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ON THE LAPLACE TRANSFORM OF THE WEINBERG TYPE SUM RULES *

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ABSTRACT

We consider the Laplace transform of various sum rules of the Weinberg type including the leading non-perturbative effects. We show from the third type Weinberg sum rules that $7.5 \sim 8.9 \lesssim \hat{m}_s/\hat{m}_d \lesssim 22.5 \sim 26.6$ for the sum rule scale value $M \simeq M_\rho \sim M_\phi$. We also deduce an upper bound to the by-product $\hat{m}_u\hat{m}_d$ of light quark invariant masses. The first Weinberg sum rule allows us to get a lower bound on f_π and on the A_1 coupling to the W boson, while the second sum rule gives an upper bound on the A_1 mass. $(M_{A_1} \lesssim 1.25$ GeV).

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There has been recent progress in extending the applicability domain of Quantum Chromodynamics (QCD) to obtain predictions on low-energy parameters (hadron masses and coupling constants). The approach is based on sum rules obeyed by the spectral functions of a specific two-point function of current operators, as a consequence of general analyticity properties. There exist a variety of QCD sum rules in the literature [1-3] depending on how these analyticity and positivity properties are exploited. Of particular interest for low-energy phenomenology are the sum rules of the Laplace transform type

$$\mathcal{F}(M^2) = \frac{1}{\pi}\int_0^\infty dt\, e^{-t/M^2}\, \mathrm{Im}\,\Pi(t) \qquad (1)$$

proposed by SVZ [1a] for the light quark system and recently revised by NR [1b]. Here $\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)$ denotes a specific spectral function (e.g. the hadronic vacuum polarization measured in the $e^+e^- \to$ hadrons); $\mathcal{F}(M^2)$ is a quantity, which in principle can be computed asymptotically in QCD provided that M^2 is larger than the QCD scale Λ which we shall take to be $\Lambda \simeq 70 \sim 210$ MeV in the dimensional renormalization $\overline{\mathrm{MS}}$ scheme [1)] as a result issued from the QCD sum rule analysis of the isovector part of the $e^+e^- \to$ hadrons total cross-section [5]. Clearly, the sum rule in Eq.(1) is more selective on the low-energy behaviour of the spectral function for small enough M than the right-hand side of the usual dispersion relation

$$\Pi(Q^2 = -q^2 > 0) = \frac{1}{\pi}\int_0^\infty \frac{dt}{t+Q^2}\,\mathrm{Im}\,\Pi(t) + \text{"subtraction"} \; . \qquad (2)$$

In this note we shall be concerned with the two-point correlation functions of the vector (axial vector) current $J^\mu_{(5)}(x) \equiv \bar\psi_i\gamma^\mu(\gamma_5)\psi_j$ (ψ_i being the quark field of flavour i) [1)]:

$$\Pi^{\mu\nu}_{(5)} = i\int d^4x\, e^{iqx}\langle 0|\mathrm{T}\,J^\mu_{(5)}(x)\left(J^\nu_{(5)}(0)\right)^\dagger|0\rangle \equiv -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\Pi^{(1)ij}_{(5)}(q^2) + g^{\mu\nu}\Delta^{ij}_{(5)}(q^2)\, , \qquad (3)$$

and to its divergence $\partial_\mu J^\mu_{(5)} = (m_i \mp m_j)\,\bar\psi_i(\gamma_5)\psi_j$:

$$\psi^{ij}_{(5)}(q^2) = i\int d^4x\, e^{iqx}\langle 0|\mathrm{T}\,\partial_\mu J^\mu_{(5)}(x)\left(\partial_\nu J^\nu_{(5)}(0)\right)^\dagger|0\rangle \, . \qquad (4)$$

$D^{ij}_{(5)}$ and $\psi^{ij}_{(5)}$ are related by the current algebra Ward identity

$$q^2 D^{ij}_{(5)} = \psi^{ij}_{(5)}(q^2) - \psi^{ij}_{(5)}(0) , \tag{5}$$

where

$$\psi^{ij}_{(5)}(0) = -(m_i \mp m_j)\Big\{\langle 0|\bar\psi_i\psi_i \mp \bar\psi_j\psi_j|0\rangle - m_i^3 \log\frac{m_i^2}{\nu^2} \pm m_j^3 \log\frac{m_j^2}{\nu^2}\Big\} + O(m_i^4, \bar\alpha_s). \tag{6}$$

