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ABSTRACT

We propose a physical picture and a non-perturbative definition of confinement in lattice gauge theories with matter fields. We argue that in the thermodynamical limit the fractional (baryon) charges become unobservable in the confinement phase for all local measurements. As an illustration we have tested the criterion in three of the possible phases of the Z(N) matter-gauge system by means of perturbation theory. We have found that the usual confinement and Higgs phases show confinement, while the free phase contains fractionally charged states contributing to local physical quantitites.

АННОТАЦИЯ

Предложены физическая картина и непертурбативное определение невылетания кварков в рамках калибровочных теорий на решетке, взаимодействующих с полями веществa. Показано, что в термодинамическом пределе дробные /барионные/ заряды становятся ненаблюдаемыми в замыкающей фазе для всех локальных измерений. Для примера наш критерий проверяется на трех из возможных фаз Z(N) системы вещество-калибровочное поле методом теории возмущений. Найдено, что обычные фазы невылетания и фаза Хиггса ведут к невылетанию, а свободная фаза содержит состояния с дробными зарядами, влияющие на локальные физические величины.

KIVONAT

A kvarkbezárás egy nemperturbativ definicióját javasoljuk olyan rács mértékelméleteknél, amelyekben anyagtér is jelen van. Megmutatjuk, hogy termodinamikai limeszben a tört (barion) töltés megfigyelhetetlenné válik lokális mérések számára, ha a bezáró fázisban vagyunk. A kritériumot a Z(N) gauge-spin modell három fázisában ellenőriztük perturbációszámitással. A confinement, Higgs és a szabad fázisokat megvizsgálva azt találtuk, hogy a tört töltés csak a szabad fázisban ad lokálisan is megfigyelhető járulékot.

I. INTRODUCTION

In pure gauge theories one can distinguish between confining and deconfining phases by measuring the expectation value of the Wilson-loop [1]. The Wilson-loop is an order parameter of the 4-dimensional statistical system and it is also related to a simple physical picture in terms of the 3+1 demensional quantum theory. Namely, the gauge field develops a straight flux tube between the external color sources producing a linear force law, as it was explained by Kogut and Susskind in the Hamiltonian formalism [2]. Unfortunately, if dynamical quark fields or any matter field is coupled to the gauge field, both the above-mentioned properties of the Wilson-loop are destroyed. The flux tube picture fails because of pair creation, and the Wilson-loop is not an order parameter any more: it behaves according to the perimeter law on both sides of the phase transition line. On the other hand the Wilson-loop still measures the force law between external color sources. However, the main problem is that we are interested in the confinement of dynamical quarks, not of the external ones, and the Wilson-loop doesn't yield a starting point to attack this problem.

The phase structure of several coupled matter-gauge systems is extensively studied in the literature. The phase transitions are usually revealed by thermodynamical considerations (singular behaviour in the correlation functions). However, if one wishes to answer to such questions as in what sense the 'confinement' phase is confining, it is unavoidable to bring in the aspect of quantum field theory. It was explained in ref. [3] that in perturbation theory the space of states of the confinement-Higgs and the free phases of Abelian matter-gauge systems are very different: the latter contains fractionally charged states while the former doesn't. That seems to be a paradox from the nonperturbative point of view because the Hilbert space of states must be the same for all values of the coupling constants at least for finite volumes. So one needs to understand what happens in the Hilbert space while taking the thermodynamical limit.

The aim of this paper is to investigate the qualitative and quantitative behaviour of the thermodynamical limit of both confining and deconfining matter-gauge systems. In this way we shall be able to define non-perturbatively what the confinement means in the presence of dynamical quarks. We restrict ourselves to lattice theories with fixed lattice spacing and will not study the possibility that the infinite volume system with finite ultraviolet cutoff still contains fractional barion charges but a certain continuum limit kills them.

Though it is the space of states where one can formulate the physical requirements of confinement, from the computational point of view, it would be very convenient to use a 3+1 dimensional quantum field theory which has a transcription to a 4-dimensional Euclidean statistical system. A few methods are known in the literature which construct such discrete time quantum field theories [4,5,6]. We will employ essentially the Osterwalder and Seiler's method, though a particular representation will be chosen for the Hilbert space instead of using the abstract factor space of [5].

In section 2 we outline the physical picture what will lead in sect. 3 to the formulation of a confinement criterion. In sect. 3 it will be proven that the confinement criterion generally holds in all orders of perturbation theory in the inverse coupling constant $1/g^2$ and in the hopping parameter K. In the remaining sections we shall apply our definition to the confining, Higgs and free-charge phases of the Z(N) matter-gauge system in perturbation theory.

2. THE PHYSICAL PICTURE

Consider a 3+1 dimensional quantum gauge theory with matter fields, defined in a finite box $\mathcal{V}$ of the three dimensional space. Let the Lagrangian have a global Abelian symmetry group $\mathcal{G}_B$ acting only on the matter fields. The generator of $\mathcal{G}_B$ is called baryon charge. The largest subgroup of $\mathcal{G}_B$ the action of which is the same as that of a certain subgroup of the global gauge group is denoted by $\mathcal{Z}$ and called the symmetry group of fractional (baryon) charges. In the physically interesting examples (QED, QCD) $\mathcal{Z}$ is the center of the global gauge group. The group $\mathcal{Z}$ has a crucial role in Mack's formulation of confinement [7]. After quantization our system may include states with integer baryon charge transforming trivially under $\mathcal{Z}$ as well as states with fractional charges. Mack speaks of confinement when the latter states don't exist.

