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TWO-QUARK ANNIHILATION VERSUS THREE-QUARK FUSION IN PROTON DECAY

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Abstract

Adopting the Bethe-Salpeter formalism for the treatment of hadrons the relative contributions of the two-quark annihilation and three-quark fusion mechanisms to baryon number violating nucleon decay are compared. The former is found to dominate over the latter by a factor of about 3 in amplitude, implying a proton lifetime of $4\cdot 10^{30}$ yr for $m_X = 4.2\cdot 10^{14}$ GeV.

I. Motivation

The violation of baryon number, implying the decay of the proton into leptons and mesons, is without doubt the most spectacular prediction of grand unified theories (GUTs). This prediction will be confronted with experiment in the near future. Therefore great efforts have been made in calculating lifetimes and branching ratios to an accuracy as large as possible [1]. The obstacle which causes most troubles in these computations is the question how to implement the baryon and lepton number violating interactions arising within the framework of GUTs into the concepts one has about confinement, i.e. the question how to translate from an interaction given in terms of quarks to a certain hadronic model.

The interactions generally considered to be most relevant for proton decay are represented by effective four-fermion operators of the structure $O \sim (q\ q\ q\ \ell)$ which respect the conservation of the quantum number (B-L) [2]. These operators give rise to three classes of diagrams which are able to induce nucleon decay [3]. The three mechanisms are shown in Fig. 1. The quark decay of Fig. 1c necessarily involves at least two mesons in the final state and is therefore suppressed by phase space. Furthermore, Jarlskog and Yndurain [3] have argued that the contribution of the three-quark fusion of Fig. 1b is proportional to the probability of observing three quarks in a nucleon at one space-time point. This probability is generally accepted to be much smaller than that of finding only two quarks at one point so that the part of Fig. 1b in proton decay can also be neglected. Therefore numerous calculations solely paid attention to the two-quark annihilation depicted in Fig. 1a [1].

However, recently there has been growing interest in the three-quark fusion process [4-7]. It has been claimed that baryon pole terms involving only the three-quark mechanism might be of comparable importance for nucleon decay. Even more, some authors speculate that Fig. 1b can perhaps dominate nucleon decay in nuclei or at least gain experimental significance there [8].

In any case, all calculations of that kind need an idea about the magnitude of the baryon-to-vacuum matrix element of the three quark fields in the effective operator responsible for nucleon decay. Meljanac et al. [6] calculate this matrix element with the aid of the MIT bag model, whereas Berezinsky et al. [5] obtain its magnitude from QCD sum rules.

The situation concerning the question of the relevance of the three-quark fusion can only be clarified by calculating the relative contributions to proton decay of the various quark-level mechanisms within one and the same conceptual framework. Thomas and McKellar [9] made a first attempt in this direction. These authors use the standard non-relativistic SU(6) quark model for hadrons and the so-called Cloudy Bag Model [10] for the estimation of the pion-nucleon coupling strength.

In this paper I try to cast some light on this question by calculating two-body proton decay via the two-quark conversion according to Fig. 2. The initial state nucleon and the final state meson are characterized by Bethe-Salpeter amplitudes. The use of the Bethe-Salpeter formalism for the treatment of quark confinement has the advantage that one deals with fully covariant expressions at every stage of the computation. The resulting matrix element is compared with the one obtained from three-quark conversion to an anti-lepton in a previous work [11].

II. The Bethe-Salpeter Models

The fundamental assumption of the Bethe-Salpeter (BS) models employed for the description of baryons B and mesons M as three-quark or quark-anti-quark bound states, respectively, is the strong binding of the constituents. This means that the effective quark mass m is much larger than the mass of the bound state, i.e. the baryon mass M_B or the meson mass M_M, $M_B/3m \ll 1$ and $M_M/2m \ll 1$. This in turn entails that one is allowed to expand the BS amplitudes into power series in 1/m.