In the Nambu-Goldstone realization of chiral symmetry, the quantity $\langle 0|\bar\psi_i\psi_i|0\rangle$ is not zero and its role becomes crucial in the sum rule involving $D^{ij}_{(5)}$ and $\psi^{ij}_{(5)}$. We shall be mainly interested in the re-analysis of the Weinberg sum rule [6] [1])

$$\int_0^\infty dt\, \frac{1}{\pi}\Big\{ Im\, \Delta^{(1)}_{ij} + Im\, \Delta^{(0)}_{ij}\Big\} = 0 \qquad \text{first sum rule ,} \tag{7a}$$

$$\int_0^\infty dt\, \frac{1}{\pi}\, Im\, t\,\Delta^{(1)}_{ij}(t) = 0 \qquad \text{second sum rule ,} \tag{7b}$$

$$\int_0^\infty dt\, \frac{1}{\pi}\, Im\, \Delta^{ij}(t) = 0 \qquad \text{third sum rule ,} \tag{7c}$$

where $\Delta^{(1)}_{ij} \equiv \Pi^{(1)ij} - \Pi^{(1)ij}_5$, $\Delta^{(0)}_{ij} \equiv \frac{1}{q^2}\{D^{ij} - D^{ij}_5\}$ and $\Delta^{ij} \equiv q^2 \Delta^{(0)}_{ij}$. FNR [2] showed that the second and third sum rules are quadratically divergent to leading order of chiral symmetry breaking terms and so they introduced the convergent sum rule

$$\int_0^\infty dt\, \frac{1}{\pi}\, Im\Big\{\Delta^{ij}(t) - \frac{m_j}{m_k}\Delta^{ik}(t) \equiv \Delta^{ijk}(t)\Big\} = 0. \tag{8}$$

The Laplace transforms of Eqs.(7) and (8) are obtained by applying to both sides of the dispersion relation of the type in Eq.(2) the operator

$$\hat{L} \equiv \lim_{\substack{q^2\to\infty \\ n\to\infty}} \; \frac{(-1)^n}{(n-1)!}\Big(\frac{\partial}{\partial q^2}\Big)^n . \qquad \frac{q^2}{n} \equiv M^2 \tag{9}$$

We shall also study the phenomenological implications of these Laplace sum rules for the light quark systems and other versions of the third Weinberg sum rules.

1. The third Weinberg sum rule

To leading order of QCD and of the non-perturbative effects, the Laplace transform of the third Weinberg sum rule reads:

$$\frac{1}{M^2}\int_0^\infty dt\, e^{-t/M^2}\frac{1}{\pi}\, Im\, \Delta^{ij}(t) = -\frac{3}{8\pi^2}\frac{\hat m_i \hat m_j}{(\log M/\Lambda)^{2\gamma_1/-\beta_1}} + \frac{2}{M^2}\langle 0|m_i\bar\psi_j\psi_j + m_j\bar\psi_i\psi_i|0\rangle , \tag{10}$$

where $\gamma_1 = 2$, $-\beta_1 = \frac{1}{2}(11 - \frac{2}{3}n)$ for $SU(3)_c \times SU(n)_F$, $\hat m_i$ is the invariant mass of the quark i. For $M^2 \to \infty$, this sum rule recovers the original Weinberg result. However, we want to saturate the spectral function by the low resonance states and so we have to choose M^2 small enough. In the $\bar u d$ channel, the sum rule is saturated by the π pole and we shall use the positivity of the scalar contribution. Using the PCAC result

$$(m_u + m_d)\langle 0|\bar\psi_u\psi_u + \bar\psi_d\psi_d|0\rangle \simeq -2 f_\pi^2 m_\pi^2 + O(m_\pi^4) \tag{11}$$