Since we want the dynamics to choose between confinement and deconfinement, we have to allow our system to contain fractionally charged states in both cases. This can be brought into harmony with local gauge invariance even in a finite volume system if we demand the Gauss-law to operate only in points inside $\mathcal{V}$ and not on the boundary $\partial\mathcal{V}$. In this way we have a state as physical state which for instance consists of one quark at the point $\underset{\sim}{x} \in \mathcal{V}$ and of a string of color flux starting from $\underset{\sim}{x}$ and ending at $\underset{\sim}{y} \in \partial\mathcal{V}$. Thus the fractionally charged sector of the physical Hilbert space is not empty. But since our Hamiltonian commutes with the baryon charge, this means at the same time that there will be fractionally charged energy eigenstates too. Consequently the confinement can not be absolute as long as the volume is finite. However there must be some difference in the shape of the 1-quark eigenfunctions comparing the free and the confinement regimes what will lead to a qualitative change in the $V \equiv |\mathcal{V}| \to \infty$ limit.

If the fractional charge is free, then the one-particle eigenstates of the fractionally charged sector will look like standing waves. Their energy spectrum doesn't change essentially when $V \to \infty$. It only becomes more and more dense but remains bounded from above due to the ultraviolet cutoff. Therefore the energy expectation value $\langle W|\hat{H}|W\rangle$ in a wave packet state $|W\rangle$ of fractional baryon charge depends only on the size of the wave packet and goes to a finite limit when $V \to \infty$.

On the contrary, if there is confinement the occurence of a long string in any low energy eigenstate has a very small amplitude. Therefore the eigenstates in the fractionally charged sector resemble very much to an appropriate eigenstate of integer baryon charge in the middle of $\mathcal{V}$, while the 'valence quark' carrying the fractional charge can be found with large probability only in a layer of finite width around $\partial\mathcal{V}$. As we go to higher and higher energy eigenvalues this width becomes larger and larger. So the largest eigenvalue in the one-particle spectrum of the fractionally charged sector depends on the size of $\mathcal{V}$ and we expect an unbounded spectrum in the thermodynamical limit. Accordingly $\lim_{V\to\infty} \langle W|\hat{H}|W\rangle = \infty$ despite of the fixed size of the wave packed $|W\rangle$.

To use more definit formulae, instead of wave packets we can speak about the expectation values of local, gauge invariant operators $\hat{F}$ in the canonical ensemble defined on the states of baryon charge having a fixed fractional part. The picture described above involves that in the confinement phase the local quantity $\hat{F}$ feels always as if the charge were an integer, and doesn't feel the fractional charge concentrated at the boundary $\partial\mathcal{V}$ far away from F. In this sense can we say that the fractional baryon charge disappears from our system in the $V \to \infty$ limit. More precisely the difference between expectation values in the fractional and in the integer sector comes from the states which have energy going to infinity with some size of the volume $\mathcal{V}$.
Thus

$$\frac{1}{Z_{frac}} \mathrm{tr}\Big|_{frac} \hat{F}\, e^{-\hat{H}/T} - \frac{1}{Z_{int}} \mathrm{tr}\Big|_{int} \hat{F}\, e^{-\hat{H}/T} = \mathcal{O}(e^{-aV^{b}}) \qquad (2.1)$$

where a and b are positive numbers, T is the temperature, Z_{frac} and Z_{int} are the partition functions of a certain fractional and the integer sectors respectively. We see that no local observation can measure the fractional part of the baryon charge if the volume is infinite. Hence a class of states with baryon charge of different fractional part looks being the same, what points to that the unitary operators representing the symmetry group $\mathcal{Z}$ on the Hilbert space can not exist in the thermodynamical limit.

Let us now turn to the case what we would call the free phase. The low temperature expectation value of $\hat{F}$ computed in the sector where $\hat{B}=\frac{1}{3}+$(integer) (for QCD) is dominated by the lowest energy 1-quark state $|1\rangle$. The free propagation of quarks means also that they are momentum eigenstates in the infinite volume limit. So $|1\rangle$ is translation invariant. Because $\hat{F}$ is local, $\langle 1|\hat{F}|1\rangle$ differs from the vacuum expectation value $\langle 0|\hat{F}|0\rangle$ by a term of $\mathcal{O}(\frac{1}{V})$. Hence for low T at least

$$\frac{1}{Z_{frac}} \mathrm{tr}\Big|_{frac} \hat{F}\, e^{-\hat{H}/T} - \frac{1}{Z_{int}} \mathrm{tr}\Big|_{int} \hat{F}\, e^{-\hat{H}/T} \simeq \langle 1|F|1\rangle - \langle 0|F|0\rangle = \mathcal{O}(\tfrac{1}{V}) \tag{2.2}$$

Comparing (2.1) and (2.2) one sees that the local measurement registers the confinement as a very fine effect. As a matter of fact it has to make distinction between $\mathcal{O}(e^{-aV^{L}})$ and $\mathcal{O}(\frac{1}{V})$. The fact that the difference is so small can be explained in the language of statistical mechanics. We will see in sect. 3 that the expectation values in the fractional and in the integer sectors can be expressed by the same path integral if one disregards certain boundary terms. Theese boundary terms give zero contribution in the $V \to \infty$ limit. This is why the r. h. s. of (2.1) and (2.2) must go to zero in the thermodynamical limit. In other words the fractional and the integer sectors have the same bulk properties. One cannot differentiate between them by studying only infinite volume expectation values. So the question of confinement is reduced to the question how theese two types of expectation values approach their common thermodynamical limit.