The structure of the integral kernels which enter into the BS equations is taken to be the same for the baryon and meson case. It has been used previously with great success for the calculation of the widths of strong and electromagnetic meson decays [12], of the strong decays of the $\frac{3^+}{2}$- baryon resonances [13], and of the electromagnetic and weak form factors of the $\frac{1}{2}^{+}$-baryons [13].

The baryonic BS amplitude χ_B is defined in terms of the renormalized quark fields ψ by

$$\chi_B(p,q;P) = \frac{1}{(2\pi)^2}\int d^4r\, d^4s\, e^{i(pr+qs)} \cdot \langle 0|T(\psi(\frac{s}{3}+\frac{r}{2})\psi(\frac{s}{3}-\frac{r}{2})\psi(-\frac{2s}{3}))|B(\vec{P})\rangle . \quad (1)$$

P, p and q are center-of-mass momentum and relative momenta of the three quarks with momenta k_i, $i = 1,2,3$:

$$P = k_1 + k_2 + k_3 ,$$
$$p = \frac{1}{2}(k_1 - k_2) , \quad (2)$$
$$q = \frac{1}{3}(k_1 + k_2 - 2k_3) .$$

Kielanowski has solved the corresponding BS equation up to order $(1/m^2)$ with the result [14]:

$$\chi_B(p,q;P) = N_B[1 + \frac{1}{2m}\sum_{i=1}^{3} \not{K}_i + \frac{1}{4m^2}(\not{K}_1\not{K}_2+\not{K}_1\not{K}_3+\not{K}_2\not{K}_3) + O(\frac{1}{m^3})]\chi_B^o(p,q) \quad ,(3)$$

where $\not{K}_i$, $i = 1,2,3$, stands here for $\not{K}_1 \times 1 \times 1$, $1 \times \not{K}_2 \times 1$, and $1 \times 1 \times \not{K}_3$, respectively. The zeroth-order amplitude χ_B^o is for the SU(3)-octet baryons given by

$$\chi_B^o(p,q) = \frac{1}{\sqrt{2}}(|\frac{1}{2}\ 0;M^+\rangle\chi_{SU(3)}^{(M^+)} + |\frac{1}{2}\ 0;M^-\rangle\chi_{SU(3)}^{(M^-)})\chi_C \exp(-\frac{p_E^2}{6\sqrt{\beta_B}} - \frac{q_E^2}{8\sqrt{\beta_B}}) \quad (4)$$

where $|\frac{1}{2}\ 0;M^{\pm}\rangle$ are the spin wave functions that transform according to the representation $(\frac{1}{2},0)$ of SU(2) × SU(2) [15] and $\chi_{SU(3)}^{(M^{\pm})}$ denote the standard SU(3)-flavour wave functions of the baryons. $M^{\pm}$ indicates the behaviour of these wave functions under permutation of the quarks. The totally antisymmetric colour part is labelled by χ_C, the index E should recall that one is dealing with Euclidean momenta, since for the solution of the BS equation a Wick rotation has to be performed. The normalization constant N_B in Eq. (3) is

$$N_B = \frac{1}{(2\pi)^3\sqrt{2P_o}} \frac{1}{\beta_B\sqrt{m}} \qquad (5)$$

according to the state normalization $\langle\vec{P}'|\vec{P}\rangle = \delta^{(3)}(\vec{P} - \vec{P}')$. A fit of the lowest lying baryon resonance masses yields for the level spacing parameter $\sqrt{\beta_B} = 0.029\ \mathrm{GeV}^2$ [14].

The mesonic BS amplitude

$$\chi_M(r;Q) = \int d^4x\, e^{irx} \langle 0|T(\psi(\tfrac{x}{2})\, \bar{\psi}(-\tfrac{x}{2}))|M(\vec{Q})\rangle \ , \qquad (6)$$

with

$$Q = k_1 + k_2 \ ,$$
$$r = \frac{1}{2}(k_1 - k_2) \ , \qquad (7)$$

has been investigated in a comprehensive analysis by Böhm, Joos and Krammer [12]. For the choice of the interaction kernel equivalent to the baryon amplitude one obtains for the pseudoscalar meson case

$$\chi_M(r;Q) = N_M[1 + \frac{1}{2m}\not{Q} + O(\frac{1}{m^3})]\ \chi_M^o(r) \ , \qquad (8)$$

where the leading term is of the form

$$\chi_M^o(r) = \frac{1}{\sqrt{3}}\ \chi_f^M\ \gamma_5\ \exp(-\frac{r_E^2}{2\sqrt{\beta_M}}) \ . \qquad (9)$$

