# THE INERTIA PARAMETER OF NUCLEAR COLLECTIVE EXCITATIONS

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In this contribution we will derive the collective mass parameter by using the generator coordinate method (GCM) and establish the connection of the result obtained by GCM to that derived within the cranking model (CM).

Suppose $|\phi_0\rangle$ is the ground state in a single particle mean field with particles filled up to Fermi energy, P is the collective generator operator and q is the corresponding variable. The state with collective variable q can then be expressed by

$$|\phi_0(q)\rangle = \exp\{\tfrac{i}{\hbar} qP\}|\phi_0\rangle. \qquad (1)$$

The subspace for low lying collective motions of q can be spanned by

$$|\psi\rangle = \int f(q)\, \exp\{\tfrac{i}{\hbar} qP\}|\phi_0\rangle dq. \qquad (2)$$

Solving the nuclear many-body problem in the above subspace by GCM[1], we obtain the effective Hamiltonian of the collective motions in the quantum mechanical version

$$H_{eff} = \langle\phi_0|H(q)|\phi_0\rangle_L + \frac{1}{2B(q)}\hat{p}^2 + \ldots \qquad (3)$$

Here

$$\hat{p} = -i\hbar\frac{\partial}{\partial q}\,, \qquad (4)$$

$$H(q) = \exp\{-\tfrac{i}{\hbar} qP\}H \exp\{\tfrac{i}{\hbar} qP\}\,, \qquad (5)$$

$$1/2B(q) = \langle\phi_0|PH(q)P + \tfrac{1}{2}(P^2H(q) + H(q)P^2)|\phi_0\rangle_L/4P_0^4\,, \qquad (6)$$

$$P_0^2 = \langle\phi_0|P^2|\phi_0\rangle. \qquad (7)$$

The subscript L in $\langle\ \rangle_L$ implies that only those terms linked with H(q) are included.

If the residual interactions in H(q) are neglected, it follows

$$B(q) = 2\cdot\frac{\left[\sum_n|\langle\phi_n(q)|\hat{P}|\phi_0(q)\rangle|^2\right]^2}{\sum_n(E_n-E_0)|\langle\phi_n(q)|\hat{P}|\phi_0(q)\rangle|^2} \qquad (8)$$

Here $\phi_n(q)$ and $E_n(q)$ are eigenfunctions and eigenvalues of $H_0(q)$, respectively.

We define an operator Q conjugate to P by

$$[Q,P] = i\hbar \qquad (9)$$

If the intermediate excited states are restricted to 1p-1h states, then

$$\langle\phi_n(q)|Q|\phi_0(q)\rangle = -i\hbar\,\langle\phi_n(q)|P|\phi_0(q)\rangle/2P_0^2 \qquad (10)$$

and eq. (6) can be cast into

$$1/B(q) = \langle\phi_0(q)|[[H,\tfrac{iQ}{\hbar}],\tfrac{iQ}{\hbar}]|\phi_0(q)\rangle_L \qquad (11)$$

This result is in agreement with that of Villars[2] obtained from ATDHF.

The effective operators of Q and P are

$$Q \rightarrow q \qquad (12)$$

$$P \rightarrow \hat{p} = -i\hbar\frac{\partial}{\partial q} \qquad (13)$$

and satisfy

$$[q,\hat{p}] = i\hbar \qquad (14)$$

From eqs. (3), (12) and (13) we have

$$\hat{p} = [H_{eff}, \frac{iB(q)q}{\hbar}] \qquad (15)$$

The corresponding relation is

$$\langle\phi_n(q)|P|\phi_0(q)\rangle = \langle\phi_n(q)|[H, \frac{iB(q)Q}{\hbar}]|\phi_0(q)\rangle. \qquad (16)$$

If the residual interactions are neglected, we obtain from eqs. (10), (11) and (16)

$$B(q) = 2\cdot\sum_{n\neq 0}\frac{|\langle\phi_n(q)|\hat{P}|\phi_0(q)\rangle|^2}{E_n(q) - E_0(q)} \qquad (17)$$

which is exactly the cranking formula[3].

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## References

1) Hsu Kung-ngou, Scientia Sinica (China), 17 (1974) 639.
Xu Gong-ou, Yang Ya-tian and Wang Shun-jin, Scientia Sinica (China), 24 (1981) 1085.
Xu Gong-ou and Wang Shun-jin, Nucl. Phys. (to be published)
2) F. Villars, Nucl. Phys., A285 (1977) 269
3) D.R. Inglis, Phys. Rev. 96 (1954) 1059.

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