3. FORMULATION OF THE CONFINEMENT CRITERION

Let $\mathcal{V}$ be a finite box of the 3-dimensional cubic lattice. Consider a gauge theory with the Wilson-action [8] whose fermion fields ψ and $\bar{\psi}$ are defined on the sites of $\mathcal{V}$ and whose gauge field U is defined on the links of $\mathcal{V}$. The (Euclidean) Lagrangian corresponding to the Wilson-action is the sum of the kinetic energy K containing the couplings between two neighbouring time slices and of the potential energy V containing the couplings in one time slice. In the $U_0 = 1$ gauge they are

$$L(x_0) = K(x_0, x_0+1) + V(x_0)$$

$$K(x_0,x_0+1) = -\frac{1}{g^2} \sum_{\underline{x}\in\mathcal{V}} \sum_{j=1}^{3} [\mathrm{tr} U_j^+(x_0,\underline{x}) U_j(x_0+1,\underline{x}) + \text{c.c.}] - \tag{3.1}$$

$$2K \sum_{\underline{x}\in\mathcal{V}} [\bar{\chi}(x_0,\underline{x})\chi(x_0+1,\underline{x}) + \bar{\theta}(x_0+1,\underline{x})\theta(x_0,\underline{x})]$$

$$V(x_o) = -\frac{1}{g^2} \sum_{\underset{\sim}{x}\in\mathcal{V}} \sum_{1\le j<k\le 3} [trU_jU_kU_j^+U_k^+ + c.c.] + \sum_{\underset{\sim}{x}\in\mathcal{V}} \bar{\psi}(x_o,\underset{\sim}{x})\psi(x_o,\underset{\sim}{x})$$

$$-2K \sum_{\underset{\sim}{x}\in\mathcal{V}} \sum_{j=1}^{3} [\bar{\psi}(x_o,\underset{\sim}{x})U_j(x_o,\underset{\sim}{x}) \frac{1-\gamma_j}{2} \psi(x_o,\underset{\sim}{x}+\hat{j})+\bar{\psi}(x_o,\underset{\sim}{x}+\hat{j})U_j^+(x_o,\underset{\sim}{x}) \frac{1+\gamma_j}{2}\psi(x_o,\underset{\sim}{x})]$$

where γ_o is the standard $\gamma_o = \mathrm{diag}(1,1,-1,-1)$ and we have introduced the upper and lower two-component spinor χ and ϑ of ψ. This is a classical (Euclidean) field theory with discrete time which can not be quantized in a natural way by specifying canonical commutation relations. However, following the method of [5] one gets the Hilbert space and the transfer matrix without using any canonical commutation relations. The Hilbert space $\mathcal{H}$ can be represented* by functionals ϕ which depend on the 3-dimensional lattice fields U, χ and $\bar{\vartheta}$. The scalar product on $\mathcal{H}$ is

$$\langle\phi_1|\phi_2\rangle = \int \mathcal{D}U_1\mathcal{D}U_2\mathcal{D}\bar{\chi}\mathcal{D}\chi\mathcal{D}\bar{\vartheta}\mathcal{D}\vartheta \quad \exp\{\frac{1}{g^2} \sum_{\underset{\sim}{x},j} [trU_{1j}^+(\underset{\sim}{x})U_{2j}(\underset{\sim}{x}) + c.c.]$$

$$+ 2K \sum_{\underset{\sim}{x}} [\bar{\chi}(x)\chi(x) + \bar{\vartheta}(x)\vartheta(x)]\} \overline{\phi_1(U_1,\chi,\bar{\vartheta})}\ \phi_2(U_2,\chi,\bar{\vartheta}) \qquad (3.2)$$

where $\overline{}$ is the antihomomorphysm on the Grassmann algebra generated by $\chi,\vartheta,\bar{\chi}$ and $\bar{\vartheta}$ (that is it is antilinear and reverses the order of multiplicands) which gives identity when squared and

$$\overline{\chi^{\alpha r}(\underset{\sim}{x})} = \bar{\chi}^{\alpha r}(\underset{\sim}{x}) \qquad (r = 1,2;\quad \alpha = \text{color index}) \qquad (3.3)$$

$$\overline{\bar{\vartheta}^{\alpha r}(\underset{\sim}{x})} = -\vartheta^{\alpha r}(\underset{\sim}{x})$$

The transfer matrix $\hat{T}$ which develops the states by one unit in the Euclidean time is given by the following integral operator:

$$\hat{T}\phi(U,\chi,\bar{\vartheta}) = \int \mathcal{D}U'\mathcal{D}\bar{\chi}\mathcal{D}\vartheta\mathcal{D}\chi'\mathcal{D}\bar{\vartheta}'\ e^{-V(U,\bar{\psi},\psi)}\ e^{-K(U,\bar{\chi},\vartheta;U',\chi',\bar{\vartheta}')}\phi(U',\chi',\bar{\vartheta}')$$

where $\qquad \psi = \binom{\chi}{\vartheta},\quad \bar{\psi} = (\bar{\chi}\ \bar{\vartheta}). \qquad (3.4)$

It is easy to see that tr $\hat{T}^\tau$ (τ is integer) can be performed as a 4-dimensional lattice Feynman path integral with the Wilson-action in the $U_o = 1$ gauge on the lattice which is the direct product of $\mathcal{V}$ and the 1-dimensional periodic lattice of τ sites $\mathcal{T}$.

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* The proof, that the Hilbert space, scalar product and transfer matrix given here are equivalent to the ones defined by Osterwalder and Seiler, will be published elsewhere.

$$\operatorname{tr}\hat{T}^{\tau} = \prod_{x\in\mathcal{V}\times\mathcal{T}} \prod_{j=1}^{3} \int dU_j(x) \prod_{x\in\mathcal{V}\times\mathcal{T}} \int d\bar{\psi}(x)d\psi(x)\; e^{-S}\Big|_{U_0=1} \tag{3.5}$$

(3.5) is meant of course with antiperiodic boundary conditions for the fermions what is equivalent to a small modification of the Wilson-action on the links connecting the slices $x_0 = \tau - 1$ and $x_0 = 0$.

In calculating physical quantities however, we need traces taken over $\mathcal{H}_{phys} \subset \mathcal{H}$ which is the subspace of locally gauge invariant states. The time independent classical gauge transformation Ω acting as $U \to U^{\Omega}$, $\chi \to \chi^{\Omega}$, $\bar{\delta} \to \bar{\delta}^{\Omega}$ has a unitary operator counterpart $\hat{\Omega}$ which acts as

$$\hat{\Omega}\phi(U,\chi,\bar{\delta}) = \phi(U^{\Omega},\chi^{\Omega},\bar{\delta}^{\Omega}) \tag{3.6}$$

Obviously $\hat{\Omega}$ commutes with $\hat{T}$ in (3.4). The operator which projects onto $\mathcal{H}_{phys}$ is

$$\hat{\Pi} = \prod_{\underset{\sim}{x}\in\mathcal{V}\setminus\partial\mathcal{V}} \int d\Omega(\underset{\sim}{x})\;\hat{\Omega} \quad . \tag{3.7}$$

Since we didn't require gauge invariance on the boundary there will be fractionally charged states in $\mathcal{H}_{phys}$.