($f_\pi \simeq 93.28$ MeV is the pion decay amplitude)
in order to estimate the quark condensate contribution to the sum rule ($SU(2)_F$ symmetry limit), we get from the sum rule, the useless inequality:

$$-\frac{3}{2\pi^2}\frac{\hat m_u \hat m_d}{(\log M/\Lambda)^{2\gamma_1/-\beta_1}} + O\Big(\frac{m_\pi^2}{M^2}\Big)^2 \geq 0 \tag{12}$$

due to the fact that the contribution of $\Delta^{ij}(0)$ cancels the pole contribution to the sum rule for $M^2 \gg m_\pi^2$. In fact, the usefulness of the sum rule could be saved by going to $M^2 \lesssim m_\pi^2$, but our present QCD theory does not permit this choice.

2. The FNR sum rule and a lower bound for $\hat m_s/\hat m_d$

Using the Broadhurst result [7] for $\Delta^{ij}(q^2)$ and $\Delta^{ij}(0)$, and the Ward identity in Eq.(5), we get the Laplace transform of the FNR sum rule to two loops and including the leading non-perturbative effects [2]

$$\frac{1}{M^4}\int_0^\infty dt\, e^{-t/M^2}\frac{1}{\pi}\,\mathrm{Im}\,\Delta^{jk}(t) = \frac{3}{2\pi^2}\frac{\bar m_i \bar m_j}{M^2}\left\{\frac{\bar m_j^2}{M^2}\log\frac{M^2}{\bar m_j^2}\left(1+2\frac{\bar\alpha_s}{\pi}\log\frac{M^2}{\bar m_j^2}\right) - (j\leftrightarrow k)\right\}$$
$$(-)\frac{2}{(M^2)^2}\; m_i\langle 0|\bar\psi_j\psi_j - \frac{m_j}{m_k}\bar\psi_k\psi_k|0\rangle . \qquad (13)$$

In the uds channel, we expect $\langle 0|\bar\psi_d\psi_d|0\rangle \simeq \langle 0|\bar\psi_s\psi_s|0\rangle < 0$ ($SU(3)_F$ symmetry), $m_d \ll m_s$. Saturating the spectral function by the π and K mesons and using the PCAC relation in Eq.(13), we get for moderate M^2 ($M^2 \simeq M_\rho^2 \sim M_\phi^2$):

$$-2f_\pi^2 m_\pi^2 + \frac{m_d}{m_s}2f_K^2M_K^2 e^{-M_K^2/M^2} \simeq -\frac{3}{2\pi^2}\bar m_u \bar m_d \bar m_s^2\log\frac{M^2}{\bar m_s^2}\cdot\left(1+2\frac{\bar\alpha_s}{\pi}\log\frac{M^2}{\bar m_s^2}\right) + f_\pi^2 m_\pi^2 \qquad (14)$$

We transform this equality into an inequality using the negativity of the perturbative QCD contribution to the sum rule to the extent that $m_d \ll m_s$. So we get the improved version of the FNR result [2] :

$$\frac{\hat m_s}{\hat m_d} \geq \frac{2f_K^2M_K^2}{3f_\pi^2 m_\pi^2}\, e^{-M_K^2/M^2}\left\{1+O\left(\frac{m_\pi^2}{M^2}\right)\right\} \simeq 7.5\sim 13.3 \qquad (15)$$

for $M^2 \simeq M_\rho^2 \sim M_\phi^2$.

3. The Broadhurst type sum rule and an upper bound for $\hat m_s/\hat m_d$

As was observed in Ref.7, a combination of flavoured two-point functions which is independent of its value at $q = 0$ is the $SU(2)_F$ symmetry limit of

$$q^2 D^{uds} \equiv q^2\left\{D^{us} - \left(\frac{m_s-m_u}{m_s+m_u}\right)D_5^{us} + \frac{m_s-m_u}{2m_u}D_5^{ud}\right\}. \qquad (16)$$

Taking the Laplace transform of $\mathrm{Im}\, D^{uds}(t)$ and in the limit ($m_u = m_d \ll m_s$), we get the sum rule [3]:

$$\int_0^\infty dt\, e^{-t/M^2}\frac{1}{\pi}\,\mathrm{Im}\, D^{uds}(t) = -\frac{3}{2\pi^2}\bar m_d \bar m_s^3\left\{1+\frac{4}{3}\frac{\bar\alpha_s}{\pi} + O\left(\frac{m_u}{m_s}, \frac{1}{M^2}, \bar\alpha_s^2\right)\right\} \qquad (17)$$

where $\bar m_i(M^2)$ is the running quark mass.

