PPPL-1920

PPPL-1920

AXISYMMETRIC TOKAMAK SCRAPEOFF TRANSPORT

By

C.E. Singer and W.D. Langer

August 1982

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

NOTICE

Printed in the United States of America.

Available from:

National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151

Price: Printed Copy $ * ; Microfische $3.50

*PAGES	NTIS Selling Price
1-25	$5.00
26-50	$6.50
51-75	$8.00
76-100	$9.50
101-125	$11.00
126-150	$12.50
151-175	$14.00
176-200	$15.50
201-225	$17.00
226-250	$18.50
251-275	$20.00
276-300	$21.50
301-325	$23.00
326-350	$24.50
351-375	$26.00
376-400	$27.50
401-425	$29.00
426-450	$30.50
451-475	$32.00
476-500	$33.50
500-525	$35.00
526-550	$36.50
551-575	$38.00
576-600	$39.50

For documents over 600 pages, add $1.50 for each additional 25 page increment.

AXISYMMETRIC TOKAMAK SCRAPEOFF TRANSPORT

Clifford E. Singer and William D. Langer

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

ABSTRACT

We present the first self-consistent estimate of the magnitude of each term in a fluid treatment of plasma transport for a plasma lying in regions of open field lines in an axisymmetric tokamak. The fluid consists of a pure hydrogen plasma with sources which arise from its interaction with neutral hydrogen atoms. The analysis and results are limited to the high collisionality regime, which is optimal for a gaseous neutralizer divertor, or to a cold plasma mantle in a tokamak reactor. In this regime, both classical and neoclassical transport processes are important, and loss of particles and energy by diamagnetic flow are also significant. The prospect of extending the analysis to the lower collisionality regimes encountered in many existing experiments is discussed.

I. INTRODUCTION

Although the isolation of the plasma from material boundaries is essential for magnetic confinement, relatively little progress has been made in understanding the transport properties of plasma near material boundaries in controlled fusion plasmas. Progress in understanding this regime has been hampered by the difficulty of making extensive measurements in such regions and to the formidable nature of the theoretical and computational problems in modeling multidimensional flows. Here we find the dominant terms in the simplest set of plasma transport equations relevant to modeling the plasma in a region of open field lines under conditions relevant to tokamak operation. The analysis is restricted to two-dimensional axisymmetric flows. We include all the relevant flows due to open field lines and analyze the contribution of sources due to interaction of plasma with neutrals. Our analysis shows that a number of important physical processes have been neglected in earlier studies of 2-d axisymmetric flow.

Previous work relevant to two-dimensional (2-d) plasma flows in tokamaks can be divided into fluid and kinetic approaches. One of the earlier treatments of flow in 2-d planar geometry is that of Boozer.[1] This work, while neglecting sources, gradients in the direction of diamagnetic current flow and finite ion temperature, showed that electrons and ions may leave the plasma edge at different locations. Later, Auerbach and Boozer[2] showed that a divertor plasma flow would concentrate on an X point in the magnetic field. However, their neglect of sources, conduction, and other physical processes precludes using this work as a basis for quantitative modeling. Emery et al.[3] studied 2-d axisymmetric plasma flow in a geometry with closed field lines and demonstrated that the problem is numerically tractable. However, the theory they used is not suitable for quantitative modeling due to neglect of sources

and oversimplification of viscosity, Ohm's law, and thermal conductivity. More recently, Petravic et al. published a treatment of flow in a 2-d scrapeoff plasma,[4] coupled to a sophisticated computation of neutral gas transport.[5] The simplified transport equations presented by Petravic et al. include an ad hoc treatment of cross-field transport, though the effects of this transport were not included in their published applications. There have been other similar treatments of plasma flow along field lines in a tokamak scrapeoff,[6,7] as well as several order-of-magnitude estimates of the effects of parallel losses on transport across magnetic flux surfaces.[8,9,10,11] Each of these models gives some insight into particular transport processes, but since they are one-dimensional treatments they cannot, in general, give a complete description of the plasma flows.

Steady-state kinetic models of the scrapeoff flow have been constructed by several authors.[12,13,14] This kind of approach is motivated by the observation that, in many existing tokamak plasmas, the mean free path of particle motion along or across field lines can be comparable to the scale heights for variation of plasma parameters. Since these models are susceptible to various instabilities, such as loss-cone driven instabilities,[14] it seems unlikely that steady-state kinetic models will give an adequate description of the transport in regimes of low collisionality.

We restrict our analysis to a fluid model valid in collision dominated plasmas where the effects of such instabilities do not necessarily invalidate a fluid treatment of plasma transport. With the exception of the Doublet-III expanded boundary,[15] results for this regime from existing tokamaks have not yet been reported. However, since a high collisionality regime is probably essential to minimize plasma contamination due to sputtering in tokamak reactors,[16] we have concentrated our efforts on this regime. It is possible

that a simple generalization of our equations may, in the future, give an adequate description of the turbulent, low-collisionality plasmas obtained in many present tokamaks (provided that an enhanced effective electron-electron frequency and/or electron-ion collision frequency gives an adequate description of the effect of turbulence). Of course, there is also a possibility that fluid instabilities, such as tearing, rippling, and resistive-g modes, may alter the plasma transport. With the exception of a recent treatment of rippling modes,[17] estimates of the magnitude of the effects of fluid instabilities on transport in the scrapeoff region suggests that they are not dominant, at least in existing experiments.[18] Unfortunately, these estimates are not based on a self-consistent analysis of the steady-state fluid equations and, thus, they do not represent perturbations around a valid equilibrium. Therefore, we perform here an analysis of the fluid equations which describe such an equilibrium and estimate the dominant terms in these equations.

Our approach is based on the methods outlined in the reviews by Sigmar and Hirshman,[19] Hinton and Hazeltine,[20] and Braginskii.[21] We apply the conservation equations of Sigmar and Hirshman (in a different notation) to a pure hydrogen, deuterium, or tritium plasma containing sources due to the interaction of the plasma with neutrals. After stating and justifying the approximations used, we generalize Hinton and Hazeltine's derivation of Pfirsch-Schlüter transport to a flow in two dimensions with sources. We then use Braginskii's expressions for the transport coefficients in order to estimate the dominant terms in the conservation equations. This approach is similar to that of Hinton and Hazeltine, except that here we cannot eliminate the component of the total momentum balance which is directed along the magnetic field, because flux surface averaging is not appropriate in systems

with open field lines. Therefore, we retain the parallel momentum balance in addition to the usual requirements for conservation of particles and ion and electron energy. The current parallel to the magnetic field is also retained since heating due to this current can be significant in an axisymmetric scrapeoff. After reduction of these equations by ordering in the inverse aspect ratio, the significance of the various terms is discussed, including a possible extension to scrapeoff plasmas with turbulence.

II. TRANSPORT EQUATIONS

II.1. Fluid equations

Our analysis of the transport of a scrapeoff plasma rests on a reduction of the hydromagnetic equations derived from moments of the Fokker-Planck equation. The Fokker-Planck equation describing the distribution function f_a for each species "a" is

$$\frac{\partial f_a}{\partial t} + \sum_\beta \left\{ v_\beta \frac{\partial f_a}{\partial x_\beta} + \frac{e_a(\vec{v} \times \vec{B})_\beta}{cm_a} \frac{\partial f_a}{\partial v_\beta} \right\} = \left(\frac{\partial f_a}{\partial t}\right)_c ,$$

where $(\partial f_a/\partial t)_c$ represents collisions which take place in a time short compared to the times of interest for changes in the fluid.[19] In general, this term contains a Coulomb collision operator, C, and a non-Coulomb collision operator, σ. The fluid equations, analogous to those in Braginskii with the addition of the non-Coulomb collision term, are:

$$\frac{\partial n_a}{\partial t} + \vec{\nabla} \cdot (n_a \vec{u}_a) = S_a \qquad \text{(continuity)}$$

$$m_a n_a \frac{d\vec{u}_a}{dt} = m_a n_a \left[\frac{\partial \vec{u}_a}{\partial t} + (\vec{u}_a \cdot \vec{\nabla})\vec{u}_a\right] = -\vec{\nabla} P_a - \sum_\beta \frac{\partial \Pi_{\alpha\beta,a}}{\partial x_\beta} + e_a n_a \left[\vec{E} + \vec{u}_a \times \frac{\vec{B}}{c}\right]$$

$$+ \vec{R}_a + \vec{F}_a - m_a \vec{u}_a S_a \qquad \text{(force balance)}$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2} n_a m_a u_a^2 + \frac{3}{2} n_a T_a\right)$$

$$+ \vec{\nabla}\cdot\left\{\left(\frac{1}{2} m_a n_a u_a^2 + \frac{5}{2} P_a\right)\vec{u}_a + \sum_\alpha \Pi_{\alpha\beta,a} u_{\alpha,a} + \vec{q}_a\right\}$$

$$= e_a n_a \vec{E}\cdot\vec{u}_a + W_a + \vec{R}_a\cdot\vec{u}_a + Q_a \quad \text{(energy transport)}.$$