Let us consider now the quantity $\operatorname{tr}\hat{\Pi}\,\hat{T}^{\tau} = \operatorname{tr}(\hat{\Pi}\hat{T})^{\tau}$. Using the formulae (3.4) and (3.7) one finds that the variable $\Omega(\underset{\sim}{x})$ in the n^{th} factor of $(\hat{\Pi}\hat{T})^{\tau}$ plays the role of a timelike link variable $U_0(n,\underset{\sim}{x})$ on the lattice $\mathcal{V}\times\mathcal{T}$. Therefore we can write

$$Z = \operatorname{tr}\hat{\Pi}\hat{T}^{\tau} = \int{}' \mathcal{D}U \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{-S} \equiv \tag{3.8}$$

$$\equiv \prod_{x\in\mathcal{V}\times\mathcal{T}} \prod_{j=1}^{3} \int dU_j(x) \prod_{x\in(\mathcal{V}\setminus\partial\mathcal{V})\times\mathcal{T}} \int dU_0(x) \prod_{x\in\mathcal{V}\times\mathcal{T}} \int d\bar{\psi}(x)d\psi(x)\,e^{-S}$$

This is the usual path integral with the Wilson-action without gauge fixing apart from the fact that the timelike U's on the boundary of $\mathcal{V}\times\mathcal{T}$, that is to say on $\partial\mathcal{V}\times\mathcal{T}$ are frozen to be 1. The prime above the integral in (3.8) refers to that specification (See *Fig. 1*).

If the gauge group is SU(N) or U(N) the baryon charge $\hat{B}$ is defined to be

$$e^{i\varphi\hat{B}}\phi(U,\chi,\bar{\delta}) = \phi(U,e^{i\frac{\varphi}{N}}\chi,\; e^{-i\frac{\varphi}{N}}\bar{\delta}) \tag{3.9}$$

The baryon symmetry group is now the $U(1)$ group $\mathcal{G}_B = \{e^{i\varphi B}\}_{\varphi=0}^{2\Pi N}$. The group of fractional baryon charges is the subgroup $\mathcal{Z}$ of $\mathcal{G}_B$ which is the intersection of $\mathcal{G}_B$ with the global gauge group. By definition a $\hat{B}$-eigenstate has integer charge if it is invariant under $\mathcal{Z}$. In the opposite case its charge is called fractional. If the gauge group is SU(N) then $\mathcal{Z} = \{e^{i\Pi 2n\hat{B}}\}_{n=0}^{N-1}$

which is isomorphic to $Z(N)$. In this case the $\hat{B} = B$ sector has integer (fractional) charge if and only if B is integer (fractional) in the usual sense. If, however, the gauge group is U(N), then $\mathcal{Z} = \mathcal{G}_B$ and any $B \neq 0$ is a fractional charge in the sense defined above. For instance, in QED the confinement of fractional charges means that only neutral states can propagate.

We shall need the following projection operator:

$$\hat{P}_B = \begin{cases} \frac{1}{N} \sum_{n=0}^{N-1} e^{-i2\pi nB} e^{i2\pi n\hat{B}} & \text{for } SU(N) \\ \frac{1}{2\pi N} \int_0^{2\pi N} e^{-i\varphi B} e^{i\varphi\hat{B}} & \text{for } U(N) \end{cases} \tag{3.10}$$

In both cases $\hat{P}_B$ projects onto the subspace of $\mathcal{H}$ where $\hat{B} = B + \{\text{integer}\}$. The partition function taken on the physical states with baryon number B+{integer} is

$$Z_B = \text{tr } \hat{\Pi}\hat{P}_B\hat{T}^\tau = \int_{\mathcal{Z}} dR(R^*)^{NB} \mathcal{Z}(R) \tag{3.11}$$

where $$Z(R) = \int' \mathcal{D}U \int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi \; e^{-S_R}$$

and $$S_R = -\frac{1}{g^2} \sum_P (\text{tr } U_{\partial P} + \text{c.c}) + \sum_x \bar{\psi}(x)[\psi(x) -$$

$$K \sum_{\mu=0}^{3} (1-\gamma_\mu)U_\mu(x)R^{\delta_{\mu 0}\delta_{x^0 0}}\psi(x+\hat{\mu}) - K\sum_{\mu=0}^{3} (1+\gamma_\mu)U_\mu^+(x)R^{-\delta_{\mu 0}\delta_{x^0 0}}\psi(x-\hat{\mu})]$$

To give the things told in the previous section a precise meaning we have to settle the notion of an observable. It is clear that the whole operator algebra is generated by the Heisenberg-operators $\hat{U}_j(x)$ (j=1,2,3), $\hat{\psi}(x)$ and $\hat{\bar{\psi}}(x)$. They can be considered as insertions of $U_j(x)$, $\psi(x)$ and $\bar{\psi}(x)$ in (3.8) or in (3.11) as far as they are alone. If more than one U's, ψ's and $\bar{\psi}$'s in the same lattice site are multiplied in the insertion it means the time ordering and a suitable normal ordering of the corresponding product of the Heisenberg operators $U_j(x)$, $\hat{\psi}(x)$ and $\hat{\bar{\psi}}(x)$. (See ref. [3] for a similar situation.) An operator F will be called an internal observable if 1) it is local, that is to say it is a function of finitely many operators $U_j(x)$, $\hat{\psi}(x)$ $\hat{\bar{\psi}}(x)$, 2) it doesn't lead out from $\mathcal{H}_{phys}$ even for one lattice unit of time. The expection value of the time ordered product of such an operator $\hat{F}$ in the canonical ensemble (3.8) is

$$\langle\hat{F}\rangle = \frac{\text{tr } \hat{\Pi}\hat{T}^\tau \mathcal{T}(\hat{F})}{\text{tr } \hat{\Pi}\hat{T}^\tau} = \frac{1}{Z} \int' \mathcal{D}U \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{-S} \; F \tag{3.12}$$

where F is local, invariant under time dependent gauge transformations and depends only on the fields U_j, ψ and $\bar{\psi}$ and not on U_o.

