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TRANSVERSE MAGNETORESISTANCE AND HALL RESISTIVITY IN Cd AND ITS DILUTE ALLOYS †

O.P. Katyal * and K.M. Khanna **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

Galvanomagnetic properties of Cd and its dilute alloys have been studied experimentally for the past several years. A number of authors have attempted to explain them, using the concept of inter-sheet scattering. None of the explanations has so far been very successful in explaining the experimental observations.

In this note we have emphasized the requirement of a theory that could more successfully explain the experimental observations and have suggested the nature of interactions that may ultimately unfold the phenomena.

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* Permanent address: Department of Physics, Indian Institute of Technology, Kanpur, India.

** Permanent address: Department of Materials Science, National Institute of Foundry and Forge Technology, Ranchi, India.

I. INTRODUCTION

Galvanomagnetic properties of Cd and its dilute alloys have been studied 1)-12) for the past several years. The negative Hall resistivity ρ_{21} in the basal plane of Cd has been explained on the basis of intersheet scattering 4),5) between the two adjacent hole Fermi surface pieces in the second Brillioun Zone (BZ). Hurd et al. 11) suggested that in the low field and intermediate-field regions a hole surface could support orbital segments of both electron-like and hole-like behaviour and the Hall effect reflects the balance between their contribution when summed over all planes perpendicular to the magnetic field $\vec{H}$. Recently Fletcher 12) has doubted the relevance of inter-sheet scattering theory 5) because the transverse resistivity of Cd was shown to be precisely quadratic in magnetic field at temperature of 1.2 °K.

The following features of $\rho_{21}(H,T)$ and transverse magnetoresistance $\rho_{11}(H,T)$ have been observed experimentally 2),7)-9),11),12).

(i) ρ_{21} is negative below 3.4 °K for a maximum magnetic field of 3.5 T.
(ii) ρ_{21} increases with H and does not decrease in the magnetic field at around 1.2 T as was observed by Grenier et al. 15). It appears it is not a general phenomena connected with Cd metal.
(iii) For a fixed value of H, ρ_{21}, shows a maximum at around 10 °K.
(iv) Though the general behaviour of $\rho_{21}(H,T)$ observed by various groups 2),7),11)12) is the same, the absolute value of $\rho_{21}(H,T)$ is different in each experiment. The purity of Cd samples are different in each case, and Kohler's rule does not apply here (See Fig.7a of Ref.2).
(v) At 1.35 °K under hydrostatic pressure of 1950 Atms. $[\rho_{21}(H)]$ becomes less negative as compared to $[\rho_{21}(H)]_{p=0}$ (See Fig.8 of Ref.2).
(vi) ρ_{11} is proportional to H^2 in the temperature range 1.4 to 4.2 °K.
(vii) The Kohler plot of $\frac{\Delta\rho}{\rho}$ vs $\omega\tau$, shows a change pf slope for $\omega\tau \simeq 1$. (See Fig.2 of Ref.3).

Using an idealized Fermi surface, the influence of small angle scattering between different sheets of Fermi surface on the galvanometric properties of metals was calculated 5). The effects of the small angle scattering were included explicitly in the solution of the Bolzmann equation. The whole discussion was semiclassical and the motion of the electron was considered to be that of a quasi-particle with a general dispersion law $\in$ (R) and which satisfied Fermi-Dirac statistics.

Effort was also made to explain the results for the galvanomagnetic properties of cadmium 1),2),16) and Cd-Zn alloys 2) which display several anomalous features which are quite sensitive to the field strength, impurity concentration and variations of temperature. It is pointed out that Hall resistivity ρ_{21} exhibits electronlike behaviour for very low temperatures ($T \simeq 2$ °K), but displays a hole-dominated character for slight larger temperatures ($T \simeq 4$ °K). For Zn-Cd alloys the resistivity changes sign as a function of temperature in a manner similar to the case of pure Cd, but for low temperatures, $T = 1$-4 °K, ρ_{21} is positive for low magnetic fields and then becomes negative, i.e., electron-like for higher fields.

