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ON A FUSION OF SUPERSYMMETRIES WITH GAUGE THEORIES *

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ABSTRACT

A scheme of unification is discussed where the sources of gravity form a supersymmetric multiplet, but gravity is not a member of this multiplet (with spins $\leqslant 3/2$). Gravity is introduced later as a gauge field, the global gauge being the 4-parametric translation group.

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In a paper entitled "A supersymmetry dynamics of hadrons" 1) an extended supermultiplet of particles with spins not higher than 3/2 was investigated and phyiscally interpreted as describing the hadronic matter. It was the case N = 4 of Table I where N is the number of generators and simultaneously the number of vector gauge fields. Our discussion was, however, not quite clear and satisfactory inasmuch as it did not stress the fact that the multiplets appearing in Table I mean essentially two-component field quantities (principally massless particles while masses appear later in consequence of spontaneous symmetry breakings). In order to describe quarks allowing for both helicities one has to introduce - besides the last column of Table I - also a mirror multiplet so that finally there should occur a doubling of the number of fields. Thus, instead of the supermultiplet {1,4,6,6} we deal finally with a set {2,8,12,12} of particles with spins 3/2, 1, 1/2 and 0, respectively. This necessitates interpretational changes.

Table I

N =	1	2	3	4
spin 3/2	1	1	1	1
spin 1	1	2	3	4
spin 1/2		1	3	6
spin 0			2	6

The number 12 of spin 1/2 fields denotes just six four-component spinors describing quarks with both helicities whereby 6 = 3.2 accounts for three colours and two flavours, i.e. for one generation of quarks. Now, the doubled number of vector fields, i.e. eight altogether, are interpretable as gluons. On the other hand, inasmuch as the above described set is a composition of two sets with different helicities, it is natural to introduce also a parity violating interaction, leaving us with a rest symmetry $SU(2)_L \times U(1)$. The rest symmetry may also be called a "hidden symmetry". The additional (parity violating) interaction may be introduced by means of the standard gauge procedure, breaking the original global symmetry, but leaving us with a local rest symmetry $SU(2)_L \times U(1)$. The additional gauge fields are just the fields $W^{\pm}$, Z^0, A of Salam and Weinberg. From this point of view it may be said that the vector fields split into two classes: those present from the very beginning in the supermultiplet and those introduced by an additional gauge procedure supplementing the original supersymmetry SU(3) by the remaining $SU(2)_L \times U(1)$.

In this way it is possible to account naturally for (one generation of) hadrons. On the other hand, the leptonic world seems to be accounted for with the help of the third column of Table I. In this case there is no need (as yet) for doubling, i.e. of introducing any additional set of fields (mirror set). The number 3 of two-component fields describing particles with spin 1/2 may denote one generation of leptons, for example e_L, ν_L, e_R. The three vector bosons appearing from the very beginning in the supermultiplet characterized by $N = 3$ denote $W^{\pm}$ and Z^0 and account for the SU(2) supersymmetry, whereas the missing fourth vector field should be introduced additionally by means of the standard group procedure to account for the remaining and not exploited yet (hidden) U(1) symmetry.

The above considerations aim at exhibiting clearly a supersymmetric structure of the hadronic and leptonic worlds. They also provide a unification of strong with electroweak interactions of hadrons whereby the sources of these interactions (in the form of fields with half-integral spins)are introduced no more "by hand" but are shown to be just unavoidable from the point of view of supersymmetries.

The above scheme does not provide us yet with a full (grand?) unification not only because the very very weak interactions responsible for the proton decay are missing, but mainly because the most common force - the gravitational interaction - is missing.One may put the question why had we not started directly with supermultiplets involving the highest spin value 2, but limited ourselves to these with the highest value 3/2. We have done so mainly for reasons of methodology. Inasmuch as quantum theory of gravity is questionable yet, we preferred to deal at first with a well defined theory of sources of the gravitational field and introduce gravity itself only later, at a second step, by admitting a proliferation of the original supersymmetric set of fields.