Let us study for a while the quantities F which are local, gauge invariant and do depend on U_o. What operators do they correspond to? Let $\hat{\Pi}_x$ project onto states which are gauge invariant at points $y \neq x$, but transforms as a quark at $y = x$ ($x \in \mathcal{V} \setminus \partial\mathcal{V}$).

$$\hat{\Pi}_x = \prod_{y \in \mathcal{V} \setminus \partial\mathcal{V}} \int d\Omega(y)\ \hat{\Omega}\ \Omega(x) \qquad (3.13)$$

If now F is such that it depends on exactly one U_o among the U_o's in the hyperplane of time x_o, namely on $U_o(x_o,x)$, then this U_o can be considered as part of an operator $\hat{\Pi}_x$ (substitute $\Omega(y) = U_o(x_o,y)$ in (3.13)). That means that an external color source is put into our system between the moments x_o and $x_o + 1$ in the point x. Generally we can say that the U_o dependent quantities F test the system in the presence of several time dependent external color charge distributions. Therefore the time evolution leads out from $\mathcal{H}_{phys}$. For this reason a quantity F will be called an external observable, if it is local, invariant to time dependent gauge transformations and does depend on U_o. For example $\langle\bar{\psi}(0,0)U_o(0,0)\ \ldots\ U_o(x_o-1,0)\psi(x_o,0)\rangle$ describes the following process: One quark is created at the origin together with an external antiquark. The latter remains in the origin but the motion of the dynamic quark is not controlled. x_o units of time later both quarks are annihilated. Obviously this correlation function is dominated by the lowest energy bound state of the quark in the field of the external antiquark whether we are in the confinement phase or in the free one. Another important example is the thermal Wilson-loop expectation value of which $\langle \mathrm{tr}\, U_o(0,x)\ \ldots\ U_o(\tau-1,x)\rangle$ is nothing but the partition function on the subspace determined by the projection $\hat{\Pi}_x$. The third example is a rectangular Wilson-loop in a timelike plane. One can see now why the Wilson-loop measures the force law between external sources even in the presence of dynamic fermions.

We are now ready to formulate our requirements for confinement in accordance with the picture described in section 2. Let $\langle F\rangle_B(\tau,V)$ denote the expectation value of the (internal or external) observable $\hat{F}$ in the canonical ensemble of states with baryon charge $\hat{B} = B + \{\text{integer}\}$ of the system of volume V at temperature $^1/_\tau$. Then

$$\langle F\rangle_B(\tau,V) = \frac{1}{Z_B} \int_{\mathcal{Z}} dR\ R^{-NB} \int' \mathcal{D}U \int D\bar{\psi}D\psi\ e^{-S_R}\ F_R \qquad (3.14)$$

where F_R, similarly to S_R, is made from F by the substitution $U_o(0,x) \to RU_o(0,x)$ for all x in $\mathcal{V}$.

We will say that the theory confines for all fractional charges if for all B and arbitrary observable F there exist a and b positive numbers such that

$$\langle F\rangle_B(\tau,V) - \langle F\rangle_0(\tau,V) = \mathcal{O}(e^{-aV^b}) \qquad (3.15)$$

Whereas if there exist an observable F such that for the fractional charge B

$$\langle F\rangle_B(\tau,V) - \langle F\rangle_O(\tau,V) = \theta(\frac{1}{V}) \tag{3.16}$$

holds, we will say that the B-charges are free.

It is possible to formulate the criterion in terms of volume 'integrals' of local observables. But we have to be careful not to violate the principle that we are never allowed to measure quantitites what extend to the whole volume $\mathcal{V}$. Let $\mathcal{V}_\rho \subset \mathcal{V}$ be a box of volume $|\mathcal{V}_\rho| = \rho V$ $(0<\rho<1)$ with center common with that of $\mathcal{V}$, and define

$$\overline{F_\rho} = \sum_{\underline{x}\in\mathcal{V}_\rho} F_{\underline{x}} \tag{3.17}$$

where $F_{\underline{x}}$ is the translation of F by $\underline{x}$. The average volume 'integral' of F for the infinite volume must be calculated in this way:

$$\lim_{\rho\to 1}\lim_{V\to\infty} \langle\bar{F}_\rho\rangle(\tau,V) \tag{3.18}$$

Following that principle the confinement criterion (3.15) can be written equivalently as

$$\lim_{\rho\to 1}\lim_{V\to\infty} [\langle\overline{F_\rho}\rangle_B(\tau,V) - \langle\bar{F}_\rho\rangle_O(\tau,V)] = 0 \tag{3.19}$$

while (3.16) is equivalent to

$$\lim_{\rho\to 1}\lim_{V\to\infty} [\langle\bar{F}_\rho\rangle_B(\tau,V) - \langle\bar{F}_\rho\rangle_O(\tau,V)] \neq 0 \tag{3.20}$$

with the same F and B as in (3.16).