For moderate M^2, we approximate $\mathrm{Im}\, D_5^{us}$ by the K pole and $\mathrm{Im}\, D_5^{ud}$ by the π pole. We use the positivity of $\mathrm{Im}\, D^{us}$ together with the negativity of the right-hand side of the sum rule. Then, we get the inequality

$$-2f_K^2M_K^2 e^{-M_K^2/M^2} + \frac{\hat m_s}{\hat m_d} f_\pi^2 m_\pi^2 \lesssim 0 , \qquad (18)$$

which for $M^2 \simeq M_\rho^2 \sim M_\phi^2$ gives

$$\frac{\hat m_s}{\hat m_d} \lesssim 22.5 \sim 26.6 \qquad (19)$$

in the limit $m_u = m_d$.

4. Upper value of $\hat m_u \hat m_d$ from a third Weinberg type sum rule independent of $\psi_{(5)}^{ij}(0)$

We can also work with the Laplace transform of the quantity $q^2\Delta^{ij}(q^2)$ which becomes independent of $\psi_5^{ij}(0) - \psi^{ij}(0)$ after the use of the Laplace transform operator. Using the BNRY result [3] for $\psi_5^{ij}(q^2)$ and the result of $\psi^{ij}(q^2)$ from Ref.8, we get the sum rule in the $\bar u d$ channel

$$\frac{1}{M^4}\int_0^\infty dt\, e^{-t/M^2}\frac{1}{\pi}\, t\,\mathrm{Im}\,\Delta^{ud}(t) = -\frac{3}{2\pi^2}\frac{\hat m_u \hat m_d}{(\log M/\Lambda)^{2\gamma_1/\beta_1}}\left(1-e^{-2t_c/M^2}\right)\cdot$$
$$\cdot\left\{1+\left(\frac{\bar\alpha_s}{\pi}\right)\left[\frac{11}{3}+2\gamma_E+\frac{2\gamma_1\beta_2}{\beta_1^2}\log\log\frac{M^2}{\Lambda^2}-\frac{2}{\beta_1}\left(\gamma_2-\gamma_1\frac{\beta_2}{\beta_1}\right)\right]\right.$$
$$\left. - \frac{\bar m_u^2+\bar m_d^2}{M^2} + \frac{\pi}{3}\frac{1}{M^4}\langle\alpha_s G^2\rangle + \frac{\pi^2}{3M^4}\left[\left(-3+\frac{m_d^2}{m_u^2}\right)\langle 0|\bar\psi_d\psi_d|0\rangle + u\leftrightarrow d\right]\right\} \qquad (20)$$

Here the continuum contribution within the QCD model is parametrized by the e^{-2t_c/M^2} factor [9] ($\sqrt{t_c} \simeq 3m_\pi$ is the continuum threshold), $\langle \alpha_s G^2 \rangle \simeq 0.04\ \mathrm{GeV}^4$ [10] is the gluon vacuum condensate $\gamma_2 = \frac{101}{12} - \frac{5}{18} n_F$, $\beta_2 = -\frac{51}{4} + \frac{19}{12} n_F$ for $SU(3)_c \times SU(n)_F$. We shall use PCAC combined with the $SU(2)_F$ symmetry argument for the estimate of the quark vacuum condensate $m_i \langle 0|\bar{\psi}_j \psi_j|0\rangle$. Using the positivity of the scalar contribution to the sum rule, we get the upper bound [4])

