Alpha-nucleus Absorptive Potential In a Nuclear Matter Approach.

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I. INTRODUCTION

In fact what is required to calculate the absorptive part of the nucleus-nucleus optical model potential is a reliable yet simple model which enables us not only to determine the various parameters of this potential but also to study systematically their behaviour as a function of energy and nuclei.

In the absence of a self-consistent dynamic description of heavy ion reactions one feels justified to follow the forward scattering amplitude approximation [1] (FSAA) to estimate the absorptive part of nucleus-nucleus optical potential. In the nucleus-nucleus case, the absorptive potential depends on the product of the two form factors which is more forward peaked than each one of them individually, thus inducing the scattering to take place in an even more forward direction.

In section II, the FSAA and impulse approximation in the framework of Kerman-McManus-Thaler (KMT) formalism, generalized to include the case of nucleus-nucleus scattering [2,3] , is used to derive an analytical expression for the α - nucleus absorptive potential. Application to $\alpha - {}^{12}C$ and $\alpha - {}^{16}O$ scattering is made in section III. Section IV contains a discussion and conclusions.

II. THE MODEL

It has been shown [3,4] that the impulse approximation in the framework of a generalized version of KMT formalism leads to the following expression for the α-nucleus optical in momentum space

$$U(Q) = -\frac{4A\hbar^2}{\mu_0(2\pi)^2} f^{\alpha}(Q) f^{A}(Q) \bar{A}(Q) \qquad (2\text{-}1)$$

where $\bar{A}(Q)$ is the average over the spin- and isospin-independent parts of the two-nucleon free scattering amplitude, $f^{\alpha}(Q)$ and $f^{A}(Q)$ are the nuclear form factors of the colliding nuclei and $\mu_0 = m/2$.

The optical in configuration space can them be obtained by taking the Fourier transform of Eq. (2-1) to be

$$U(R) = -\frac{n\hbar^2}{\mu_0(2\pi)^2} \int d\vec{Q} f^{\alpha}(Q) f^{A}(Q) \bar{A}(Q) \exp(i\vec{Q}\cdot\vec{R}) \qquad (2\text{-}2)$$

with $n = 4A$

To derive the imaginary part of α-nucleus optical potential from Eq. (2-2), one may use the FSAA. With this approximation the imaginary part of the α-nucleus optical potential reads :

$$W_{\alpha\text{-}A}(R) = -\frac{n\hbar^2}{\mu_0(2\pi)^2} \, Im\,\bar{A}(0)\, F(R) \qquad (2\text{-}3)$$

where

$$F(R) = \int d\vec{Q}\, f^{\alpha}(Q) f^{A}(Q) \exp(i\,\vec{Q}\cdot\vec{R}) \qquad (2\text{-}4)$$

Using the optical theorem, the imaginary part of the forward scattering amplitude reads :

$$Im\,\bar{A}(0) = \frac{K\langle\sigma\rangle}{4\pi} \qquad (2\text{-}5)$$

Where k is the wave number of the projectile (target) inside the compound system and $\langle\sigma\rangle$ is the total nucleon-nucleon cross section averaged over the relative momenta of the interacting nucleus.

Combining Eq. (2-3) and (2-5) yields the following expression for the α-nucleus absorptive potential.

$$W_{\alpha-A}(E,R) = -\gamma \frac{\hbar^2}{2\mu_0} K \langle\sigma\rangle \rho_{CS}(R) \qquad (2\text{-}6)$$

where the scaling factor γ is given by

$$\gamma = \frac{n}{N} \equiv \frac{4A}{4+A} \qquad (2\text{-}7)$$

and

$$\rho_{CS}(R) = \rho^0_{CS} F_{CS}(R) = \frac{N}{(2\pi)^3} F(R) \qquad (2\text{-}8)$$

The quantity $\rho_{CS}(R)$ is interpreted as the nuclear matter density distribution of the compound system with the following normalization condition:

$$\int d\vec{R}\, \rho_{CS}(R) = N \qquad (2\text{-}9)$$

Therefore, it is assumed that once the two colliding nuclei start to overlap, a new nuclear matter conditions are established. Such conditions are described by Eqs. (2-8) and (2-9). Then, the α-nucleus absorptive potential will be considered to be equivalent to the absorptive potential of a nucleon moving in nuclear medium with uniform density distribution $\rho_{CS}(R)$ multiplied by scaling factor γ. This picture will enable us to make the maximum use of the nuclear matter theory.