Though these criteria contain the temperature we expect that if τ is large enough they will not depend on τ giving information only about the Hamiltonian of the system

Closing this section we show that (3.15) and (3.19) are fulfilled in any order of perturbation theory with respect to $1/g^2$ and K. Introducing the new integration variable $U_O'(0,\underline{x}) = RU_O(0,\underline{x})$ in (3.14) and also in (3.11) (remember that R is a centrum element of the gauge group) F_R will be F again and S_R changes to S_R'. S_R' is nearly the same action as S is in (3.8) except that the frozen links in the hyperplane $x_O = 0$ and the plaquettes touching these frozen links are multiplied by R or R^*. So the functional

$$e^{-\delta S_B} = \int_{\mathcal{Z}} R^{-NB} e^{-(S_R'-S)} dR \tag{3.21}$$

depends non-trivially only on variables being at most one lattice spacing distance from $\partial \mho$. By means of δS_B (3.14) can be written in the following way:

$$\langle F\rangle_B(\tau,V) = \frac{\langle e^{-\delta S_B} F\rangle}{\langle e^{-\delta S_B}\rangle\langle F\rangle} \cdot \langle F\rangle \tag{3.22}$$

Here the expectation $\langle . \rangle$ is taken with respect to the measure (3.12). The first term in the r.h.s. of (3.22) is equal to 1 up to the order of $\left(\frac{1}{g^4} + K^2\right)^{d-1}$, where d is the distance of F from $\partial \mho$. Thus up to an arbitrary large order term

$$\langle F\rangle_B(\tau,V) - \langle F\rangle = \mathcal{O}(1) \times \exp\{-V^{1/3} \tfrac{1}{3} \ln\left(\tfrac{1}{g^4} + K^2\right)^{-1}\} \tag{3.23}$$

if V is large enough. Therefore (3.15) holds for all values of the temperature $1/\tau$.

Summing up on $\underset{\sim}{x}$ in $\mho_\rho$ in (3.23) with F replaced by $F_{\underset{\sim}{x}}$, we get

$$\langle \bar{F}_\rho\rangle_B(\tau,V) - \langle \bar{F}_\rho\rangle(\tau,V) = \mathcal{O}(1) \times \rho V \exp\{-(1-\rho^{1/3})V^{1/3} \tfrac{1}{3} \ln\left(\tfrac{1}{g^4} + K^2\right)^{-1}\} \tag{3.24}$$

what vanishes in the thermodynamical limit. So (3.19) is verified in all orders of perturbation theory.

To justify that (3.15) and (3.19) hold also non-perturbatively in the neighbourhood of the point $\frac{1}{g^2} = K = 0$ needs a sophisticated proof because the usual bounds for the cluster decomposition of $e^{-\delta S_B}$ and F available in the literature are not strong enough for this purpose. (See ref. 9 and the references therein.)

4. THE HIGH TEMPERATURE PHASE OF THE Z(N) MODEL

In this section we wish to discuss in detail why (3.15) is true in the high temperature (i.e. large g, small K) phase of the Z(N) matter-gauge model. Let σ_x be the matter field U_ℓ the gauge field both having values in Z(N) = $\{e^{i\frac{2\Pi}{N}n}\}_{n=0}^{N-1}$. The action is

$$S = -\frac{\alpha}{2}\sum_\ell [U_\ell \sigma(\partial \ell) + c.c] - \frac{\beta}{2}\sum_p [U(\partial p) + c.c] \tag{4.1}$$

The 'baryon' symmetry transformations $\sigma_x \to e^{i\frac{2\Pi}{N}n}\sigma_x$ are identical to the global gauge transformations. Therefore we expect that in the case of confinement the observable 'baryon' charge is zero.

An arbitrary observable quantity is a linear combination of the quantities $F^{(\Gamma)} \equiv U(\Gamma)\sigma(\partial\Gamma)$, where Γ is a mod N integer valued function on links (i.e. a 1-chain) and we have introduced the notation

$$U(\Gamma) = \prod_{\ell} U_{\ell}^{\Gamma_{\ell}} \qquad \text{for any 1-chain } \Gamma$$

and

$$\sigma(\gamma) = \prod_{x} \sigma_x^{\gamma_x} \qquad \text{for any 0-chain } \gamma.$$

The allowed Γ's have finite support, that is: $\sum_{\ell}(1-\delta_{\Gamma_{\ell}0})$ = finite. Let R_{ℓ} be the $R_{\ell} \equiv R_{(x,x+\hat{\mu})} = R^{\delta_{\mu o}\delta_{x^o 0}}$ configuration what we have been used in the previous section. Then the index R in (3.14) concerning the quantity $F = F^{(\Gamma)}$ means: $F_R^{(\Gamma)} = R(\Gamma)U(\Gamma)\sigma(\partial\Gamma)$.

Let us transform the path integral (3.14) from variables (U,σ) to variables (μ,ν) where μ and ν are mod N integer valued 2-chain and 1-chain variables respectively. The transformation is the same as if we wanted to do a duality transformation like in ref. [10]. We get

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B(\tau,V) = \frac{1}{Z_B} \sum_{R\in Z(N)} R^{-NB} \sum_{\{\nu_{\ell}\}} \sum_{\{\mu_p\}} I_{\alpha}[\nu] I_{\beta}[\mu] \times \tag{4.2}$$
$$\times \sum_{\{U_{\ell}\}}{}' \sum_{\{\sigma_x\}} U(\nu+\partial\mu+\Gamma)\sigma(\partial\nu+\partial\Gamma)R(\nu+\Gamma)$$

where $I_{\alpha}[\nu] = \prod_{\ell} I_{\nu_{\ell}}(\alpha)$ and $I_{\nu}(\alpha) = \frac{1}{N}\sum_{V\in Z(N)} V^{-\nu} e^{\frac{\alpha}{2}(V+V^*)}$,

and a similar expression for $I_{\beta}[\mu]$. The crucial point is now the prime above the U-sum in (4.2) what means that we don't sum up over the U_{ℓ}'s with ℓ from the set of frozen links $\mathcal{F}$. This is why, performing the sums on the U's and the σ's, we get Kronecker-delta constraints on the ν having the general solution

$$\nu = -\Gamma - \partial\mu + \varphi \tag{4.3}$$

where φ is an arbitrary 1-cycle (i.e. $\partial\varphi = 0$) which differs from zero only on links in $\mathcal{F}$. In this way we reduced (4.2) to

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B(\tau,V) = \frac{\frac{1}{N}\sum_R R^{-NB} \sum_{\varphi}\sum_{\mu} I_{\beta}[\mu] I_{\alpha}[-\Gamma-\partial\mu+\varphi] R(\varphi)}{\frac{1}{N}\sum_R R^{-NB} \sum_{\varphi}\sum_{\mu} I_{\beta}[\mu] I_{\alpha}[-\partial\mu+\varphi] R(\varphi)} \tag{4.4}$$

where use was made of the identity $R(-\partial\mu+\varphi) = R(\varphi)$. (4.4) is the suitable form for doing high temperature expansion. The graphs will be labelled by the (μ,φ) configurations.