Here χ_f^M denotes the flavour part of the meson wave function. The normalization factor N_M is given by

$$N_M = \frac{1}{(2\pi)^{3/2}\sqrt{2Q_o}} \frac{4\pi}{\sqrt{\beta_M}} \ . \qquad (10)$$

The mass splittings between the 0^-, 1^+, 2^- mesons fix the parameter β_M to the value $\sqrt{\beta_M} = 0.17\ \mathrm{GeV}^2$ [12].

Finally, the effective quark mass m is extracted from the prediction of the weak pion decay constant: m = 1 GeV [12].

III. Comparison

In order to obtain a reliable estimation of the relative importance of two-quark annihilation and three-quark fusion I compute the S-matrix element for two-body proton decay in three different ways. These three points of view differ by how much they involve the two quark-level mechanisms under consideration. It is sufficient for the purpose of comparison to restrict this discussion to one specific decay channel. For simplicity I choose the decay mode $p \to \nu_e^c + \pi^+$, which appears in most of the calculations of nucleon decay with a branching ratio in the range 10 - 20% in SU(5) [1] and which is expected to be dominant in SO(10) not broken down to SU(5) [11,16].

The effective four-fermion operator responsible for this decay is [2]

$$O^{\nu_e} = \epsilon_{ijk}[\overline{\nu_e^c}\,\gamma_\mu(1-\gamma_5)d_i][\overline{u_k^c}\,\gamma^\mu(1+\gamma_5)d_j] \,, \tag{11}$$

i,j,k being colour indices. I parametrize its matrix element by an amplitude A in the form

$$\langle \nu_e^c(\vec{K})\pi^+(\vec{Q})|O^{\nu_e}(0)|p(\vec{P})\rangle = -i\,\frac{1}{(2\pi)^{9/2}}\,\frac{1}{\sqrt{2E_Q}}\sqrt{\frac{m_\nu_e}{E_K}}\sqrt{\frac{m_p}{E_P}}\,A\,\bar{u}(K)(1+\gamma_5)u(P) \,. \tag{12}$$

The three prescriptions for the calculation of the amplitude A mentioned above are the following:

(i) The two-quark annihilation diagram of Fig. 1a is sandwiched by a baryonic and a mesonic BS amplitude in the way indicated in Fig. 2, the BS amplitudes serving to transform from hadronic to quark level and vice versa. Of course, one has to take the sum over all permutations of the internal quark lines connected with different quark legs of the BS amplitudes. (In fact, only one permutation contributes in our case.) In lowest non-vanishing order in 1/m one obtains for the two-quark annihilation amplitude

$$A_{2q} = \frac{12\sqrt{2}}{\pi}\,\frac{1}{\sqrt{\beta_M}}\sqrt{\frac{M_p}{m}}\left(\frac{1}{4\sqrt{\beta_B}} + \frac{1}{\sqrt{\beta_M}}\right)^{-2}\exp\left[-\frac{2M_p^2 - 3M_{\pi^+}^2}{72(4\sqrt{\beta_B} + \sqrt{\beta_M})}\right] \,, \tag{13}$$

where M_p and M_{π^+} denote the masses of proton and pion, respectively. Note that in Eq. (13) terms of order O(m), which could have been introduced by the inverse propagator of the spectator quark in Fig. 2, vanish due to the chirality and Lorentz structure of the operator in Eq. (11). The exponential term in Eq. (13) is of minor importance for the magnitude of A_{2q}, since its numerical value 0.92 is close to unity.