$$(\hat{m}_u \hat{m}_d)^{1/2} \lesssim \begin{pmatrix} 19 \pm 5 \\ 13 \pm 3 \\ 9 \pm 2 \end{pmatrix} \mathrm{MeV} \ \text{for}\ \Lambda \simeq \begin{pmatrix} 70 \\ 140 \\ 210 \end{pmatrix} \mathrm{MeV}, \tag{21}$$

where the error is the square of the calculated QCD corrections. Note that up to the non-leading chiral symmetry breaking terms and to the correction factor due to the inclusion of the continuum contribution, the analytical expression for the upper bound of $(\hat{m}_u \hat{m}_d)^{1/2}$ and for the lower bound of $\frac{1}{2}(\hat{m}_u + \hat{m}_d)$ obtained by NR [1b] are the same. The result in Eq.(21) combined with the results from other sources is useful for the determination of the absolute values of the u and d quark masses.

5. First Weinberg sum rule and coupling constant of mesons

The Laplace transform of the first Weinberg sum rule including the leading non-perturbative effects has been discussed recently by the authors of Ref.11 and reads

$$\frac{1}{M^2}\int_0^\infty dt\, e^{-t/M^2} \frac{1}{\pi}\left\{ \mathrm{Im}\,\Delta_{ij}^{(1)}(t) + \mathrm{Im}\,\Delta_{ij}^{(0)}(t) \right\} \simeq \left(\frac{\bar{\alpha}_s}{\pi}\right)\frac{1}{\pi^2}\left\{ -\frac{\bar{m}_u \bar{m}_d}{M^2} + \frac{8}{3}\pi^2 \frac{\left(\bar{m}_d \langle \bar{\psi}_u \psi_u \rangle + \bar{m}_u \langle \bar{\psi}_d \psi_d \rangle\right)}{M^4} \right\}. \tag{22}$$

In the $\bar{u}d$ channel, we saturate the integral by the π, ρ and A_1 using a narrow width approximation. Using the PCAC relation in Eq.(11) ($SU(2)_F$ symmetry limit), the relation $\bar{m}_u \bar{m}_d \simeq \frac{4}{3}\pi^2 f_\pi^2 m_\pi^4/M^4$, from Eq.(20) and the positivity of the scalar contribibition to the sum rule, we get

$$e^{-M_\rho^2/M^2}\frac{M_\rho^2}{2\gamma_\rho^2} - e^{-M_{A_1}^2/M^2}\frac{M_{A_1}^2}{2\gamma_{A_1}^2} - e^{-m_\pi^2/M^2}\, 2f_\pi^2 \lesssim -\left(\frac{\bar{\alpha}_s}{\pi}\right)\frac{4}{3}\frac{m_\pi^2 f_\pi^2}{M^2}. \tag{23}$$

where γ_i is the coupling of the meson i to the vector boson.

Clearly for $M \simeq M_\rho$, the A_1 contribution is suppressed compared to the π and ρ one [12] and we get a relation between f_π and the ρ meson coupling

$$f_\pi \gtrsim \frac{M_\rho}{2\gamma_\rho}\frac{1}{\sqrt{e}}\left\{ 1 - \frac{M_{A_1}^2}{M_\rho^2}\frac{\gamma_\rho^2}{\gamma_{A_1}^2} e^{1 - M_{A_1}^2/M_\rho^2} \right\}^{1/2} \simeq (92 \pm 7)\ \mathrm{MeV}, \tag{24}$$

which can be reached using an alternative method [12]. For $M \simeq M_{A_1}$, the A_1 contribution to the sum rule is optimized. Then, we deduce the A_1 coupling to the W boson

$$\frac{\pi M_{A_1}^2}{2\gamma_{A_1}^2} \geqslant e\left\{ \frac{\pi M_\rho^2}{2\gamma_\rho^2} e^{-M_\rho^2/M_{A_1}^2} - 2\pi f_\pi^2 \left(1 + \mathcal{O}\left(\frac{\bar{\alpha}_s}{\pi}\frac{m_\pi^2}{M_{A_1}^2}\right)\right)\right\} \simeq 60.7\ 10^3\ \mathrm{MeV}^2 \tag{25}$$

which we consider as an improvement of the usual result [6]

$$\frac{\pi M_{A_1}^2}{2\gamma_{A_1}^2} \simeq \frac{\pi M_\rho^2}{2\gamma_\rho^2} - 2\pi f_\pi^2 . \tag{26}$$

