation.

IC/80/41 INT.REP.* Z.A. KHAN: Elastic scattering of intermediate energy protons on ^{4}He and ^{12}C.

IC/80/44 A. OSMAN: Two-nucleon transfer reactions with form factor models.

IC/80/45 INT.REP.* E.W. MIELKE: Towards exact solutions of the non-linear Heisenberg-Pauli-Weyl spinor equation.

IC/80/46 INT.REP.* N.S. CRAIGIE: Catastrophe theory and disorders of the family system.

IC/80/53 R. BECK, M.V. MIHAILOVIĆ and M. POLJSAK: Calculation of nuclear reaction parameters with the generator co-ordinate method and their interpretation.

IC/80/58 INT.REP.* S. YOKSAN: Spatial variations of order parameter around Kondo impurity for $T \leq T_c$.

IC/80/59 INT.REP.* J.K.A. AMUZU: Sliding friction of some metallic glasses.

IC/80/60 INT.REP.* Zi-Zhao GAN and Guo-Zhen YANG: A theory of coherent propagation of light wave in semiconductors.

IC/80/61 INT.REP.* K.G. AKDENIZ and O. OGUZ: A new class of meronic solutions.

IC/80/63 INT.REP.* H.B. SINGH and H. HAUG: Optical absorption spectrum of highly excited direct gap semiconductors.

IC/80/64 INT.REP.* W. KLONOWSKI: Living protein macromolecule as a non-equilibrium metastable system,

IC/80/65 INT.REP.* H.Q. REIK: An approximation to the ground state of $E \otimes \varepsilon$ and $\Gamma_8 \oplus \tau_2$ Jahn-Teller systems based on Judd's isolated exact solutions.

IC/80/66 INT.REP.* A.N. ERMILOV, A.N. KIREEV and A.M. KURBATOV: Spin glasses with non-Gaussian distributions. Frustration model.