(ii) The Born approximation to the matrix element of Eq. (12) involves only the three-quark fusion graph of Fig. 1b. In this pole model the nucleon is assumed to decay into the meson showing up in the final state and into a virtual baryon which is transformed to an on-shell antilepton by the baryon number violating interaction. The calculation of the effective baryon-lepton transition matrix elements entering in this approach has been described in detail in Ref. [11]. Concerning the question of the number of baryon poles to be taken into account it has been estimated that the contribution of higher resonances does not exceed 10% [6] so that they can safely be neglected. Retaining only the nucleon pole the Born amplitude is found to be

$$A_B = + \frac{24\sqrt{3}}{\pi^2} \frac{\beta_B}{\sqrt{mM_p}} \frac{g_{\pi NN}}{m} . \tag{14}$$

Here $g_{\pi NN}$ labels the strong pion-nucleon coupling constant.

(iii) The most elaborate treatment involves the use of a suitably adapted soft pion formalism [17]. Within this framework the pion is reduced from the final state vector by the LSZ reduction technique. PCAC allows then to replace the pion field operator by the divergence of the axial current j_5^μ carrying the quantum numbers of the pion. After a partial integration one ends up with the low-energy theorem [11]

$$\langle \nu_e^c \pi^+(\vec{Q})|O^{\nu_e}(0)|p(\vec{P})\rangle = - \frac{i}{f_\pi} \frac{1}{(2\pi)^{3/2}\sqrt{2E_Q}} \langle \nu_e^c|[Q_5^-(0), O^{\nu_e}(0)]|p\rangle +$$

$$+ \lim_{Q\to 0} \{\frac{1}{f_\pi} \frac{1}{(2\pi)^{3/2}\sqrt{2E_Q}} Q^\mu \int d^4x\, e^{iQx} \langle \nu_e^c|T(j_{5\mu}^-(x), O^{\nu_e}(0))|p\rangle - B(Q)\} +$$

$$+ B(Q) . \tag{15}$$

Here f_π is the weak decay constant of the pion, introduced into this formula by PCAC, and Q_5^- is the axial charge corresponding to $j_{5\mu}^-$, from which the pion pole already has been subtracted. B(Q) is the Born term contribution to the matrix element on the left-hand side of Eq. (15). Its appearance is necessary to make the low-energy limit $Q \to 0$ well-defined.

This soft pion theorem, Eq. (15), represents in some sense a synthesis of both of the mechanisms to be compared. The limit $Q \to 0$ selects from the expression in braces just those pole terms where the intermediate baryon is degenerate in mass with the decaying nucleon. The conversion of this baryon to the emitted anti-lepton takes place by the three-quark fusion process. On the other hand the first term on the right-hand side of Eq. (15), which contains the equal time commutator of the axial charge and the operator mediating the decay, can be interpreted as the contribution of the two-quark annihilation graph.

The amplitude emerging from the soft pion theorem, Eq. (15), is given by

$$A_{SP} = \frac{12\sqrt{3}}{\pi^2} \frac{g_B}{m} \sqrt{\frac{M_p}{m}} \frac{1}{f_\pi} \left(1 + \frac{f_\pi\, g_{\pi NN}}{M_p}\right) , \qquad (16)$$

where again only the leading term of the power series in $1/m$ has been retained.

One is now in the position to be able to compare the relative importance of the baryon number violating mechanisms of Fig. 1 for proton decay. The ratio of soft pion to Born amplitude depends only on the weak axial vector coupling constant g_A:

$$\frac{A_{SP}}{A_B} = + \frac{1}{2} \left(1 + \frac{M_p}{f_\pi\, g_{\pi NN}}\right) = + \frac{1}{2} \left(1 + \frac{1}{g_A}\right) . \qquad (17)$$