Eq.(25) can be translated into a bound to the $\tau \to \nu_\tau A_1$ branching ratio

$$R_{\tau A_1} \equiv \frac{\Gamma(\tau \to \nu_\tau A_1)}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} \gtrsim 0.47 \tag{27}$$

to be compared to the FNR result $R_{\tau A_1} \gtrsim 0.48$ [2], to the result in [13] $R_{\tau A_1} \lesssim (0.7 \pm 0.2)$ and to the data $R_{\tau A_1} \simeq (0.54 \pm 0.16)$ [14]. In the $\bar{u}s$ channel, we saturate the spectral function by the K and K^* meson. An analysis similar to the above gives the bound

$$R_{\tau K^*} \equiv \frac{\Gamma(\tau \to \nu_\tau K^*)}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} \gtrsim 0.06 . \tag{28}$$

6. Second Weinberg sum rule and the A_1 mass

As was shown by FNR [2], the second Weinberg sum rule is divergent. However, its convergence can be saved working with its Laplace transform which reads in the $\bar{u}d$ channel:

$$\frac{1}{M^2}\int_0^\infty dt\, t\, e^{-t/M^2}\frac{1}{\pi}\,\mathrm{Im}\,\Delta^{(1)}_{ud}(t) \simeq \frac{3}{2\pi^2}\,\frac{\hat m_u \hat m_d}{(\log M/\Lambda)^{2\gamma_1/\beta_1}}\left\{1 + O\left(\frac{\bar\alpha_s}{\pi},\ \frac{1}{M^2}\right)\right\}. \tag{29}$$

A saturation of the sum rule by low mass resonances requires to choose a moderate M^2 and so for the light quark u,d, the right-hand side of the sum rule remains yet a correction term. We optimize the A_1 contribution to the sum rule for $M^2 \simeq M^2_{A_1}$. Using the positivity of the right-hand side of Eq.(29), we get the inequality:

$$e^{-M_\rho^2/M_{A_1}^2}\cdot\frac{M_\rho^4}{2\gamma_\rho^2} - e^{-1}\cdot\frac{M_{A_1}^4}{2\gamma_{A_1}^2} \geq 0 \tag{30}$$

which combined with Eq.(23) gives

$$M_{A_1}^2 \lesssim M_\rho^2\cdot\frac{1}{1 - 2f_\pi^2\left(2\gamma_\rho^2/M_\rho^2\right)e^{M_\rho^2/M_{A_1}^2}} \tag{31}$$

i.e.

$$M_{A_1} \lesssim 1.62\, M_\rho \simeq 1.25\ \mathrm{GeV} . \tag{32}$$

An inclusion of higher ρ states to Eq.(31) could be done in order to improve the result but it seems, then, natural to include higher A_1 states as a companion of the ρ-like one.

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FOOTNOTES

1) We shall follow the notation in BNRY [3]. In particular, the notation for the non-transverse part of the two-point function is slightly different than that used by FNR [2].

2) In this paper we shall only take into account the contribution of the leading non-perturbative terms to the sum rule. It is known from some examples of Ref.1 that the contribution of the operators $\bar\psi\Gamma\psi\bar\psi\Gamma\psi$ and $\bar\psi\sigma_{\mu\nu}\frac{1}{2}\lambda_a\psi G^{\mu\nu}_a$ remains a few correction of the leading QCD contribution from $M \gtrsim M_\rho$ and so, their effect which is proportional to $\bar\alpha_s(M^2)\ \langle 0|\bar\psi\psi|0\rangle^2\ \frac{1}{M^6}$ does not play a crucial role for the results given in this paper.

3) Notice that the coefficient of our $(\bar\alpha_s/\pi)$ term differs from the result in Ref.7. In fact, our result comes from the $(\bar\alpha_s/\pi)\log - q^2/\nu^2$ term of the quantity B-C of Ref.7.

4) I thank E. de Rafael for a discussion on this point.

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