Most of the symbols have their standard definition, but, to avoid any confusion, we list some of these below.

$$n_a = \int f_a d^3v \qquad \text{(density)}$$

$$\vec{u}_a = \frac{1}{n_a}\int \vec{v} f_a d^3v \qquad \text{(fluid velocity)}$$

$$S_a = \int \sigma_a d^3v \qquad \text{(particle source)}$$

$$P_a = \frac{1}{3} m_a \int |\vec{v} - \vec{u}_a|^2 f_a d^3v \qquad \text{(pressure)}$$

$$\Pi_{\alpha\beta,a} = n_a m_a \int \left(v_\alpha^R v_\beta^R - \frac{1}{3}\delta_{\alpha\beta}|v^R|^2\right) f_a d^3v \qquad \text{(viscosity tensor)}$$

where

$$\vec{v}^R = \vec{v} - \vec{u}_a \qquad \text{(random velocity)}$$

$$\vec{R}_a = m_a \int C_a \vec{v}^R d^3v \qquad \text{(Coulomb friction)}$$

$$\vec{F}_a = m_a \int \sigma_a \vec{v}^R d^3v \qquad \text{(non-Coulomb scattering force)}$$

$$T_a = P_a/n_a \qquad \text{(temperature)}$$

$$\vec{q}_a = \frac{1}{2} m_a \int |\vec{v} - \vec{u}_a|^2 \vec{v} f_a d^3v \qquad \text{(heat flux)}$$

$$W_a = \frac{1}{2} m_a \int |\vec{v}|^2 \sigma_a d^3v \qquad \text{(net energy source)}$$

$$Q_a = \frac{1}{2} m_a \int |\vec{v} - \vec{u}_a|^2 C_a d^3v \qquad \text{(collisional heating)} .$$

These fluid equations must be augmented with the equations for the electric field, $\vec{E}$,

$$\vec{\nabla}\cdot\vec{E} = -4\pi \sum_a e_a n_a ,$$

and current continuity

$$\vec{\nabla}\cdot\vec{J} = \sum_a e_a S_a .$$

Equations for the magnetic field are dropped under the assumption that external currents prescribe the magnetic field in the region of interest. For the analysis below, it can be shown that this assumption follows from assuming a limit on the pressure ratio,

$$\beta \equiv P/(B^2/8\pi) \ll 1 ,$$

where $P = \sum_a P_a$ is the total plasma pressure.

In this paper, we consider only the equations which describe the ion and electron fluids. The neutral source terms are probably best computed with a Monte-Carlo algorithm which can follow a large number of neutral particles through a realistic geometry.[22] Here we assume that such an algorithm exists and can be used to sum over all plasma-neutral collisions in order to compute the source integrals in the plasma fluid equations. We also implicitly assume that the expressions given by Braginskii[21] for plasma transport coefficients are not significantly changed by the existence of neutral sources. For special neutral distribution functions this assumption is valid, but general quantitative modeling studies which rely on this assumption should be examined carefully for possible errors.

II.2. Geometry and restrictive assumptions

To carry out an analysis of the fluid equations, it is necessary to define a convenient coordinate system relevant to an axisymmetric scrapeoff. For this purpose, we use the flux coordinates defined by Hinton and Hazeltine, (ψ, θ, ζ), where ψ is constant on a magnetic flux surface and θ and ζ are the poloidal and toroidal angles. We can write the magnetic field in terms of the toroidal, $\vec{B}_T$, and poloidal, $\vec{B}_p$, components as

$$\vec{B} = \vec{B}_T + \vec{B}_p = RB_T\vec{\nabla}\zeta + \vec{\nabla}\zeta \times \vec{\nabla}\psi ,$$

as discussed on page 251 of Hinton and Hazeltine.[17] In this form, the major radius $R = |\nabla\zeta|^{-1}$ and the metric determinant (i.e., the inverse of the Jacobian) is

$$g^{-1/2} = |\vec{\nabla}\psi \cdot \vec{\nabla}\theta \times \vec{\nabla}\zeta| .$$

We usually resolve vectors into components along three orthonormal unit vectors $\hat{u}_{\parallel} = \hat{B} = \vec{B}/B$ parallel to the magnetic field, $\hat{u}_{\phi} = \hat{\nabla}\phi = \vec{\nabla}\phi/|\vec{\nabla}\phi|$ normal to the magnetic flux surfaces (referred to here as the "radial" direction), and $\hat{u}_d = \hat{u}_{\parallel} \times \hat{u}_{\phi}$ in the direction of the diamagnetic current. These vectors are illustrated in Fig. 1. Our analysis of the fluid equations for the axisymmetric scrapeoff regime will be carried out in terms of ordering parameters which measure the scale lengths, velocities, and collisionality of the plasma. The scale lengths for the plasma are $L_{\parallel}$, L_d, and L_{ϕ}, where the subscripts denote the parallel, diamagnetic, and radial directions (these directions are mutually perpendicular as indicated in Fig. 1.). The length of the region analyzed is L_{ℓ} as measured along a field line, and $L_d = (\varepsilon/q)L_{\parallel}$ by definition of the safety factor, q. The Mach number is

$$M \equiv u_{\parallel,i}/v_{th,i} ,$$

where $v_{th,i}$ is the thermal velocity of the ions and $u_{\parallel,i}$ is their fluid velocity in the direction parallel to the magnetic field. The toroidal and poloidal gyroradii are assumed to be small compared to the perpendicular scale length, so that

$$\delta_i = \rho_{L_i}/L_{\phi} \ll 1 ,$$

$$\delta_{p_i} = \frac{B}{B_p}\,\delta_i \approx \frac{q}{\varepsilon}\,\delta_i \ll 1 ,$$

where the ion Larmor radius is $\rho_{L_i} = v_{th,i}/\Omega_i$ and $\Omega_i = eB/(m_i c)$, q is the safety factor, and $\varepsilon \equiv a/R$ is the inverse aspect ratio. Finally, the symbol Δ denotes the mean free path compared to the parallel scale length for

the ions and is given by

$$\Delta = v_{th,i}/(\nu_i L_\parallel) ,$$

where $\nu_i = 4\pi^{1/2} \ell n\Lambda_i e^4 Z_i^4 n /(3m_i^{1/2} T_i^{3/2})$, is the ion-ion collision frequency, $\ell n\Lambda_i$ is the Coulomb logarithm,[24] and Z_i is the ion charge (which we take to be unity).

In the analysis to follow, we assume steady state, toroidal symmetry, and quasineutrality

$$\frac{\partial}{\partial t} = 0 ,$$

$$\frac{\partial}{\partial \zeta} = 0 ,$$

$$n_e = n_i .$$

As a result, we can write $\vec{E} = -\vec{\nabla}\phi$, where ϕ is the electrostatic potential. In addition, we adopt the following basic ordering assumptions:

$$L_\phi \sim L_{\phi,e} \sim L_{\phi,i} ,$$

$$L_\phi << \varepsilon L_d ,$$

$$L_d/L_\parallel = \varepsilon/q << 1 .$$

Here $L_{\psi,i}$ and $L_{\psi,e}$ are the scale lengths for variation of ion and electron plasma parameters in the $\hat{\nabla\psi}$ direction.

For convenience, we choose the inverse aspect ratio as a fundamental ordering parameter. (Here, the expression $a \ll b$ means $\lim_{\varepsilon\to 0} \varepsilon^n |b/a| = 1$ where n is a number satisfying $1/2 > n > -1/2$. The expression $a \sim b$ means $\lim_{\varepsilon\to 0} \varepsilon^n |b/a| = 1$ where $n > -1/2$. The symbols $<$ and $=$ have their usual algebraic meaning: $a \lesssim b$ means that either $a \ll b$ or $a \sim b$.) We assume

$$q \sim 1/\varepsilon ,$$

$$(m_e/m_i)^{1/2} \sim \varepsilon^4 ,$$

which is a reasonable approximation for typical tokamak parameters. We shall comment further on the assumption $\varepsilon \sim (m_e/m_i)^{1/8}$ after deriving our final results.