Now, it can be shown [4,5] that

$$\langle\sigma\rangle = 27.3567 \cdot \frac{2\mu_0}{\hbar^2} K(R) P(E,R) \cdot \frac{1}{\rho_{cs}(R)} \qquad (2\text{-}10)$$

Where the Pauli function

$$P = \left[\frac{1}{3\sqrt{2}}(1+x^2)^{1/2}(1+x^2-\xi^2)\ln\frac{(1+3x^2)+2x[2(1+x^2)]^{1/2}}{(1+3x^2)-2x[2(1+x^2)]^{1/2}} \qquad (2\text{-}11)\right.$$

and

$$\left. -\left(\frac{4}{3}-2\frac{\xi^2}{5}\right)\ln\left(\frac{1+x}{1-x}\right)+\frac{4}{3}x\right]_a^b$$

$$\xi^2 = 2\,K_F^2(R)/K^2(R) = 2\,t_m^2$$

with $K_F(R)\{=[1.5\,\pi^2 \rho_{cs}(R)]^{1/3}\}$ being the local Fermi momentum. For the expression (2-11), the limits are

$$a = 0 \quad , \; b = t_m \qquad \text{for } \xi^2 \leqslant 1$$

$$a = [\xi^2-1]^{1/2} \quad , \; b = t_m \qquad \text{for } \xi^2 \geqslant 1 \qquad (2\text{-}12)$$

Inserting expression (2-10) in Eq. (2-6) yields the following formula for the α-nucleus absorptive potential.

$$W_{\alpha\text{-}A}(E,R) = -\,27.3567 \cdot \gamma \; K^2(E,R)\; P(E,R) \qquad (2\text{-}13)$$

Where E is the centre of mass energy.

It is evident from the last equation that the determination of α- nucleus absorptive potential is now reduced to the problem of calculating $K(R)$. In the framework of the previous picture this be performed by using the local density approximation together with an extended version of Cugnon's local separation energy assumption [6] . Accordingly, the momentum distribution of the nucleon within the nucleus is given by

$$K^2(r) = \frac{2m}{\hbar^2} e_{inc} + [1.5\,\pi^2 \rho(r)]^{2/3} + \frac{2m}{\hbar^2} S(\rho) \qquad (2\text{-}14)$$

with ϵ_{inc} being c-m energy per nucleon. In Cugnon's approach (C-approach hereafter). The separation energy has been assumed to be linear function of

$$S(\rho) = S_{n.m}\, \rho(r)/\rho(0) \qquad (2\text{-}15)$$

where $S_{n.m} = 16\,MeV$. Thus the expression for K reads;

$$K^2(R) = 0.0482\, \epsilon_{inc} + 6.0292\, \rho_{CS}^{2/3} + 0.7717\, E_{CS}(R) \qquad (2\text{-}16)$$

Using Hugenholtz - Van Hove condition [7] for the infinite nuclear matter at equilibrium, namely,

$$S = -\bar{E}_B \qquad (2\text{-}17)$$

two further approaches will be introduced. The first is is based on the semiemprical formula proposed by Brueckner [8] . In this approach (B-approach hereafter), the wave number of the nucleon within the compound system reads [4]:

$$K^2(R) = 0.0482\, \epsilon_{inc} + 2.4117\, \rho_{CS}^{2/3} + 39.4642\, \rho_{CS} - 66.1262\, \rho_{CS}^{4/3} + 26.8424\, \rho_{CS}^{5/3} \qquad (2\text{-}18)$$

In the second , the wave number K is calculated by using the energy functional derived by Vautherin and Brink [9] for the Skyrme interaction. This leads to the following expression for K [4] (V-B approach hereafter).

$$K^2(R) = 0.0482\, \epsilon_{inc} + 2.4117\, \rho_{CS}^{2/3} + 20.4149\, \rho_{CS} - 42.2013\, \rho_{CS}^{2} - 7.7422\, \rho_{CS}^{5/3} \qquad (2\text{-}19)$$

III. Application of the Model :

For the sake of simplicity, the model is used to calculate the absorptive potential for $\alpha - {}^{12}C$ and $\alpha - {}^{16}O$ systems.

Using the Gaussian form factor for the α -particle [10] and choosing the form factors for C-and O- nucleus to be consistent with an oscillator shell-model [11] lead to the following expression of the nuclear matter density distribution $\rho_{CS}(R)$ of the compound system.

$$\rho_{CS}(R) = \rho^{0}_{CS} \left[1 + \frac{\varsigma}{\lambda^2} R^2\right] \exp(-R^2/\lambda^2),$$

$$\rho^{0}_{CS} = \frac{(4+A)}{(\pi\lambda^2)^{3/2}(1+1.5\,\varsigma)}, \qquad (2\text{-}20)$$

$$\lambda^2 = a_\alpha^2 + a_A^2,$$

$$\varsigma = 4\,a_A^2/(9\lambda^2 - 6\,a_A^2)$$

Having determined $\rho_{CS}(R)$, the absorptive potential can be determined from Eqs. (2-11), (2-13), (2-16), (2-18) and (2-19). The results are shown in Figs. (1) and (2).