The second equality in this equation holds by the Goldberger-Treiman relation. The factor 1/2 is brought about by the modification of the standard low-energy theorem by the inclusion of the Born term contribution in the way indicated in Eq. (15). Destructive interference between the second and the third term on the right-hand side of Eq. (15) lead to a reduction of the whole pole term amplitude by a factor 1/2 compared to the Born approximation B(q). Therefore the employment of the soft pion formalism results in a slight suppression of the amplitude. Using the experimental numbers [18] $g^2_{\pi NN}/4\pi = 14.4$ and $f_\pi = 93.2$ MeV one obtains

$$\left|\frac{A_{SP}}{A_B}\right| = 0.874 ,$$

which enlarges the theoretical proton lifetime of Ref. [11] by roughly 4/3. Quite similarly, Tomozawa [4], Berezinsky et al. [5], and Claudson et al. [19] report for the ratio A_{SP}/A_B a proportionality to $(1+g_A)$. In either case the matrix elements are practically of the same magnitude.

The appropriate measure for the relative significance of the two- and three-quark diagrams for proton decay is the ratio two-quark annihilation amplitude A_{2q} to Born approximation amplitude A_B. In their treatment of this question Thomas and McKellar [9] found for this ratio values in the range 0.4 to 2, depending critically on the adopted value of the proton radius which in turn is influenced by the method used to extract it from different experiments. The favoured result of these authors is $A_{2q}/A_B = 0.44$ for the decay $p \to e^+ + \pi^0$. In this work I obtain from Eqs. (13) and (14)

$$\left|\frac{A_{2q}}{A_B}\right| = 2.7 ,$$

i.e. the two-quark annihilation dominates the three-quark fusion, and similarly the low-energy result, by a factor of about 3. The reason for this is mainly the fact that the momentum cutoff parameter $\sqrt{\beta} = 4\sqrt{\beta_B} = \sqrt{\beta_M}$ enters in Eqs. (14) and (16) on the one hand and in Eq. (13) on the other hand with different powers. The two-quark annihilation amplitude A_{2q} is only proportional to $\sqrt{\beta}$, whereas both Born and soft pion amplitude are of order $(\sqrt{\beta})^2$, so that the smallness of $\sqrt{\beta}/m^2$ 0.12 causes the observed differences. The different powers of $\sqrt{\beta}$ reflect the different powers of the two-quark wave function $\psi(0)$ at the origin in proton decay calculations using nonrelativistic SU(6) quark models. There the two-quark annihilation is proportional to $|\psi(0)|$, the three-quark fusion to $|\psi(0)|^2$, which allows in principle to determine $|\psi(0)|$ from electromagnetic nucleon decay where only pole graphs play a role [11].

IV. Summary

I have calculated the S-matrix element for proton decay using three different methods to connect the baryon number violating interaction, given in grand unified theories at quark level, with the external hadronic states. It is shown that corrections suggested by current algebra do not modify significantly the amplitude resulting from a simple Born approximation. However, the two-quark annihilation is found to be about 3 times larger in amplitude than the three-quark fusion. Assuming a roughly equal enhancement for all decay modes, the effect of employing the two-quark mechanism is a reduction of the predicted proton lifetime by almost an order of magnitude. Scaling the results of Ref. [11] by this factor one obtains within SU(5)

$$\tau_p = 4.4 \cdot 10^{30} \text{ yr}$$

and

$$\tau_n = 3.9 \cdot 10^{30} \text{ yr}$$

for a grand unified gauge boson mass $m_X = 4.2 \cdot 10^{14}$ GeV [20]. For comparison, the Kolar Gold Field collaboration quotes an experimental value of $\tau_N \approx 7 \cdot 10^{30}$ yr for the nucleon lifetime [21].

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References

[1] P. Langacker, Phys. Rep. 72, 185 (1981); University of Pennsylvania preprints UPR-0180 T (1981) and UPR-0186 T (1982).

[2] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979);
F. Wilczek, A. Zee, Phys. Rev. Lett. 43, 1571 (1979).

[3] C. Jarlskog, F.J. Ynduráin, Nucl. Phys. B149, 29 (1979).

[4] Y. Tomozawa, Phys. Rev. Lett. 46, 463 (1981); University of Michigan preprint UM HE 81-51 (1981).

[5] V.S. Berezinsky, B.L. Ioffe, Y.I. Kogan, Phys. Lett. 105B, 33 (1981).