We also assume that the parallel Mach number is small

$$M \lesssim \varepsilon ,$$

since 1-d analyses of scrapeoff flows[4,6,7] suggest that this is true for the large region of plasma which is not near material boundaries. In addition, we assume that the parallel Mach number is comparable to or larger than the dimensionless poloidal gyroradius,

$$\delta_{p_i} \lesssim M .$$

This condition turns out to apply to the high collisionality regimes of

interest here and it greatly simplifies the analysis of the fluid equations.

We further assume that the plasma is "magnetized"[21]

$$(\nu_i/\Omega_i) \ll 1 \ ,$$

which should be valid for any tokamak plasma.

In addition, there are restrictions related to the neutrals in the fluid and the source function of ions, namely:

$$S_a \sim \delta_s \frac{n_a u_{\parallel,a}}{L_\parallel} \ ,$$

where the source function scale factor, $\delta_s \lesssim 1$; and

$$(\vec{u}_o - \vec{u}_i)_\parallel \lesssim u_{i,\parallel}$$

where $\vec{u}_o$ is the velocity of the neutrals. We also assume

$$\sigma_{cx} V_{max} \sim \langle \sigma v \rangle_{eo} \ ,$$

where σ_{cx} is the charge-exchange cross section, V_{max} the maximum average relative velocity between ions and neutrals in any direction, and $\langle \sigma v \rangle_{e-o}$ the rate of ionization of neutrals by electrons; this relation is valid for $T_e \gtrsim 10$ eV.[23] (Below about 10 eV, contributions from electron impact ionization should be deleted from our equations.) Furthermore, the difference between the ion and electron sources is

$$S_i - S_e \lesssim \varepsilon^{1/2} \delta_{p_i} S_i \ ,$$

which assumes that the difference results from particle drifts over at most a banana orbit width.[20] In addition, we assume that charge exchange is the dominant non-Coulomb source of friction on ions, and that electron impact ionization of neutrals is the dominant source of ions; these conditions restrict the electron temperature to $T_e \lesssim 2$ keV.[23]

Finally, since we are explicity interested in 2-d transport, we assume that particle transport across flux surfaces is comparable to particle loss along field lines

$$\frac{u_{\psi,a}}{L_\psi} \sim \frac{u_{\parallel,a}}{L_\parallel} ;$$

we also assume that neither of these terms is dominated by diamagnetic fluxes. Thus, we have

$$\frac{u_{d,a}}{L_d} \lesssim \frac{u_{\psi,a}}{L_\psi} \sim \frac{u_{\parallel,a}}{L_\parallel} \sim S_a/(\delta_s n_a)$$

where we assume $\delta_s \lesssim 1$.

To summarize, we make four classes of assumptions. First, there are the assumptions essential for the validity of a fluid theory; namely, $\Delta \ll 1$ and $\delta_{p_i} \ll 1$. Second, there are assumptions which (fortuitously) hold for typical tokamak parameters: $q \sim 1/\varepsilon$; $(m_e/m_i) \sim \varepsilon^8$; $(\nu_i/\Omega_i) \ll 1$, and $L_d/L_\parallel = \varepsilon/q \ll 1$. Third, there are those which can be justified a posteriori: $L_\psi \sim L_{\phi,e} \sim L_{\psi,i}$ and $L_\psi \ll \varepsilon L_d$. The remaining assumptions are made for convenience and are chosen to be sufficiently restrictive to make the theory relatively simple but not so restrictive as to exclude the parameter regimes of most interest in tokamak reactors. The first three classes of

assumptions should probably be retained in a future development of the transport theory while those in the fourth class should be relaxed in order to extend the regime of validity of the theory.

II.3. Reduced fluid equations

Using the basic assumptions listed above, we can form a set of equations to describe the fluid. These consist of (1) the continuity equation; (2) the parallel component of the total momentum transport; (3) an equation for the radial particle fluxes; (4) the total heat balance equation; (5) the ion heat balance equation, and (6) continuity of current density.

Under the assumptions discussed above, the reduction of the fluid equations to a form where the dominant terms can be identified and their order determined is a tedious algebraic problem. We will not present this procedure in detail, but sketch briefly how this was accomplished and give one simple example here and a more complicated example in the Appendix. Each equation was first cast in an appropriate dimensionless form. Next, the order of each term was determined, and a term by term comparison was made to identify the (potentially) leading terms. For example, the continuity equation for ions,

$$\nabla\cdot\left[n\,\vec{u}_{\parallel,i} + n\,\vec{u}_{d,i} + n\,\vec{u}_{\psi,i}\right] = S_i \ ,$$

when multiplied by $L_\parallel/(n\,v_{th,i})$, yields the following term by term ordering,

$$\left[M,\ \frac{q}{\varepsilon}\frac{u_{d,i}}{v_{th,i}}\ ,\ \frac{L_\parallel}{L_\psi}\frac{u_{\psi,i}}{v_{th,i}}\right] \gtrsim \frac{L_\parallel S_i}{n v_{th,i}} \ ,$$

where M is the parallel Mach number defined above. (Here the $\sim$ sign applies if the neutral density is high enough for sources to be significant, i.e.,

when $\delta_s \sim 1$ in our terminology. The > sign applies when sources are insignificant.) From an analysis of the radial ion momentum balance, it can be shown that

$$\left(\vec{u}_{d,i} \times \vec{B}\right)_\psi / c \sim \nabla_\psi P \ ,$$

under the assumptions listed above and, hence, the second term in the continuity equation is of the order $(q/\epsilon)\, u_{d,i}/v_{th,i} \sim \delta_{p_i}$. According to the assumptions at the end of Section II.2, the ordering for the continuity equation becomes.

$$\left[M,\ \delta_{p_i}\ ,\ M\right] \sim M\delta_s \ .$$

Here the terms in square brackets give the relative size of the gradients of the parallel, diamagnetic, and radial fluxes, respectively. The parallel and radial terms are always included because we are interested in 2-d flows. The diamagnetic term makes no significant contribution if $\delta_{p_i} \ll M$.

Applying similar considerations to all the remaining equations yielded a self-consistent ordering. The resulting equations are presented below, where the ordering is written to the right of each term.

Total parallel momentum balance $\qquad\qquad \times \frac{L_\parallel}{P}$ + ordering

$$0 = -\nabla_\parallel P \qquad \text{(pressure gradient)} \qquad \lambda_p$$

$$- m_i u_{\parallel,i} S_i \qquad \text{(momentum dilution)} \qquad \delta_s M^2$$

$$+ F_{\parallel,i} \qquad \text{(neutral friction)} \qquad \delta_s \left(\frac{E_{io}}{T}\right)^{1/2} M^2$$

$$-\sum_{\beta}\left(\frac{\partial \Pi_{\alpha\beta,i}}{\partial x_{\beta}}\right)_{\parallel} \qquad \text{(viscosity)} \qquad \delta_{p_i} M,\ \delta_i^2 \frac{M}{\Delta}$$

$$-\frac{n}{\Sigma} m_i \nabla_{\parallel} u_i^2 \qquad \text{(ram pressure)} \qquad M^2$$

$$-n m_i \left[\vec{u}_i \times \left(\vec{\nabla} \times \vec{u}_i\right)\right]_{\parallel} \qquad \text{(Coriolis forces)} \qquad M^2,\ \delta_{p_i} M\ ,$$

where terms of order (m_e/m_i) have been omitted, $\lambda_p \equiv \nabla_{\parallel} P/(P/L_{\parallel})$, $\vec{\nabla}_{\parallel} P = \hat{B} \nabla_{\parallel} P$, and E_{io} is the average energy lost by the ions per charge-exchange event.

The significance of the ordering denoted above for the parallel momentum balance is as follows. First, the pressure gradient must be balanced by terms which, under the assumptions adopted here, turn out to be of the order M^2 where $M^2 << 1$. This is consistent with detailed numerical calculations of parallel flow distant from a material boundary.[4,5,7] Second, neutral friction and momentum dilution due to electron impact ionization of cold neutrals can be significant unless the neutral density is small (i.e., unless $\delta_s << 1$).