In order to account for gravity, we apply the following principle: always whenever there appears a global symmetry of a Lagrangian, one should apply a suitable gauge procedure converting it into a local symmetry. A question arises as to whether, in addition to the above described symmetries, there appears any other symmetry of the Lagrangian describing the sets of particles with spins 3/2, 1, 1/2 and 0. The answer is affirmative: there appears a global symmetry under the group of translations in space and time

$$X^{\mu} \rightarrow X^{\mu} + \varepsilon^{\mu} \quad ,$$

where ε^{μ} are four parameters assumed as infinitesimal for simplicity. It is this symmetry that constitutes a starting point for introducing gravity as a gauge field by converting the global into a local symmetry $\varepsilon^{\mu} \rightarrow \varepsilon^{\mu}(x)$. In this way four arbitrary gauge functions have been introduced accounting for the freedom of choosing an arbitrary system of co-ordinates.

In order to present the theory of gravity most of the modern authors needed a tetrad technique. Without denying that tetrads mean a useful tool for many investigations within general relativity, we should like to point out that sometimes it may be useful and convenient not to use the tetrad formalism. In our opinion, it is just the case if one is in search of a formulation of the general relativity as a gauge theory in a methodologically satisfactory way. Tetrads seemed to be necessary because they provide one with two sets of indices: a co-ordinate index, say μ, and an index, say $\underset{\sim}{a}$, enumerating the tetrad vectors and accounting for a Lorentz group (i.e. tetrad "four rotations"). The appearance of two indices μ and $\underset{\sim}{a}$ is common in gauge theories where the sets of vector fields A^{a}_{μ} come into play. But the point is that neither the Lorentz group nor the whole Poincaré group form the global counterpart of the group of general co-ordinate transformations, but it is simply the group of translations ε^{μ} . The case of gravity is exceptional just because there are not two sets of indices necessary but only one and the same set μ playing simultaneously both roles: this of a co-ordinate index and that of the gauge group index.

In the other cases of gauge theories the local gauge group is Abelian if the global group is so, but in the case of gravity the situation is different: in spite of the fact that the global group (translation group) is Abelian, the local group turns out to be non-Abelian. This is due to the fact that the index μ in ε^{μ} plays simultaneously the role of the co-ordinate index, i.e. of the co-ordinate x being an argument of the functions $\varepsilon^{\mu}(x)$. Expanding these functions into a power series in X^{ν} in the neighbourhood of any point assumed arbitrarily as the co-ordinate origin

$$\varepsilon^{\mu}(x) = \varepsilon^{\mu} + \varepsilon^{\mu}_{\nu} X^{\nu} + \ldots$$

it is seen that - besides translations - also Lorentz transformation dilatations, etc. have been taken into account so to say automatically (as the "higher moments"). The local group is, nevertheless, a group of translations, but not the Poincaré group as stated often erroneously in the literature.

In order to present the theory of gravity as a gauge theory based on the global group of translations it is only necessary to replace $\eta_{\mu\nu}$ by $g_{\mu\nu}$ and to replace partial derivatives by covariant derivatives in the Lagrangian of material sources of gravity and to add a Lagrangian for the gauge field $g_{\mu\nu}$ itself. From the requirements that this Lagrangian should be a scalar under the local gauge transformations, i.e. under the group of general local co-ordinate transformations, and that it should yield Lagrange equations of the second order, this Lagrangian follows almost uniquely: it must be the scalar curvature, possibly supplemented by a cosmological term. In this way the theory of sources may be enlarged to incorporate into its framework the gravitational field, being another gauge field.

Let us conclude with a remark about the problem of quantization of the gravitational field. Apart from the problems of an apparent non-renormalizability there appears the following difficulty: there are indications of the existence of unitarily and physically inequivalent representations of the quantized theory of gravity whereby it is meant that the results of quantization in one system of co-ordinates (one gauge) and then transforming the resulting operators to another system of co-ordinates is not always equivalent to quantizing it directly in the latter co-ordinate system. But it would not be correct to say that quantization violates general covariance, but rather that to one and the same classical theory of gravity there correspond several inequivalent versions of quantum theory, all of them "generally covariant" in the sense of being expressible finally in arbitrary co-ordinates, but still inequivalent because quantization performed in a particular gauge (a particular co-ordinate system) means also a choice of a particular frame of reference, i.e. a choice (separation) of what belongs to the system to be observed and measured, and what to the apparatus and to the conditions of measurement. These problems are discussed in two papers by one of us, one already published and the other will soon be published in GRG Journal [2].

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