In Fig. (3) the present potentials for $\alpha - {}^{12}C$ scattering at E_α = 166 MeV are compared with that of Tatischeff and Brissud [12].

The energy dependence of the volume integral per nucleon pair J_W and the root means square radius of the present potentials are shown in Figs. (4) and (5).

IV. DISCUSSION AND CONCLUSIONS:

It is clear from Fig. (4) that J_w , although increasing with energy at first, tends to decrease slowly for high energies. Such behaviour is expected since in evaluating $\langle\sigma\rangle$, the cross section has been assumed to vary essentially as E^{-1} even at high energies. However, this result differs from that obtained from the phenomenological analysis which assumes linear energy dependence [13]. It can also be seen from Fig. (5) that the root mean square radius $R_{r.m.s}$, while decreases rapidly with energy at first, tends to saturate to constant value for high energies. A similar result has been obtained by Vinh Mau [14] for $\alpha - {}^{40}Ca$ system. Furthermor, the discrepancy between the values of J_W and $R_{r.m.s.}$ obtained for the three different approaches at lower disappears at higher ones.

In conclusion, while the present model for α-nucleus absorptive potential is free from any adjustable parameters, we regard it in view of the assumptions made only as a step towards a more exact microscopic description.

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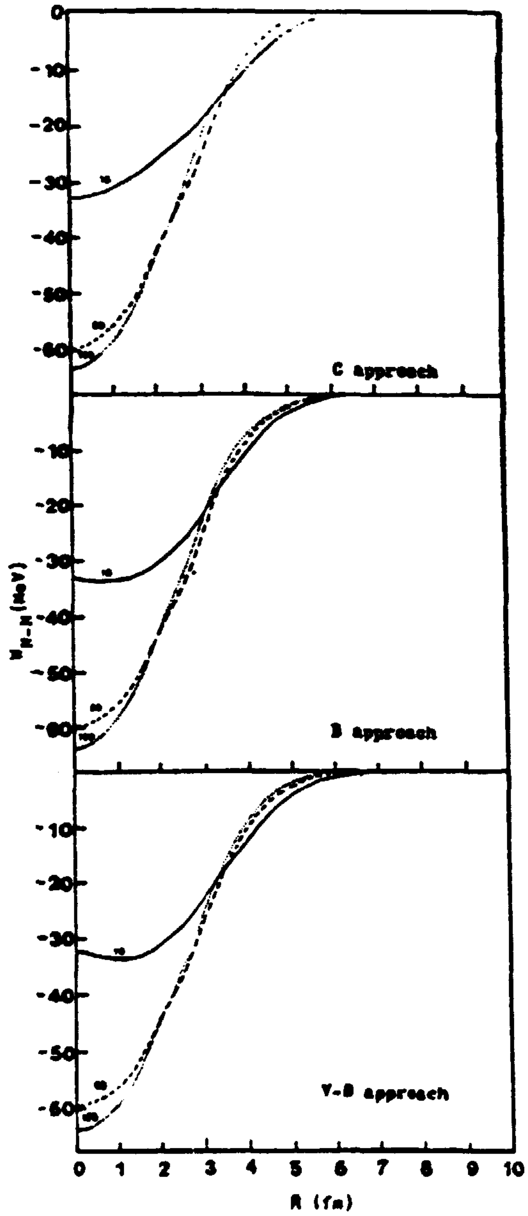

Fig. 1 - The α*-*^{12}C *imaginary potential (the numbers on the curves denote the incident energy per nucleon).*

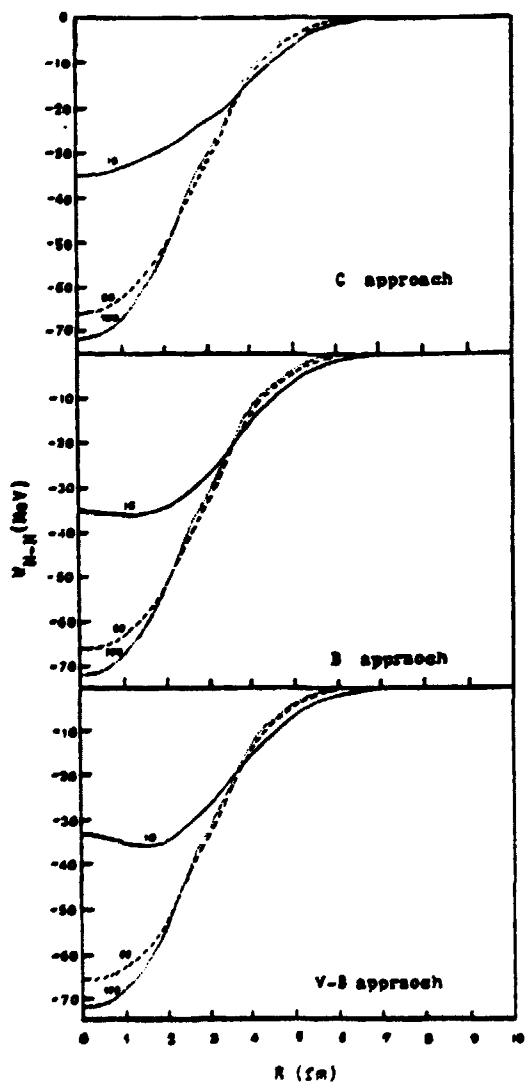

Fig. 2 - The α*-*^{16}O *imaginary potential (the number on the curves denote the incident energy per nucleon)*