Third, there are two viscous forces of order $\delta_{p_i} M$ and one of order $\delta_i^2 M/\Delta$. The only significant viscous forces in this equation are due to shearing of the parallel ion flow velocity, $u_{\parallel,i}$. One term of $\mathcal{O}(\delta_{p_i} M)$ corresponds to the gradient in the diamagnetic direction of the radial shear of the parallel flow. The other term of $\mathcal{O}(\delta_{p_i} M)$ corresponds to the radial gradient of the diamagnetic shear of the parallel flow. The familiar ("classical") term of $\mathcal{O}(\delta_i M^2/\Delta)$ corresponds to the radial gradient of the radial shear of $u_{\parallel,i}$.[9] The viscous forces are never small compared to the other forces in the total parallel momentum balance at very large collisionality where $\delta_{p_i} \lesssim M$.

Finally, the important inertial terms include ram pressure and

components of the vorticity $\vec{\omega}_i = \vec{\nabla} \times \vec{u}_i$. These latter "Coriolis forces" arise from radial and diamagnetic gradients of the parallel flow velocity. Like the viscous forces, the term involving $\vec{\omega}_{\phi,i} = \vec{\nabla} \times \vec{u}_{d,i}$ is negligible at very large collisionality where $\delta_{p_i} \ll M$.

Radial ion transport $\times \dfrac{1}{n\nu_{th,i}} \rightarrow$ ordering

$$
\begin{aligned}
n u_{\psi,i} \qquad & \text{(radial ion flux)} && \delta_{p_i}^{-1} M \\
= \frac{1}{m_i \Omega_{p_i}} \frac{\vec{B}_T}{B_T} \cdot \Big[& \vec{R}_i && \text{(Coulomb friction)} && \lambda_T, \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\delta_{p_i}}{\Delta} \\
& + \vec{F}_i && \text{(neutral friction)} && 0 \\
& - m_i \vec{u}_i S_i && \text{(momentum dilution)} && 0 \\
& + m_i n \vec{u}_i \times (\vec{\nabla} \times \vec{\ }) && \text{(Coriolis forces)} && 0 \\
& - \sum_\beta \frac{\partial \Pi_{\alpha\beta,i}}{\partial x_\beta} \Big] && \text{(viscosity)} && 0
\end{aligned}
$$

where

$$\lambda_T \equiv \nabla_\parallel T_e / \left(T_e / L_\parallel\right) ,$$

and the poloidal gyrofrequency is $\Omega_{p_i} = eB_p/m_i c$, and 0 in the last column indicates terms which do not contribute to highest order in ε. Three terms, $\vec{B}_T \cdot \vec{\nabla} P$, $\vec{B}_T \cdot \vec{\nabla} \phi$, and $\vec{B}_T \cdot \vec{\nabla} (u^2/2)$, are identically zero for toroidally symmetric systems and, therefore, have been explicitly excluded above. The dominant contributions on the right-hand side are neoclassical terms and arise from the parallel thermal and frictional forces. That is

$$\vec{B}_T \cdot \vec{R}_i \sim \vec{B}_T \cdot \vec{R}_{\parallel,i} ,$$

where

$$\vec{R}_{\parallel,i} = \beta_o n \vec{V}_\parallel T_e + m_e n \nu_e \frac{\alpha_o \vec{J}_\parallel}{(en)} ,$$

with $\nu_e = 8(2\pi)^{1/2} ne^4 \ell n \Lambda_e m_e^{1/2} / (3 m_i T_e^{3/2})$, [24] the electron collision frequency, $\alpha_o = 0.5129$ and $\beta_o = 0.7110$.[21]

Heat balance for the ions

		$\times \frac{L_\parallel}{P v_{th,i}} \rightarrow$ ordering
$0 = \left[\frac{3}{2} n \vec{u}_i \cdot \vec{\nabla} T_i$	(advection	$M \lambda_{T_i}$, $\delta_{p_i} \lambda_{T_i}$, M
$+ P_i \vec{\nabla} \cdot \vec{u}_i$	and work)	M, δ_{p_i}, M
$+ \vec{\nabla} \cdot \vec{q}_i$	(conduction)	0, δ_{p_i}, $\frac{\delta_i^2}{\Delta}$
$- W_i$	(heating by fast neutrals)	$\frac{E_{io}}{T} \delta_s$ M
$+ \sum_{\alpha\beta} \Pi_{\alpha\beta,i} \frac{\partial u_{\alpha,i}}{\partial x_\beta}$	(viscous heating)	0
$+ Q_i$	(Coulomb interchange)	$(\frac{m_e}{m_i})^{1/2} \frac{\Delta_T}{\Delta}$
$+ \vec{u}_i \cdot \vec{F}_i$	(neutral friction heating)	0
$- \frac{1}{2} m_i u_i^2 S_i$	(ram energy dilution and work)	0
$+ \frac{3}{2} T_i S_i$	(energy dilution)	δ_s M

where

$$\lambda_{T_i} \equiv \nabla_{\parallel} T_i / (T_i / L_{\parallel}) ,$$

$$\Delta_T \equiv 3n(T_e - T_i)/P .$$

As shown in the Appendix, the highest order contributions to $\vec{\nabla}\cdot\vec{q}_i$ involve the classical and diamagnetic fluxes, corresponding to the thermal forces:

$$\vec{q}_i \sim -\kappa_{\perp,i} \vec{\nabla}_{\perp} T_i + \kappa_{\wedge,i} [\frac{\vec{B}}{B} \times \vec{\nabla} T_i] ,$$

discussed by Braginskii, where κ denotes conductivity. The collisional heat exchange between electrons and ions, is

$$Q_i = \frac{3}{2} n \nu_e (T_e - T_i) .$$

The significance of the ordering of the terms in the ion heat balance equation is as follows. As in the continuity equation, the diamagnetic contributions to advection and work are negligible for very large collisionality, when $\delta_{p_i} \ll M$. The radial ("classical") and diamagnetic gradients of the ion heat flux make important contributions to the heat balance. When $\delta_{p_i} \ll M$, only the classical term contributes. We have avoided cases where the neutral energy is large compared to the ion energy, so the W_i term is only important for neutral densities sufficient to effect particle continuity ($\delta_s \sim 1$). Since the collisional energy interchange, Q_i, must be balanced by other terms in this equation, a limit is set on the dimensionless measure of the temperature difference by $\Delta_T \lesssim (m_i/m_e)^{1/2} \Delta M$. Finally, the dilution of ion energy by the creation of neutrals must be considered whenever

neutral sources are important in the particle balance.

Heat balance for the electrons		$\times \frac{L_\parallel}{P v_{th,i}} \rightarrow$ ordering
$0 = (\frac{3}{2} n \vec{u}_e \cdot \vec{\nabla} T_e$	(advection	$M\lambda_T$, $\delta_{p_i} \lambda_T$, M
$+ P_e \vec{\nabla} \cdot \vec{u}_e)$	and work)	M, δ_{p_i}, M
$+ \vec{\nabla} \cdot \vec{q}_e$	(divergence of heat flux)	$[\delta_{p_i}, (\frac{m_i}{m_e})^{1/2} \lambda_T \Delta], 0, \delta_{p_i}$
$- W_e$	(ionization and radiation)	$\frac{E_{eo}}{T} \delta_s$ M
$- Q_i$	(Coulomb interchange)	$(\frac{m_e}{m_i})^{1/2} \frac{\Delta_T}{\Delta}$
$+ \frac{3}{2} T_e S_e$	(energy dilution)	M
$+ \vec{R}_e \cdot \vec{J}/(en)$	(Joule and Thompson heating)	$(\frac{m_e}{m_i})^{1/2} \frac{\delta^2_{p_i}}{\Delta}$, $\delta_{p_i} \lambda_T$

Here E_{eo} is the energy loss per electron impact ionization event. To evaluate this equation, we used $\vec{u}_e = \vec{u}_i - \vec{J}/(en)$, where it can be shown that $u_{\parallel,e} \approx u_{\parallel,i}$, $u_{\psi,e} \approx u_{\psi,i}$, and $u_{d,e} + u_{d,i} \ll u_{d,i}$. (The symbol $\approx$ is defined by $a \approx b \Rightarrow a \sim b$ and $|a-b| \ll |a|$.) The dominant terms in the heat balance equations are as follows: (1) for $\vec{u}_e \cdot \vec{\nabla} T_e$, the parallel and perpendicular heat flows both contribute, where the terms are listed in the order $\parallel$, d, and ψ; (2) similarly, all three components are important for the term $P_e \vec{\nabla} \cdot \vec{u}_e$; (3) the divergence of the electron heat flux is dominated by the effects of parallel currents and thermal conductivity, and by diamagnetic heat flow; and (4) the $\vec{R}_e \cdot \vec{J}$ term is dominated by Joule heating and Thompson heating due to

current flow in the parallel direction.

