RELAXATION IN FINITE FERMION SYSTEMS

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Abstract: The derivation of a collision term extending time-dependent mean-field theories to describe the equilibration in finite fermion systems due to the residual interaction is discussed. Numerical results based on a relaxation ansatz for the collision term exhibit its qualitative effect. The equation for the time-dependent occupation numbers of the s.p. orbits is reduced to a non-linear partial differential equation which is solved analytically. In the equilibrium limit, a Fermi-type distribution for the occupation numbers is attained.

To include both one-body and two-body dissipation in nuclear collisions in a microscopic dynamical model, TDHF calculations have recently been supplemented by phenomenological collision terms[1,2]). On the other hand, microscopic theories have been developed to derive collision terms which are due to the incoherent effect of the residual force[3]). We give a survey of the formulation presented in [3]). A random-matrix model has been used to account for the irreversible behaviour due to nucleon-nucleon collisions. Carrying out an ensemble-average we have derived the time-irreversible equation of motion for the average A-body density matrix $\overline{\rho^{(A)}}(\vec{x}_1,\ldots\vec{x}_A;\vec{x}_1',\ldots\vec{x}_A';t)$.

The reduction to the equation of motion for the average one-body density matrix is done by tracing over (A-1) variables. The elements of $\overline{\rho^{(1)}}$ in a diabatic basis of time-dependent single-particle states[3]) are governed by

$$i\dot{\overline{\rho}}^{(1)}_{\mu\nu} = \left[h_{HF}(\overline{\rho^{(1)}}), \overline{\rho^{(1)}} \right]_{\mu\nu} + i C_{\mu\nu}(\overline{\rho^{(2)}}, \overline{\rho^{(3)}}). \qquad (1)$$

The mean field-part is reminiscent of the Hartree-Fock equation. The collision term contains gain and loss contributions. It is still a function of the average two- and three-body densities. Upon factorization into anti-symmetrized products of $\overline{\rho^{(1)}}$, the diagonal elements of the occupation number matrix obey

$$\frac{\partial}{\partial t} n_\mu = \sum_{\alpha\beta\gamma} \overline{V^2}_{\mu\gamma\alpha\beta} \cdot G(\varepsilon_\mu + \varepsilon_\gamma, \varepsilon_\alpha + \varepsilon_\beta) \times$$

$$\times \left[(1-n_\mu)(1-n_\gamma) n_\alpha n_\beta - (1-n_\alpha)(1-n_\beta) n_\mu n_\gamma\right]. \quad (2)$$

Whereas TDHF corresponds to occupation numbers $n_\mu = 1$ or 0, the collision term(2) describes the equlibration due to the action of the residual force. In a finite system, the energy conserving function $G(\varepsilon_\mu + \varepsilon_\gamma, \varepsilon_\alpha + \varepsilon_\beta)$ is not a δ-function as in the Boltzmann equation but it acquires a width of several MeV such that collisions between nukleons in s.p. levels lying far apart become possible [3].

To obtain a transparent treatment of the equilibration process in the s.p. occupation numbers we have transformed [4] eq.(2) into a nonlinear partial differential equation for $n \equiv n(\varepsilon_\mu, t)$

$$\frac{\partial}{\partial t} n = -\frac{\partial}{\partial \varepsilon}\left[v n(1-n) + n^2 \frac{\partial D}{\partial \varepsilon}\right] + \frac{\partial^2}{\partial \varepsilon^2}\left[D n\right]. \quad (3)$$

The transport coefficients v,D determine the speed of the relaxation process as well as the diffuseness of the equilibrium distribution. In the limit of constant v,D we have solved (3) analytically through a nonlinear transformation. The equilibrium distribution is of the Fermi type $n_{\infty}(\varepsilon) = \left[1 + \exp\left[-v(\varepsilon - \varepsilon_F)/D\right]\right]^{-1}$ where the temperature has been replaced by - D/v. The equilibration time is derived as $\tau_{equ} = 4D/v^2$ which is $\propto 1/\overline{V^2}$, $\overline{V^2}$ the second moment of the residual force. An example for the analytical results is shown in Fig. 1.

The mean field-equations for the Wigner transform f(x,k) of the one-body density have been solved in [1] with the phenomenological collision term based on a relaxation ansatz. Typical results are shown in Fig. 2. It will be of interest to replace the ansatz for the collision term by the microscopic form derived above in order to obtain a consistent picture of the equilibration process[5].