The following step is to decompose every configuration μ in a unique way in the form $\mu = \mu_\Gamma + \mu'$ where all connected component of $|\mu_\Gamma|$ (=support of μ_Γ) are connected to $|\Gamma|$ which is the support of Γ. Two sets Q_1 and Q_2 formed from links and plaquettes are called to be connected if for $i = 1,2$ there exist a link ℓ such that: 1.) either $\ell \in Q_i$ 2.) or there is a plaquette $p_i \in Q_i$ possessing ℓ as a boundary link. Keeping μ_Γ fixed the sum on μ' is just like as if we were calculating Z_B on $\Lambda \setminus \overline{|\mu_\Gamma| U |\Gamma|}$ instead of Λ at least for $|\mu_\Gamma|$ which is not connected to $\mathcal{T}$. Here $\bar{Q}$ is the closure of Q and Λ is the lattice $\mathcal{V} x \mathcal{T}$. Hence we can write

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B(\tau,V) = \sum_{\mu_\Gamma} I_\alpha[-\partial\mu_\Gamma-\Gamma] I_\beta[\mu_\Gamma] \frac{Z_B(\Lambda\setminus\overline{|\Gamma| U |\mu_\Gamma|})}{Z_B(\Lambda)} + \\ + \mathcal{O}((\tfrac{1}{T})^{d(|\Gamma|,\mathcal{T})}) \tag{4.5}$$

where $d(|\Gamma|,\mathcal{T})$ is the distance of $|\Gamma|$ from $\mathcal{T}$ which determines the minimal size of $|\mu_\Gamma|$ which connects Γ to φ, or in other words the minimal order when the factorization $I_\alpha[-\partial\mu-\Gamma+\varphi]/_{I_\alpha[0]} = I_\alpha[-\partial\mu_\Gamma-\Gamma]/_{I_\alpha[0]} \times I_\alpha[-\partial\mu'+\varphi]/_{I_\alpha[0]}$ breakes down. $1/T$ is a certain polinomial in α and β : $\frac{1}{T} = (\frac{\alpha}{2})^2 \frac{\beta}{2} + (\frac{\beta}{2})^4$, which comes from the two types of building minimal order diagram extending over a distance d.

Let Q be an arbitrary set of links and plaquettes from Λ. Then $Z_B(\Lambda\setminus Q)$ has an expansion similar to (4.5):

$$Z_B(\Lambda\setminus Q) = \frac{1}{N}\sum_R R^{-NB} \sum_\varphi R(\varphi) \sum_{\mu_\mathcal{T}} I_\alpha[-\partial\mu_\mathcal{T}+\varphi] I_\beta[\mu_\mathcal{T}] \times \\ \times Z(\Lambda\setminus Q\setminus\overline{|\mu_\mathcal{T}| U \mathcal{T}}) \tag{4.6}$$

where Z denotes the partition function on the whole $\mathcal{H}_{phys}$. (See (3.8) as an analog). It can be shown that for any $Q_1, Q_2 \subset \Lambda$

$$\frac{Z(\Lambda\setminus Q_1\setminus Q_2)}{Z(\Lambda)} = \frac{Z(\Lambda\setminus Q_1)}{Z(\Lambda)} \times \frac{Z(\Lambda\setminus Q_2)}{Z(\Lambda)} + \quad ((\tfrac{1}{T})^{d(Q_1,Q_2)}) \tag{4.7}$$

Substituting $Q_1 = \overline{|\Gamma| U |\mu_\Gamma|}$ and $Q_2 = \overline{|\mu_\mathcal{T}| U \mathcal{T}}$ if follows from (4.7) and (4.6) that

$$Z_B(\Lambda\setminus\overline{|\Gamma| U |\mu_\Gamma|}) = \frac{Z(\Lambda\setminus\overline{|\Gamma| U |\mu_\Gamma|}}{Z(\Lambda)} \; Z_B(\Lambda) + \mathcal{O}((\tfrac{1}{T})^{d(|\Gamma|,\mathcal{T})}). \tag{4.8}$$

Comparing (4.8) and (4.5) we get

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B = \langle U(\Gamma)\sigma(\partial\Gamma)\rangle + \mathcal{O}((\tfrac{1}{T})^{d(|\Gamma|,\mathcal{T})}) \tag{4.9}$$

Thus the confinement criterion (3.15) is satisfied in the perturbation theory with respect to α and β in the following sense. Let a positive integer m be given. Then there exist a volume V such that for any box $\mho$ with folume $|\mho|>V$ (3.15) holds up to terms of order $\beta^m\alpha^m$. The (3.19)-type property for $F=F^{(\Gamma)}$ is realized as a trivial consequence of (4.9). One finds that the r.h.s. of (3.19) vanishes in any finite order of the high temperature expansion.

5. THE FREE PHASE OF THE Z(N) MODEL

The eigenstates of the system (4.1) calculated in the perturbation theory around $\beta = \infty$, $\alpha = 0$ contain globally non-Z(N)-invariant, that is to say fractionally charged states. These states are present neither in the high temperature expansion nor in the perturbation theory around $\beta = \infty$, $\alpha = \infty$, i.e. in the Higgs phase. (The other possible phases, if N>4, are not discussed here.) We have to show that the non-perturbative criterion described in section 3 is consistent with this perturbative picture. In this section we will show that (3.16) and (3.20) are satisfied if they are calculated in lowest order perturbation theory in the large β, small α phase.