The ordering of the electron energy balance is similar to that in the ion energy balance except for thermal conductivity and frictional heating. In contrast to ion conduction, a parallel gradient of electron temperature is important, but a radial gradient is insignificant. When $\delta_{p_i} \sim M$, there are also various contributions due to diamagnetic conductivity and the Pfirsch-Schlüter currents. In addition, there are significant contributions from Joule heating due to parallel currents and Thompson heating due to the parallel temperature gradient $\vec{\nabla}_{\parallel} T_e$. These arise from thermoelectric forces along open field lines even in the absence of an electromagnetically induced electric field.

Continuity of current density

$$\vec{\nabla}\cdot\vec{J} = e(S_e - S_i)$$

$$\vec{\nabla}\cdot(\vec{J}_{\parallel} + \vec{J}_d) \sim 0,$$

where the expression $\vec{J}_d \approx \frac{c}{B}\hat{B} \times \vec{\nabla}_{\phi} P$, since $\vec{\nabla}\cdot\vec{J}_{\psi}$ can be shown to be small.

The equations above summarize the terms which should be retained in a numerical solution of the fluid equations (under the restriction $\delta_{p_i} \lesssim M$, discussed below).

III. DISCUSSION

The solutions to the ordering formulation of the fluid equations restrict the parameter space over which the equations are valid. The radial ion balance equation requires that the temperature gradient and mean free path parameters are

$$\lambda_T \lesssim 1 ,$$

$$\Delta \lesssim \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\delta_{p_i}^2}{M} ,$$

since we assume

$$\frac{M}{\delta_{p_i}} \gtrsim 1 .$$

Using these results in the ion heat balance equation yields limits on the ion-electron temperature difference

$$\Delta_T \lesssim \delta_{p_i}^2 \ll 1 ,$$

which implies that

$$T_i \approx T_e .$$

Therefore, the ion and electron temperature gradients are comparable;

$$\lambda_{T_i} \sim \lambda_T .$$

Similarly, the electron heat balance equation limits the size of the source function since, to avoid excess ionization and radiation losses, we require

$$\frac{E_{eo}}{T} \delta_s \lesssim 1 .$$

Finally, the parallel momentum balance equation leads to an estimate of the pressure gradient

$$\lambda_p \sim M^2 \ .$$

The density and temperature regime where these equations are valid is determined by the machine parameters ε, q, and B. The boundary of this regime is given by $M \sim \delta_{p_i}$ and $\Delta \sim (m_e/m_i)^{1/2}\, \delta_{p_i}$ and corresponds to small pressure gradients, $\lambda_p << 1$, and large temperature gradients, $\lambda_T \sim \lambda_{T_i} \sim 1$. We have evaluated the density and temperature space where these equations are valid for typical conditions, such as those expected in Tokamak Fusion Test Reactor TFTR (see caption in Fig. 1). For any given temperature, we determine Δ and, from our upper limit on the dimensionless poloidal gyroradius $\delta_{p_i} \sim M$, the perpendicular scale height, L_ψ. At T = 10 eV and n = 10^{14} cm^{-3}, we find

$$\Delta = 1.73 \times 10^{-3} \ ,$$
$$\delta_{p_i} = 0.105 \ ,$$
$$L_\psi = 1.2 \text{ cm} \ .$$

For the case $\delta_{p_i} \sim \varepsilon$ (consistent with the limitation on $\delta_{p_i} \lesssim M << 1$), the limitation on n and T is shown in Fig. 2 with valid solutions restricted to n and T below the solid line. (Below T $\simeq$ 10 eV, the reaction rate coefficient for electron impact ionization becomes much smaller than the rate coefficient for charge exchange, and the sources due to electron impact ionization become negligible.) Along the upper boundary of the regime of validity in Fig. 2, the perpendicular scale height is given by

$$L_\psi = 1.28 \times 10^{-2} \frac{q}{\varepsilon} \frac{(0.1 \ell n \Lambda_e) L_\parallel (cm)(n/10^{14})}{(B/10^4)\ [T_i(eV)]^{3/2}} ,$$

where B is in gauss, and $\ell n \Lambda_e$ is the Coulomb screening factor for electrons.[24] If there are sources present, then the solutions are further restricted to $T > \delta_s E_{eo}$; this restriction is indicated in Fig. 2 for $\delta_s = 1$, ε and ε^2 where $E_{eo} \sim 40$ eV for temperatures larger than a few eV.

It is instructive to compare our formulation of the fluid equations with previous calculations and models of the axisymmetric scrapeoff region. We will use the 2-d transport model of Petravic et al.[4] as an example. These authors presented a simplified set of equations for axisymmetric plasma flow. Their motivation for using these equations was the need for a numerically tractable system with sufficient physical content to give insight into scrapeoff flows at relatively low collisionality. Not surprisingly, some modifications and additions to this simplified set of equations are necessary to give a complete description of collision dominated flows. To make a comparison, it is necessary to rewrite our equations in the form of Petravic et al. and replace their arbitrary cross-field diffusion term $D(\partial n/\partial y)$ by $(-nu_{\psi,i})$, which can be determined from our radial ion transport equation. Below, we discuss which terms in the equations of Petravic et al. have to be modified and which terms must be added.

(1) In the continuity equation, the diamagnetic flow, $\vec{\nabla} \cdot \vec{u}_{d,i}$ should be included in their equations.

(2) In the parallel momentum balance, the diamagnetic and radial divergence of the viscosity terms, $\Pi_{\parallel,d}$ and $\Pi_{\parallel,\psi}$, respectively, must be included. Furthermore, Coriolis force terms proportional to $(\vec{u}_i \times \vec{\nabla} \times \vec{u}_i)$ and arising from the diamagnetic flows are missing from their equations.

(3) An ion energy transport equation (i.e., a Bernoulli's equation including mechanical energy) can be derived from our conservation equations, and a comparison with that of Petravic et al. shows significant differences. Our full equation is as follows.

Ion energy transport		$\times \frac{L_I}{Pv_{th,i}} \rightarrow$ ordering
$0 = -\vec{\nabla}\cdot\left(\frac{5}{2} T_i n\vec{u}_i\right)$	(thermal energy)	M, δ_{p_i}, M
$- \vec{\nabla}\cdot\left(\frac{1}{2} m_i u_i^2 n\vec{u}_i\right)$	(ram energy)	0
$- \vec{\nabla}\cdot\vec{q}_i$	(conduction)	0, δ_{p_i}, δ_i^2/Δ
$+ W_i$	(heating by fast neutrals)	$\frac{E_{eo}}{T}$ M δ_s
$- \sum_{\alpha\beta} \frac{\partial}{\partial x_\beta} \left(\Pi_{\alpha\beta,i} u_{\alpha,i}\right)$	(viscous heating)	0
$+ Q_i$	(Coulomb interchange)	$\left(\frac{m_e}{m_i}\right)^{1/2} \frac{\Delta_T}{\Delta}$
$- \vec{u}_i\cdot\vec{\nabla}P_e$	(work, and	M, δ_{p_i}, M
$+ \frac{\vec{J}}{en}\cdot\vec{\nabla}P_e$	dissipation	0
$+ \vec{J}\cdot\vec{E}$	due to	$\delta_{p_i} \frac{e\phi}{T_e}$
$- \frac{\vec{J}\cdot\vec{R}_e}{en}$	Coulomb friction)	$\left(\frac{m_e}{m_i}\right)^{1/2} \frac{\delta_{p_i}^2}{\Delta}$, $\delta_{p_i} \lambda_T$

where we have set $\vec{E} = -\vec{\nabla}\phi$ and again noted the order of each potentially significant term. The corresponding equation of Petravic et al. can be written as

$$\vec{\nabla}_o \cdot \left[n\vec{u}\left(\frac{5}{2} T_i + \frac{1}{2} m_i u_\parallel^2\right)\right] = S_{E_i} - u_{\parallel,i} T_e \frac{\partial n}{\partial x_\parallel} \quad ,$$

where S_{E_i} is an energy source rate, the term $D(\partial n/\partial y) + -nu_{\phi,i}$, Petravic et al.'s coordinates ξ and y are replaced by $\parallel$ and ϕ, respectively, and we define $\nabla_o = \hat{u}_\parallel(\partial/\partial x_\parallel) + \hat{u}_\phi(\partial/\partial x_\phi)$. A number of significant differences are apparent. First, their equation applies only if

$$\lambda_T << 1 \ ,$$

$$\delta_{p_i} << M \ ,$$

and

$$\frac{e\phi}{T_e} \lesssim 1 \ ,$$

where the electrostatic potential ϕ is chosen to be zero at some reference point within the region of interest. Second, even within these limits, this equation omits the electron-ion exchange term and the radial component of $\vec{u}_i \cdot \vec{\nabla} P_e$. Unfortunately, the assumptions required to obtain the formulation of Petravic et al. are not generally valid since diamagnetic terms may be important, and λ_T may be ~ 1. In such cases, ion conductivity perpendicular to the magnetic field has to be included as well.