In the calculations done, the basic concept was the so-called "intersheet scattering", and C and Δ (see Ref.5 definition of C and Δ) were so adjusted that $\Upsilon^{-1} = C\Delta^2$ remained finite. It was shown that the small-angle scattering effects can provide an explanation for the deviation from the expected behaviour of the magnetoresistance. Also the behaviour of the Hall resistivity ρ_{21} showed the transition from electron-like to hole-like behaviour as the magnetic field was increased. On a qualitative basis, various results for ρ_{21} corresponding to different values of τ (relaxation time) and Υ (the typical scattering time) are given in detail in Ref.1. The results emphasize the following points:

(i) Impurity scattering is the principal mechanism which gives rise to intersheet scattering.

(ii) There is a possibility of phonon scattering and particularly near the hot-spot regions a phonon with very small wave vector $q(|q| \ll k_F)$ can scatter holes in such a way as to randomize the formerly conserved velocity component and thus weaken the coherence properties of the channel. This weakening of the coherence of the interchannel scattering is reflected in a decrease of Υ^{-1} as the temperature is increased. However, it is mentioned in Ref.1 that a detailed study will require the study of electron-phonon interaction in Cd.

It is well known 10) that the galvanomagnetic properties of Cd at low temperatures are peculiar and complicated and this is due to the fact that different scattering mechanisms dominate the galvanomagnetic properties at different temperatures. The behaviour below 4 °K is said to be well understood 5) in terms of impurity-enhanced intersheet scattering. It is believed that at higher temperatures, other scattering mechanisms take over and various local maxima and minima in the Hall coefficient are observed.

The important concept introduced in Ref.10 to explain various experimental results is the scattering by a very flat, almost Einstein like, phonon mode. It is assumed that such a mode is capable of scattering quasi elastically electrons from any points in the Brillouin zone to any other point. Particularly, it can scatter electrons from anywhere in the Fermi surface to any other Fermi-surface point. The energy of the Einstein-like phonon mode, is very small compared with typical electronic energies (in the order of Fermi energy). Yet, it is assumed that it can supply a very large momentum transfer. Consequently, it is not only effective in causing further interelectronic and interhole scattering but also capable of scattering electrons into hole pockets and vice versa, as well as causing transitions between different hole pockets.

The matrix elements for three processes, $C_{\text{intra-e}}$, $C_{\text{intra-h}}$, $C_{\text{e-h}}$ and $C_{\text{inter-h}}$ are, in general, different. In the calculations presented in Ref.10, these matrix elements have been assumed to be equal and constant $= C_E$.

The following are the results, achievements and shortcomings of the calculations given in Ref.10.

(i) Some experimental results can be understood qualitatively and quantitatively by the inclusion of anisotropic scattering of electrons by a very flat, almost Einstein-like phonon mode, a hybrid of TA and TO branches.

(ii) Within the isotropic scattering model, the combined effect of Debye and Einstein phonons is to cause a sharp rise in the Hall coefficient in the neighbourhood of 10-20 °K with a line shape which compares favourably to the experimental one. Bloch T^5 terms alone caused too slow a rise and the Einstein term alone drops too steeply. If the impurity term is too large so as to be dominant the rise is again very slow; in such a case, the impurity contributions dominate both in numerator and denominator and tend to maintain the constant value,

$$C_e' - C_h' \, [\, nec(c_e' + c_h')]^{-1} ,$$

where, $C_e' = m_e c_e$

$C_h' = m_h c_h$

C is a constant independent of T. C_e and C_h are constant impurity terms and these are adjustable parameters.

The fully isotropic model, however, cannot explain the drop in the Hall coefficient beyond 20 °K. The calculation yields a constant

(ii) plateau in that range, in disagreement with the experimental results.

(iii) It is suggested that the phenomena of a local maximum in the Hall coefficient can be sufficiently accounted for if anisotropic scattering is properly incorporated. In fact, the experiments provide a direct measure of such anisotropy.

(iv) $R(T)/r(T)$ as plotted against $100/T$ in a semilog scale also appears to be linear in the temperature range 4-40 ^{o}K for both the isotropic and anisotropic scattering models. A plot of $1/_{r(T)}$ alone deviates considerably from the straight line but not as drastically as observed experimentally. For the values A', B', C'_e and C'_h that have been used, the same linear behaviour is observed and the slope varies from 42 to 48 ^{o}K, which compares favourably to the one observed at 43 ^{o}K.

