ON THE DYNAMICAL SINGLE-PARTICLE BASIS IN LARGE AMPLITUDE COLLECTIVE MOTIONS

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We aim at describing large amplitude collective nuclear motion in a dynamical single-particle basis which is the most reasonable approximation to the time-dependent single-particle Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha = h(x,q)\,\psi_\alpha. \qquad (1)$$

Eq. (1) usually is solved within the adiabatic representation $|\phi_n\rangle$

$$h_0(x,q)\;|\phi_n(x,q)\rangle = e_n(q)\;|\phi_n(x,q)\rangle \qquad (2)$$

which, however, is only applicable for small collective velocities $\dot q$ and if no quasicrossings of the adiabatic levels $e_n(q)$ occur.[1] For somewhat larger collective velocities $\dot q$, i.e. within the range of collective kinetic energy per nucleon of

$$\frac{1}{40}\,\mathrm{MeV} \le E_{coll}/A \ll 40\,\mathrm{MeV}, \qquad (3)$$

a diabatic basis[2] is more physical because in the vicinity of quasicrossings it includes jump probabilities close to 1 in a nonperturbative way. Assuming $q(t)$ to be a given function of time, eq. (1) can be cast into the form

$$[h_0(x,q) - i\hbar\dot q\tfrac{\partial}{\partial q}]|\tilde\psi_\alpha(q)\rangle = 0, \qquad (4)$$

which allows further interesting approximations with respect to a static basis.

For small $\dot q$ an optimal representation of $\tilde\psi_\alpha(q)$ is given by

$$|\tilde\psi_\alpha\rangle = \sum_n C_{\alpha n}|\phi_n\rangle, \qquad (5)$$

where the coefficients $C_{\alpha n}$ are determined by the eigenvalue equation

$$\sum_n \langle\phi_m|h_0 - i\hbar\dot q\tfrac{\partial}{\partial q}\,|\phi_n\rangle\, C_{\alpha n} = E_\alpha C_{\alpha m} \qquad (6)$$

and meet the limiting conditions

$$\lim_{\dot q\to 0} C_{\alpha m} = \delta_{\alpha m} \qquad (7a)$$

$$\lim_{\dot q\to 0}\frac{\partial C_{\alpha m}}{\partial q} = \lim_{\dot q\to 0}\{\frac{\partial a_{\alpha m}}{\partial q}\dot q + O(\dot q^2)\} = 0 \qquad (7b)$$

with $a_{\alpha m} = \frac{\partial C_{\alpha m}}{\partial \dot q}$.

Returning to the initial value problem (1) and expanding

$$|\psi_\alpha\rangle = \sum_\beta d_{\alpha\beta}|\tilde\psi_\beta\rangle\; e^{-i/\hbar\int_0^t E_\beta(t')dt'} \qquad (8)$$

we obtain

$$\dot d_{\alpha\beta} = -\dot q^2\sum_\gamma d_{\alpha\gamma}\,(\sum_m C_{\beta m}\frac{\partial a_{\gamma m}}{\partial q})\; e^{-i/\hbar\int_0^t(E_\gamma - E_\beta)dt'}, \qquad (9)$$

which is an improvement of the cranking model (CM) because the time evolution of the expansion coefficients is proportional to $\dot q^2$ (instead of $\dot q$ in the CM).

For collective velocities within the range (3) the adiabatic s.p. wavefunctions $\phi_n(q)$ in eqs. (5,6) should be replaced by the diabatic s.p. wavefunctions $\chi_n(q)$[2] and the s.p. Hamiltonian h_0 by a diabatic Hamiltonian $h = h_0 + h'$ in order to account for jump probabilities close to 1 in the neighborhood of quasicrossings. Eq. (9) again holds in this case except an extra term proportional to matrix elements of h'.

For the collective degree of quadrupole deformation eq. (6) can be solved analytically in the deformed harmonic oscillator model[3], which is well suited for a study of diabatic motion because $\dot h$=0 in this case, i.e.

$$h_0 - i\hbar\,\dot q\,\frac{\partial}{\partial q} = \sum_{i=x,y,z}\hbar\Omega_i\,(b_i^+ b_i^+ + \tfrac{1}{2}) \qquad (10)$$

with

$$\Omega_i = \omega_i\sqrt{1-\xi_i^2} \qquad (i = x,y,z) \qquad (11)$$

$$\xi_i^2 = (\frac{K_i\dot q}{2\omega_i})^2,\quad K_x = K_y = \frac{1}{2q},\quad K_z = -\frac{1}{q},$$

$$q = \frac{\omega_0}{\omega_z}$$

and

$$\tilde\phi_n = N_n\, e^{-(\alpha_x^2x^2+\alpha_y^2y^2+\alpha_z^2z^2)}\cdot e^{-i\frac{m}{\hbar}\Phi(x,y,z;\dot q)}$$

$$\cdot\, \overline{H}_n(x,y,z) \qquad (12)$$

with $\alpha_i^2 = \frac{m\Omega_i}{\hbar}$.

In eq. (12) $\overline{H}_n$ is a polynomial of order n with complex coefficients and does not contain significant phases as a function of $\dot q$, $\dot q^2$..... The essential phase is given by the velocity potential

$$\Phi(x,y,z) = \frac{\dot q}{8q}\,(x^2+y^2-2z^2) \qquad (13)$$

and implies incompressible irrotational flow. The mass parameter for quadrupole motion

$$B(q,\dot q) = \frac{\hbar}{4}\sum_{i=x,y,z}\frac{K_i^2 N_i}{\Omega_i} \qquad (14)$$

with

$$N_i = \sum_{j=1}^{A}\langle\,|b_i^+b_i^+ + \tfrac{1}{2}|\,\rangle_j \qquad (15)$$

reduces to the cranking formula for $\dot q$ = 0. For large velocity $\dot q$ and extreme oblate deformation (small q) the quantities ξ_i^2 become comparable to 1 and $B(q,\dot q)$ increases significantly. The variations of $B(q,\dot q)$ with respect to q and $\dot q$ in the diabatic approximation (i.e. $N_i \neq N_i(q,\dot q)$) are demonstrated in Fig. 1 via the ratio of $B(q,\dot q)$ with the cranking mass parameter B_{CM}.

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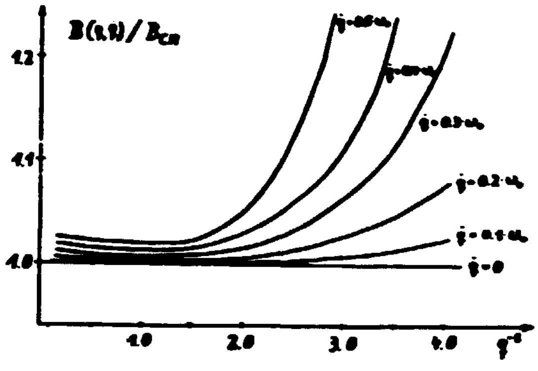

Fig. 1

For reasonable deformations $0.4 \leq 1/q = \omega_z/\omega_0 \leq 1.5$ the deviation between $B(q,\dot{q})$ and the cranking mass is less than 3% up to collective velocities $\dot{q} \cdot R$ which are half of the typical s.p. velocity $\omega_0 \cdot R$ (R = nuclear radius). This demonstrates that the cranking mass with respect to diabatic s.p. motion does not cause problems in case of nuclear deformation.

References:

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2) A. Lukasiak, W. Cassing and W. Nörenberg, Microscopic Calculation of Diabatic Single-Particle Levels, Proceedings of the X International Workshop on Gross Properties of Nuclei and Nuclear Reactions, Hirschegg, Jan. 1982
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