A close inspection of the ion energy transport equation suggests that the solution for the plasma properties should be based on the ion heat balance; the ion energy transport equation is useful and necessary only if one wants to determine the electrostatic potential, ϕ. The ordering equations yield an

estimate for ϕ of

$$\frac{e\phi}{T_e} \sim \frac{M}{\delta_{p_i}} ,$$

which is $\gtrsim 1$ for the conditions discussed here. Accordingly, the expression $\left[-\vec{u}_i \cdot \vec{\nabla} P_e + \vec{J} \cdot (\vec{\nabla} P_e - en\vec{E} - \vec{R}_e)\right]$ cannot be replaced by $-u_{\parallel,i} T_e \partial n/\partial x_\parallel$, as in the formulation of Petravic et. al.

(4) The comparison of electron equations is essentially identical to the comparison of ion equations except for the thermal conductivity, $\vec{\nabla} \cdot \vec{q}_e$, where the diamagnetic term must again be included.

Finally, we note that, in general, the classical (radial) collisional contributions ($\sim \delta_i^2/\Delta$) can be neglected in these transport equations only for very small aspect ratios where $\varepsilon >> (m_e/m_i)^{1/8}$, assuming $q \sim \varepsilon^{-1}$. (We have not investigated the consequences of orderings for the safety factor other than $q \sim \varepsilon^{-1}$, but this should require only another application of the methodology outlined above.)

IV. SUMMARY

We have determined the relative importance of the contributions to the fluid transport equations for an axisymmetric scrapeoff region including sources where diamagnetic flows contribute to, but do not dominate, the continuity equation. We discovered that a number of important physical processes have previously been neglected. The physics which leads to these results is as follows. The parameter regime of interest is dominated by end losses in the open field line geometry. These losses are balanced by radial transport only when the perpendicular scale height, L_ϕ, becomes so small that sufficient radial transport can be driven by the dominant neoclassical

frictional forces (see Section III). At large collisionality, classical ion conduction and viscosity produced by large variations across the radial direction can dominate plasma transport (hatched region in Fig. 2). Unless the collisionality is extremely high, well within the hatched region in Fig. 2, the diamagnetic flows, driven by relatively large radial pressure gradients, can be as important as the flow along open field lines. In some cases, where the fluid theory is still valid, diamagnetic flows can even dominate the particle and energy balances ("diamagnetic flow" region in Fig. 2). All of these physical processes must be considered to obtain a correct picture of plasma transport in scrapeoff plasmas where they make significant contributions to the transport of particles, energy, and momentum.

The parameter space of density and temperature where these estimates are valid is a function of the collisionality through the restriction $\Delta \lesssim (m_e/m_i)^{1/2}\ \delta_{p_i}$. Since Δ is inversely proportional to the collision rate, any turbulent process not included in the transport coefficients as derived by Braginskii,[21] which increases the effective collision frequency, should decrease the effective value of Δ. With such a modification, our reduction of the fluid equations might be valid for lower densities and higher temperatures than indicated in Fig. 2. In the future, it will be important to extend our analysis to regions of higher temperature and lower density where $\delta_{p_i} > M$. It would also be desirable to look at larger parallel pressure gradients where the parallel flow may approach the sound speed; i.e., $M \sim 1$.

ACKNOWLEDGMENTS

We would like to thank D. Larrabee, M. Petravic, for useful comments.

This work was supported by the U. S. Department DE-AC02-76-CHO-3073.

APPENDIX

Here we demonstrate the method of deriving the ordering equations by evaluating, as an example, the conductivity term in the equation for heat balance for ions,

$$\vec{\nabla}\cdot\vec{q}_i \quad . \tag{A1}$$

The heat flux has the following three components defined by Braginskii[21]

$$\vec{q}_i = -\kappa_{\parallel,i}\vec{\nabla}_\parallel T_i - \kappa_{\perp,i}\vec{\nabla}_\perp T_i + \kappa_{\wedge,i}\left[\frac{\vec{B}}{B}\times\vec{\nabla}\,T_i\right] , \tag{A2}$$

where $\kappa_{\parallel,i} = 3.9\,\kappa_o$, $\kappa_{\perp,i} = 2\,(\nu_i/\Omega_i)^2\kappa_o$, and $\kappa_{\wedge,i} = 2.5\,(\nu_i/\Omega_i)\,\kappa_o$, and $\kappa_o = nT_i/(m_i\nu_i)$. To begin the ordering procedure, each of the terms of Eq. (A1) is evaluated by substituting explicitly for the conductivity and multiplying by $L_\parallel/(Pv_{th,i})$, which provides a convenient scale for the ion heat balance equation. The divergences and gradients in Eqs. (A1) and (A2) are replaced by the inverse of the characteristic lengths over which the variables change, as follows:

(1) Parallel term

$$\left(\frac{L_\parallel}{Pv_{th,i}}\right)\frac{\kappa_{\parallel,i}}{L_\parallel}\nabla_\parallel T_i \sim \frac{1}{Pv_{th,i}}\frac{nT_i}{m_i\nu_i}\left(T_i/L_\parallel\right)\lambda_{T_i} , \tag{A3}$$

where $\lambda_{T_i} \equiv \nabla_\parallel T_i/(T_i/L_\parallel)$. We now replace P by $\sim nT_i$, and T_i by $\sim m_i v_{th,i}^2$. Using the definition of the inverse collisionallity, Δ, in Eq. (A3), then yields

$$\left(\frac{L_\parallel}{Pv_{th,i}}\right)\vec{\nabla}\cdot\left(\kappa_{\parallel,i}\vec{\nabla}_\parallel T_i\right) \sim \frac{v_{th,i}}{L_\parallel\nu_i}\lambda_{T_i} \sim \Delta\lambda_{T_i} \quad . \tag{A4}$$

Similarly, the remaining contributions can be written as a "perpendicular term" and a "diamagnetic term."

(2) Perpendicular term

$$\frac{L_\parallel}{Pv_{th,i}} \kappa_{\perp,i} \nabla_\perp T_i \sim \frac{L_\parallel}{Pv_{th,i}} \frac{nT_i}{\nu_i m_i} \left(\frac{\nu_i}{\Omega_i}\right)^2 \frac{T_i}{L_\phi^2}$$

$$\sim \frac{v_{th,i}}{L_\parallel \nu_i} \left(\frac{\nu_i}{\Omega_i}\right)^2 \left(\frac{L_\parallel}{L_\phi}\right)^2 \sim \frac{\delta_i^2}{\Delta} \quad , \tag{A5}$$

(3) Diamagnetic term

The diamagnetic contribution to the divergence operator is replaced by L_d^{-1}. Contributions to the action of this operator on the $\kappa_{\wedge,i}$ term in $\vec{q}_i$ can arise from gradients in both the parallel and radial direction; however, only the latter is important because $L_\phi \ll L_\parallel$ The diamagnetic contribution becomes

$$\frac{L_\parallel}{Pv_{th,i}} \left(\kappa_{\wedge,i} \frac{T_i}{L_\phi L_d}\right) \sim \frac{L_\parallel}{Pv_{th,i}} \frac{nT_i}{m_i \nu_i} \frac{\nu_i}{\Omega_i} \frac{T_i}{L_\phi L_d} \quad ,$$

which can be rewritten after replacing P and T_i as

$$\sim \frac{L_\parallel^2}{L_\phi L_d} \frac{\nu_i}{\Omega_i} \frac{v_{th,i}}{L_\parallel \nu_i} \sim \frac{L_\parallel}{L_d} \frac{v_{th,i}}{\Omega_i L_\phi} \sim \delta_{p_i} \quad . \tag{A6}$$