$R(T)/r(T) = be^{-\frac{42.9}{T}}$; where $R(T)$ is the deduced Hall coefficient at temperature T, b is a constant independent of temperature and $r(T) = \rho_{11}(H=0, 273\ ^{o}K)/\rho_{11}(H=0,T)$.

(v) The observed deviation of the zero-field resistivity from that of a simple Bloch model is due to (a) the existence of both electron and hole pockets of different sizes which have very different scattering parameters, (b) the existence of a term due to Einstein-like phonon mode, and (c) the very small value of the mass ratio m^*_h/m^*_e. All these factors combine to cause an apparent fit to a single Bloch I_5 function with scaling parameters approximately equal to $\frac{1}{2}\ \Theta_D$ as observed experimentally.

(vi) It is suggested that instead of using a more realistic treatment of the Fermi surface, it is more sensible to use variational technique.

II. PROPOSED QUALITATIVE EXPLANATION OF EXPERIMENTAL RESULTS

We shall consider the case where the magnetic field H is along the hexagonal axis and the electric current is in the basal plane. The resistivity tensor ρ has the following form for H along the hexagonal-axis taken the Z-axis,

$$\rho = \begin{vmatrix} \rho_{11} & \rho_{21} & 0 \\ \rho_{21} & \rho_{11} & 0 \\ 0 & 0 & \rho_{33} \end{vmatrix}$$

where $\rho_{33} = \rho_0$ is the zero-field resistivity. For a compensated metal $(n_e = n_h)$ like Cd and those with all closed orbits as in the case of Cd, it is expected that ρ_{11} and ρ_{21} vary as H^2 and H, respectively. The Hall resistivity ρ_{21} is expected to be hole-like because,

(i) The effective mass 14),15) associated with the hole-surface is much lower than that for the electron ($m_e \cong 1.2\ m_0$ and $m_h \cong 0.17\ m_0$ where m_0 is the rest mass of electron).

(ii) The velocity vectors 11) of electrons on the lens of the third BZ centred upon Γ are directed principally along the C-axis, therefore not a major contribution to the conduction in the basal plane. The major contribution to ρ_{21} will come from the second-zone hole monster 13) which consists of the six separate pieces, and the hole pockets in the first-zone. (See Fig.7.5 of Ref.13.)

In this note, we have given qualitative arguments and reviewed likely quantitative approaches resistivity and transverse magnetoresistance of Cd by considering the electron-phonon scattering. The negative part of the Hall resistivity observed below 3.5 ^{o}K does not show change in slope for high magnetic fields as expected by the inter-sheet scattering 1) of electrons between the two adjacent pieces of Fermi surface in the second zone caused by electron-impurity scattering. Fletcher 12) measured the Hall resistivity in a sample of Cd which has a purity of r(T=1.4 K) equal to 91000 ± 5 %, and this purity is much higher compared to the one used by Katyal and Geritsen 2). In spite of this, the curves ρ_{21} against H show a similar behaviour up to the magnetic field of 3.5 T. This means that the contribution of the electron-impurity scattering to the Hall resistivity is negligible.

It has been observed recently 17) that at temperatures $T < 3.9$ ^{o}K, the oscillations of the hole doppleron were increased by a factor of 40 as the temperature was reduced from 3.6 to 1.6 ^{o}K. This gives an indication of the existence of strong electron-phonon interaction in the collisions.

Recent observations 18) on resistivity ρ_0 against T have strongly indicated the existence of an Einstein type of phonon mode in Cd. This mode with frequency $\omega \simeq 0.8 \times 10^{13}$ radians Sec^{-1} in the K-M direction in BZ has been observed experimentally 19). The inter-sheet scattering between the two adjacent pieces of the Fermi surface will also take place in the K-M direction.

We come to the conclusion that a successful explanation which may ultimately explain various experimental aspects, and that should be valid for all fields and temperatures may be found by considering the following interactions:

(i) Electron-phonon interaction.

(ii) Interaction between electrons near the Fermi surface.

Introducing these interactions we shall build the requisite theory which will be published later.

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