References:

[1] L. Richert, D.M. Brink, and H.A. Weidenmüller, Phys. Lett. 87B (1979) 6
P. Grangé, J. Richert, G. Wolschin, and H.A.Weidenmüller, Nucl. Phys. A356 (1981) 260

[2] H.S. Köhler, Nucl. Phys. A343 (1980) 315

3) P.Grangé, H.A. Weidenmüller, and G. Wolschin, Ann. Phys. (N.Y.) 136 (1981) 190 and references contained therein

4) G. Wolschin, Proc. XIV. Masurian Summer School in Physics, Mikołajki, Poland, Sept. 1981 and to be published

5) P. Grangé and G. Wolschin, in progress.

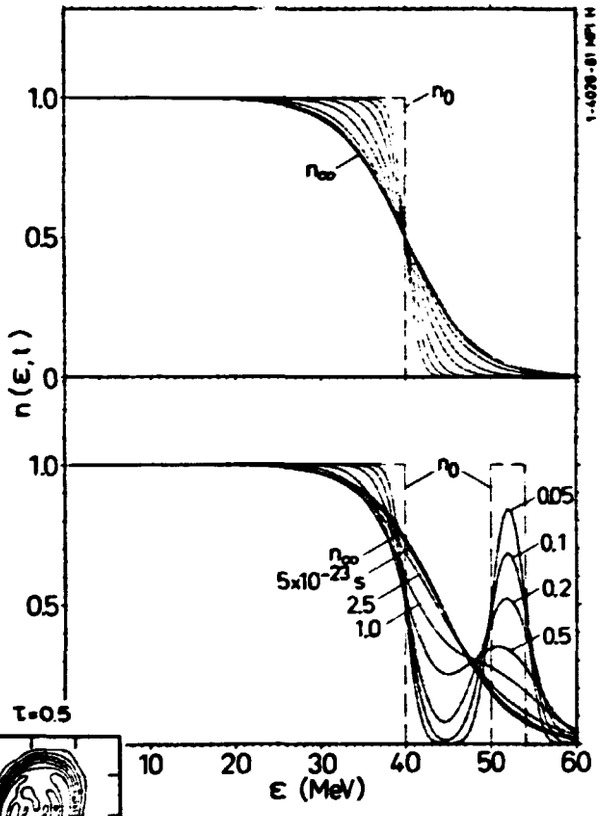

Fig.1.:Analytical solutions of the non-linear differential equation for the occupation-number distribution.

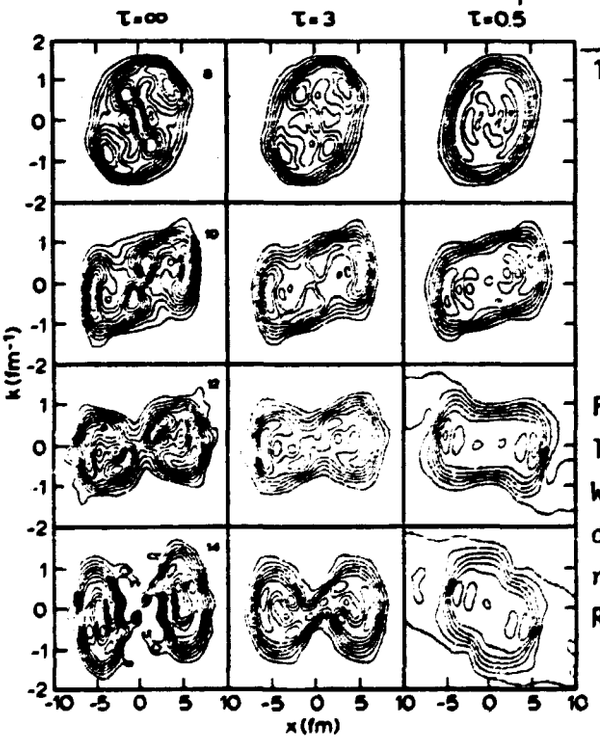

Fig.2.:Effect of a phenomenological collision term on the Wigner-transform of the one-body density matrix for different relaxation times τ_{rel} (10fm/c), Ref.1.