Therefore, the ordering terms for the ion conductivity term in the heat balance equation are

$$\left(\frac{L_\parallel}{Pv_{th,i}}\right) \vec{\nabla} \cdot \vec{q}_i + [\Delta\lambda_{T_i}, \delta_{p_i}, \delta_i^2/\Delta]. \tag{A7}$$

We will now show that the parallel contribution can be neglected in the ion heat transport equation. This demonstration is made by evaluating Δ in terms of δ_{p_i}, a relationship which can be found from the ordering of the radial ion transport equation. First, we project out the toroidal component of the total momentum balance equation,

$$m_a n_a (\vec{u}\cdot\vec{\nabla})\vec{u}_a = n_a u_a [\nabla(u_a^2/2) - \vec{u}_a \times (\vec{\nabla} \times \vec{u}_a)]$$

$$= -\vec{\nabla}p_a - \Sigma \frac{\partial \Pi_{\alpha\beta,a}}{\partial x_\beta} + e_a n_a \left(\vec{E} + \frac{1}{c}\vec{u}_a \times \vec{B}\right) + \vec{R}_a + \vec{F}_a - m_a \vec{u}_a S_a, \quad \text{(A8)}$$

using[19]

$$- \frac{c}{e_a} \frac{g^{1/2}}{q} \vec{B}_T . \quad \text{(A9)}$$

Applying (A9) to $(1/c)\vec{u}_a \times \vec{B}$, yields $n_a \vec{u}_a \cdot \vec{\nabla}\psi$, and the resulting equation can be written as

$$n_a \vec{u}_a \cdot \vec{\nabla}\psi = - \frac{c}{e_a} \frac{g^{1/2}}{q} B_T \frac{\vec{B}_T \cdot \vec{A}_a}{B_T} , \quad \text{(A10)}$$

where

$$\vec{A}_a \equiv \vec{R}_a + \vec{F}_a - m_a \vec{u}_a S_a + m_a n_a \vec{u}_a \times (\vec{\nabla} \times \vec{u}_a) - \Sigma_\beta \frac{\partial \Pi_{\alpha\beta,a}}{\partial x_\beta} . \quad \text{(A11)}$$

For a toroidally symmetric system, the terms involving $\vec{\nabla}p$, $\vec{E}$, and $\vec{\nabla}(u^2/2)$ are identically zero in (A10) and have been explicitly omitted from $\vec{A}_a$ in Eq. (A11). The ion transport form of Eq. (A10) can be cast in a more convenient form by dividing both sides by $nv_{th,i}|\vec{\nabla}\psi|$ and, according to the definition

of $\vec{B}$ in Section 2, using $|\nabla\psi| = RB_p$ and $g^{1/2} = qR/B_T$, [19]

$$\frac{u_{\phi,i}}{v_{th,i}} = -\frac{c}{e}\frac{1}{v_{th,i}B_T}\frac{B_T}{B_p}\left[\frac{\vec{B}_T\cdot\vec{A}_i}{B_T}\right] ,$$

$$= -\frac{1}{\Omega_i}\frac{1}{m_i v_{th,i}}\frac{q}{\epsilon}\left[\frac{\vec{B}_T\cdot\vec{A}_i}{B_T}\right] ,$$

$$= -\frac{\delta_{p_i} L_\phi}{m_i v_{th,i}^2}\frac{\vec{B}_T\cdot\vec{A}_i}{B_T} . \qquad \text{(A12)}$$

To write an ordering equation for (A12), we replace $m_i v_{th,i}^2$ by $\sim P$ and use the ordering for the continuity equation, $u_{\phi,i}/L_\phi \sim u_{\parallel,i}/L_\parallel$, in Eq. (A12) which then yields

$$\frac{M}{\delta_{p_i}} \sim -\frac{L_\parallel}{P}\frac{\vec{B}_T\cdot\vec{A}_i}{B_T} . \qquad \text{(A13)}$$

It can be shown that all the terms in $\vec{B}_T\cdot\vec{A}_i$ on the right-hand side are small compared to $\vec{B}_T\cdot\vec{R}_i$, and only the parallel component of $\vec{R}_i$ is important. This parallel friction term is taken from page 249 of Braginskii[21] and can be written

$$\frac{L_\parallel}{P} R_{\parallel,i} = \frac{L_\parallel}{P}\left(-\beta_o n\nabla_\parallel T_e - \alpha_o m_e n\nu_e \frac{J_\parallel}{en}\right)$$

$$= -\beta_o\lambda_T - \alpha_o \frac{L_\parallel}{m_i v_{th,i}^2} m_e\left(\frac{m_i}{m_e}\right)^{1/2}\nu_i \frac{J_\parallel}{en}$$

$$= -\beta_o\lambda_T - \alpha_o\left(\frac{m_e}{m_i}\right)^{1/2}\frac{1}{\Delta}\frac{J_\parallel}{env_{th,i}} , \qquad \text{(A14)}$$

where $\alpha_o = 0.5129$ and $\beta_o = 0.711$. To evaluate this further, we need to estimate $J_\parallel$. From the equation for current continuity, assuming as in the main text that the $S_i - S_e$ term is small, we have the ordering equation

$$\frac{J_\parallel}{L_\parallel} \sim \frac{J_d}{L_d} . \tag{A15}$$

assuming as can be shown a posteriori from the difference between the ion and electron radial flux equations, that

$$\frac{J_\psi}{L_\psi} \ll \frac{J_\parallel}{L_\parallel} . \tag{A16}$$

The ordering equation for Eq. (A13) then becomes

$$\frac{M}{\delta_{p_i}} \sim \text{Max}\left\{\lambda_r, \left(\frac{m_e}{m_i}\right)^{1/2} \frac{(q/\varepsilon)}{\Delta} \frac{J_d}{env_{th,i}}\right\} . \tag{A17}$$

The diamagnetic current can be estimated by using the radial ion momentum balance to find $u_{d,i}$. We will not derive this result here but, as noted in the text, a self-consistent analysis of all the terms in this equation (for the assumptions adopted in this paper) yields the usual result,[19]

$$J_d \sim \frac{c}{B} \left|\hat{B} \times \vec{\nabla}_\psi P\right| \sim \frac{c}{B} \frac{P}{L_\psi}$$

$$\sim \frac{cnm_i v_{th,i}^2}{BL_\psi} \sim env_{th,i}\delta_i . \tag{A18}$$

Substituting Eq. (A18) into (A17), we have the ordering for Δ in terms of other parameters given in the text,

$$\frac{M}{\delta_{p_i}} \sim \text{Max}\{\lambda_T, \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\delta_{p_i}}{\Delta}\}.$$

It can be shown, again a posteriori, that i
consistent with continuity and the total energy b

$$\Delta \sim \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\delta_{p_i}^2}{M} \lesssim \left(\frac{m_e}{m_i}\right)^{1/2} \delta_{p_i} ,$$

and $\lambda_{T_i} \sim \lambda_T \lesssim 1$ (by which we mean either $\lambda_T \ll$
into Eq. (A7) gives the ordering solution preser
heat flow,

$$\frac{L_\parallel}{pv_{th,i}} \vec{\nabla}\cdot\vec{q}_i \sim [0, \delta_{p_i}, \delta_i^2/\Delta] ,$$

since the parallel term $\Delta\lambda_{T_i} \lesssim \left(\frac{m_e}{m_i}\right)^{1/2} \delta_{p_i} \lambda_{T_i} \ll$

REFERENCES

[1]A. H. Boozer, Phys. Fluids 19, 1210 (1976).

[2]S. P. Auerbach and A. H . Boozer, Phys. Fluids 23. 1297 (1980).

[3]M. Emery, J. Garner, M. Fritts, J. Boris, and N. Winsor, Naval Research Laboratory Memorandum Report, NRL-3744 (1978).

[4]M. Petravic, D. Post, D. Heifetz, and J. Schmidt, Phys. Rev. Lett. 48, 326 (1982).

[5]D. Heifetz, D. Post, M. Petravic, J. Weisheit, and G. Bateman, Princeton Plasma Physics Laboratory Report PPPL-1843 (1981).

[6]J. G. Morgan and P. J. Harbour, in Proceedings of the Eleventh Symposium on Fusion Technology (Oxford, England, 1980) Vo. 2, p. 1187.