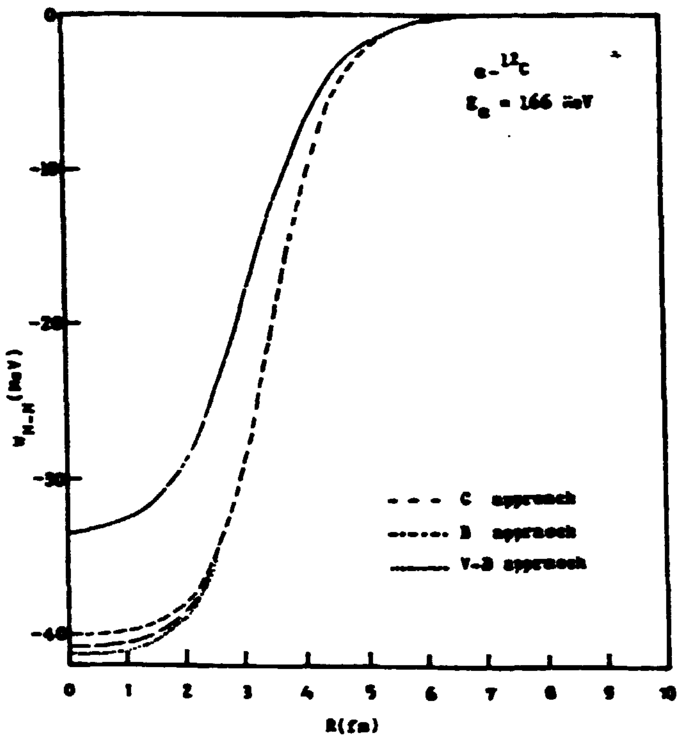

Fig. 3 - The TB equivalent WS potential (solid) compared with those of the present model.

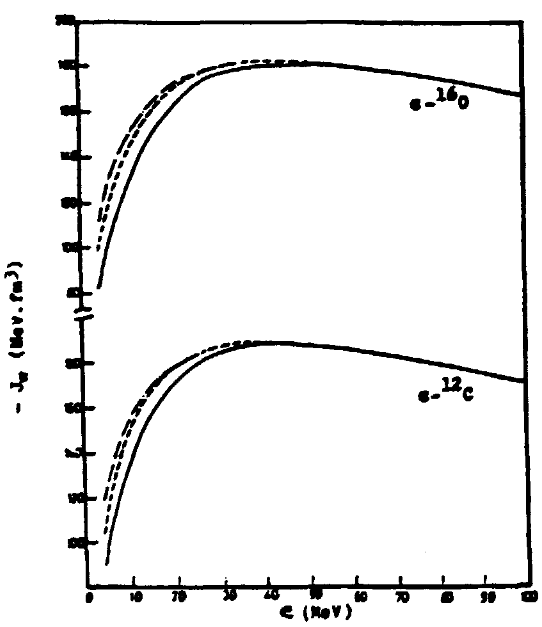

Fig. 4 - Dependence on the incident energy per nucleon of the volume integral per nucleon pair of the present imaginary potential (--C approach, .-.-B approach, ---V-V approach).

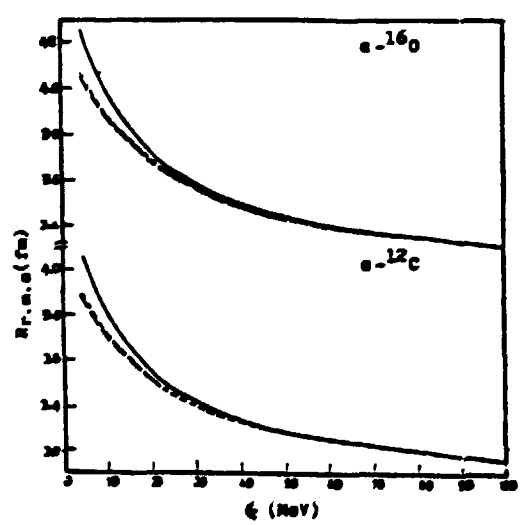

Fig. 5 - Energy dependence of the root mean square radius of the present imaginary potential (—C approach, ---B approach, .-.-V-B approach).