[6] S. Meljanac, D. Palle, I. Picek, D. Tadić, "Rudjer Bosković" Institute preprint (1981).

[7] J.O. Eeg, I. Picek, Phys. Lett. 112B, 59 (1982).

[8] J.M.F. de Labastida, F.J. Ynduráin, Phys. Rev. Lett. 47, 1101 (1981).
J. Abad, A.F. Pacheco, Phys. Lett. 109B, 279 (1982).
R.F. Alvarez-Estrada, J.L. Sánchez-Gómez, University of Madrid preprint (1982).

[9] A.W. Thomas, B.H.J. McKellar, CERN preprint TH.3376-CERN (1982).

[10] A.W. Thomas, CERN preprint TH.3368-CERN (1982).

[11] W. Lucha, H. Stremnitzer, University of Vienna preprint UWThPh-1982-20 (1982).

[12] M. Böhm, H. Joos, M. Krammer, Acta Phys. Austr. Suppl. XI, 3 (1973).
M. Krammer, Acta Phys. Austr. 40, 187 (1974).
D. Flamm, F.F. Schöberl, Nuovo Cim. 46A, 61 (1978).
H. Lasek, Acta Phys. Austr. 52, 107 (1980).

[13] P. Falkensteiner, Acta Phys. Austr. 53, 175 (1981).

[14] P. Kielanowski, Z. Phys. C3, 267 (1980).

[15] R.F. Meyer, University of Bonn preprint BONN-HE-75-14 (1975).

[16] M.E. Machacek, Nucl. Phys. B149, 29 (1979).

[17] L.S. Brown, C.M. Sommerfield, Phys. Rev. Lett. 16, 751 (1966).
Riazuddin, B.W. Lee, Phys. Rev. 146, 1202 (1966).

[18] M.M. Nagels et al., Nucl. Phys. B147, 189 (1979).

[19] M. Claudson, M.B. Wise, L.J. Hall, Harvard University preprint HUTP-81/A036 (1981)

[20] T.J. Goldman, D.A. Ross, Phys. Lett. 84B, 208 (1979);
Nucl. Phys. B171, 273 (1980).

[21] M.R. Krishnaswamy et al., Phys. Lett. 115B, 349 (1982).

Figure Captions

Fig. 1: Effective four-fermion interaction diagrams causing proton decay:

a) Annihilation of two quarks into an anti-quark and an anti-lepton

b) Three-quark fusion to an anti-lepton

c) Phase space suppressed decay of a quark into two anti-quarks and an anti-lepton.

Fig. 2: Proton decay by two-quark annihilation: The baryonic BS-amplitude χ_B decomposes the nucleon N into three quarks. Two of them are converted with interaction strength G to an anti-quark and an anti-lepton. The anti-quark and the spectator quark are fused to the meson M by the mesonic BS-amplitude χ_M.