[7]M. F. A. Harrison, P. J. Harbour, and E. S. Hotson, Modelling of Plasma Exhaust and Gas Pumping Properties of the Single-Null Divertor of INTOR, IAEA Divertor Workshop (Garching, July 1981).

[8]J. M. Ogden, C. E. Singer, D. E. Post, R. V. Jensen, and F. G. P. Seidl, IEEE Trans. Plasma Sci. PS-9, 274 (1981).

[9]A. V. Nedospasov and M. Z. Tokar, Nucl. Fus. 21, 465 (1981).

[10]A. T. Mense and G. A. Emmert, Nucl. Fus. 19, 361 (1979).

[11]D. Anderson, M. Lisak, and H. Wilhemsson, Nucl. Fus. 19, 1523 (1979).

[12]U. Daybelge, Nucl. Fus. 21, 1589 (1981).

[13]H. A. Claassen and H. Repp, Nucl. Fus. 21, 589 (1981).

[14]F. L. Hinton and R. D. Hazeltine, Phys. Fluids 17, 2236 (1974).

[15]N. Ohyabu, Nuclear Fusion 21, 519 (1981).

[16]D. Post, Fifth International Conference on Plasma Surface Interactions, Gatlinburg, 1982.

[17]B. A. Carreras, J. D. Callen, P. W. Gaffney, and H. R. Hicks, Oak Ridge National Laboratory Report, ORNL-TM-7989, Oak Ridge, TN (1982).

[18]J. D. Callen, G. A. Emmert, A. M. Bailey, M. E. Benchikhtechosin, J. N. Davidson, R. Hulse, A. Boozer, L. Foote, R. Hawryluk, D. Heifetz, W. Hsu, D. Mikkelsen, J. Ogden, K. Owens, M. Petravic, D. Post, P. Rutherford, J. Schmidt, F. Seidl, C. Singer, S. Suckewer, J. Weishiet, and M. Yamada, in Eighth International Conference on Plasma Physics and Controlled Nuclear Fusion Research (IAEA, Brusseles, 1980) IAEA-CN-38.

[19]S. P. Hirschman and D. J. Sigmar, Nucl. Fus. 21, 1079 (1981).

[20]F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. 48, 239 (1976).

[21]S. I. Braginskii, in Review of Plasma Physics, edited by M. A. Leontovich (Consultant Bureau, New York, 1965) Vol. 1, p. 205.

[22]D. Heifetz, D. Post, M. Petravic, J. Weisheit, and G. Bateman, A Monte Carlo Model of Neutral Particle Transport in Diverted Plasmas, Princeton Plasma Physics Laboratory Report, PPPL-1843 (to appear J. Comp. Phys.).

[23]E. L. Freeman and E. M. Jones, Culham Laboratory Report, CLM-R137 (1974).

[24]D. L. Book, NRL Plasma Formulary, Office of Naval Resesarch (Washington, DC, 1978).

FIGURE CAPTIONS

Fig. 1. Field lines and coordinate systems used to describe the axisymmetric scrapeoff region. The solid line indicates the regime in which the equations derived in the text are valid. While the dashed line indicates the region near the limiter where the parallel scale height is too small for the validity of our assumptions. The geometry shown corresponds to TFTR parameters with R = 254 cm, a = 91 cm, $R_{limiter}$ = 163 cm, and $B_T = 4 \times 10^4$ Gauss. For a deuterium plasma ($A_I = 2$) the value of $\epsilon = (m_e/m_i)^{1/8} = 0.358$ and with $q = \epsilon^{-1}$ the coordinate lengths are: $L_\parallel = 1390$ cm and $L_d = 178$ cm. Only a portion of the axisymmetric limiter is shown.

Fig. 2. The density and temperature range over which our solutions are valid are shown here for the parameters listed in the caption for Figure 1. The boundary corresponds to $\delta_{p_i} = M$ and thus $\Delta = (m_e/m_i)^{1/2}\, \delta_{p_i}$; within the shaded region $\delta_{p_i} \lesssim M$. The dashed lines indicate the lower bounds on the temperature when sources are present corresponding to different values of the source function scale factor, δ_s.

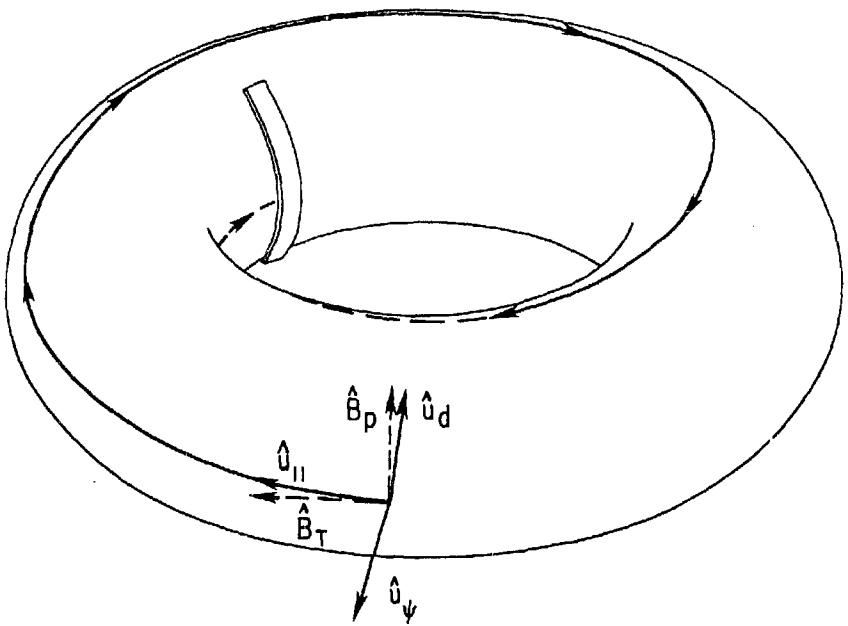

Figure 1

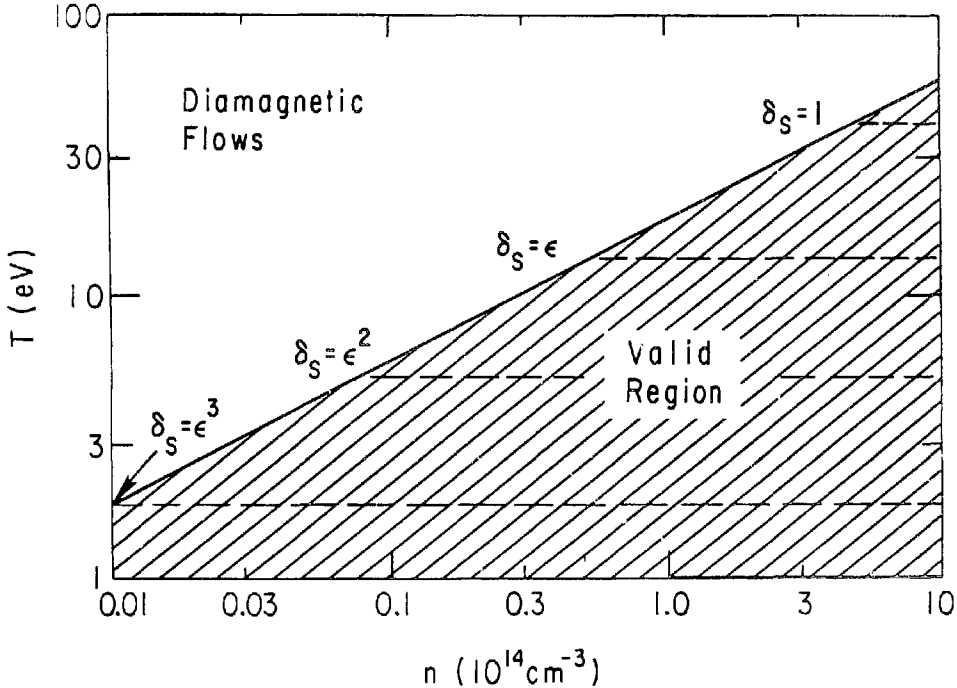
100
30
10
3
1
T (eV)
Diamagnetic
Flows
$\delta_S = 1$
$\delta_S = \epsilon$
$\delta_S = \epsilon^2$
$\delta_S = \epsilon^3$
Valid
Region
0.01
0.03
0.1
0.3
1.0
3
10
n (10^{14}cm^{-3})

Figure 2