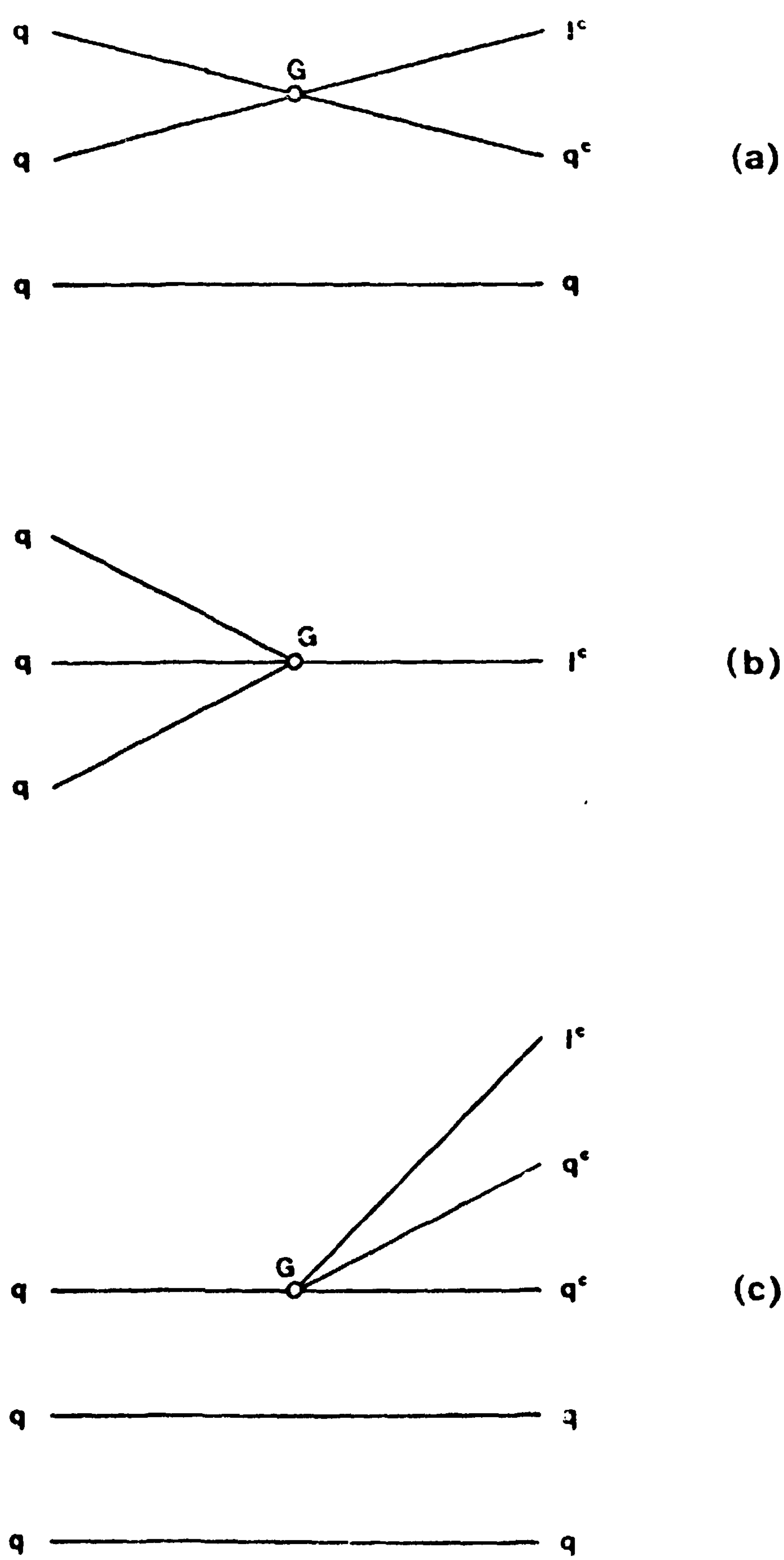

Fig. 1

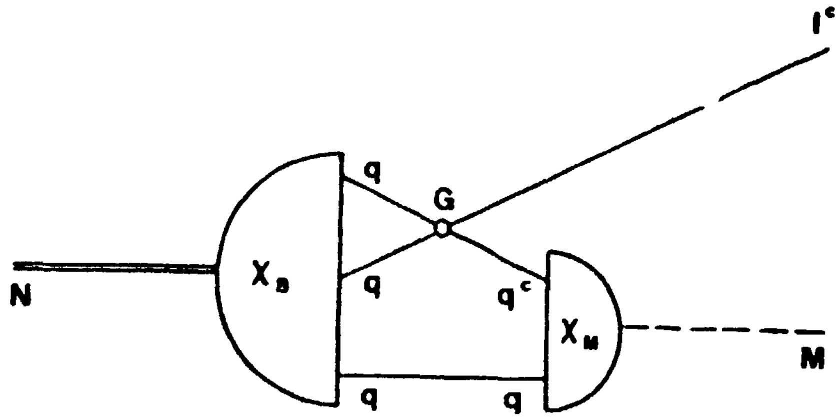

Fig. 2