If $\beta = \infty$, only pure gauge U-configurations remain in the system, or in other words $I_\beta[\mu]$ becomes independent of μ. Thus

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B\Big|_{\beta=\infty} = \frac{\frac{1}{N}\sum\limits_R R^{-NB}\sum\limits_\varphi R(\varphi)\sum\limits_\mu J_\alpha[-\Gamma-\partial\mu+\varphi]}{\frac{1}{N}\sum\limits_R R^{-NB}\sum\limits_\varphi R(\varphi)\sum\limits_\mu J_\alpha[-\partial\mu+\varphi]} \tag{5.1}$$

Here $J_\alpha[\nu]$ stands for $I_\alpha[\nu]/I_\alpha[0]$. In lowest order in α the sum on μ and φ can be written as a sum over oriented curves having winding numbers on the torus $\mho\times\Upsilon$ precisely equal to NB and having the same end points than that of Γ. Let Γ be the straight line connecting the point $(0,\ldots,0,r)$ to the origin and $B = \frac{1}{N}$. Then the perturbative expansion of the denumerator of (5.1) starts with diagrams shown in *Fig. 2.a* and the diagrams of the denominator are shown in *Fig. 2.b*. We get

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B(\tau,V)\Big|_{\beta=\infty} = \left(\frac{\alpha}{2}\right)^r + \frac{1}{V}\left[\binom{\tau+r}{r} - r-1\right]\left(\frac{\alpha}{2}\right)^r + \mathcal{O}(\alpha^{r+2}) \tag{5.2}$$

Because the first term in the r.h.s of (5.2) is just the average in the B=0 sector, we arrive at an expression fulfilling the condition (3.16):

$$\langle U(\Gamma)\sigma(\partial\Gamma)\rangle_B - \langle U(\Gamma)\sigma(\partial\Gamma)\rangle_0 = \frac{1}{V}\left(\frac{\alpha}{2}\right)^r\left[\binom{\tau+r}{r} - r-1\right] \tag{5.3}$$

This implies that (3.20) is valid with a r.h.s. what is V times the r.h.s. of (5.3). Though these expressions blow up in the zero temperature limit $(\tau\to\infty)$, it must be the feature of the perturbation theory, because $U(\Gamma)\sigma(\partial\Gamma)$ corresponds to a unitary operator and therefore its expectation value is always bounded.

6. THE HIGGS PHASE

In the Higgs phase it is convenient to use the original (U,σ) representation of the path integral. If $\beta = \infty$

$$\sum_U{}' \sum_\sigma \exp\{\frac{\alpha}{2}\sum_\ell [R_\ell U_\ell \sigma(\partial\ell) + c.c] + \frac{\beta}{2}\sum_p [U(\partial p) + c.c.]\}R(\Gamma)U(\Gamma)\sigma(\partial\Gamma) = \tag{6.1}$$

$$= \text{const.} \times \sum_\sigma \exp\{\frac{\alpha}{2}\sum_\ell [R_\ell \sigma(\partial\ell) + c.c.]\}R(\Gamma)\sigma(\partial\Gamma)$$

If $R = 1$, that is to say $R_\ell = 1$ for all links, then the action (i.e. the exponent in the r.h.s. of (6.1)) is minimal at σ_x = const. In this case the action is $-\alpha V\tau\mathcal{D}$, where $\mathcal{D}$ is the number of spacetime dimensions. If $R \neq 1$, the minimal action is larger than $(-\alpha V\tau\mathcal{D})$ with the quantity $\alpha V(1-\cos\frac{2\Pi}{N})$. Indeed

$$S_{R\neq 1}(\sigma) - (-\alpha V\tau\mathcal{D}) \geq V(-\frac{\alpha}{2})\sum_{n=0}^{\tau-1}(\sigma_n^* R_{nn+1}\sigma_{n+1} + c.c-2) \geq \alpha V(1-\cos\frac{2\Pi}{N}) \tag{6.2}$$

Consequently, the B dependent contributions which come from the $R\neq 1$ terms are suppressed by the factor $e^{-\alpha V(1-\cos\frac{2\Pi}{N})}$ relative to the $R = 1$ term which gives a B independent contribution. Therefore

$$\langle F\rangle_B(\tau,V) = \langle F\rangle(\tau,V) + \mathcal{O}(e^{-\alpha V(1-\cos\frac{2\Pi}{N})}) \tag{6.3}$$

We see that (3.15) is true also in the Higgs phase although with numbers a and b different from those in the confinement phase. This is in accordance with the well known fact that the Higgs and confinement 'phases' form a single phase if the matter field is in the fundamental representation.

Summarizing, the high temperature and the Higgs regions don't show observable fractional charges while the free phase does in the sense of (3.15) and (3.16). Although the number of spacetime dimensions were not specified we tacitly assumed that the phase under consideration does exist. For instance in a two-dimensional spacetime and with $N = 2$ the results of the sections 5 and 6 have no relevance because the free and the Higgs pahses do not exist.

7. CONCLUSION AND OUTLOOK

We have derived a criterion how to distinguish a phase which confines the fractional (baryon) charges from a phase which doesn't. Although the criterion was defined non-perturbatively we were able to check it only in perturbation theory. The phase structure determined by the criterion was consistent with the presence or absence of fractionally charged states in the perturbative spectrum. We restricted our attention to the expectation values of local quantities. We could not derive a necessary singular behaviour of the infinite volume expectation values at the deconfining transition. Instead, we found that there is a difference in the volume dependence of the finite volume averages between the confining and the free phases. In other words the boundary condition has an important role: one must keep track of the volume dependence in case of various boundary conditions (corresponding to various total baryon charges) in order to get a signal of confinement.

These results raise three questions. I. What would have happened if we have suppressed the Gauss-law for example only in one point of the boundary $\partial\mathcal{V}$? This is enough for having fractionally charged states but the boundary condition is changed. The answer is probably that in that case, too, we could derive the same criterion for confinement except that for example the exponent b in (3.15) is not necessarily the same. In this paper we have chosen the most symmetric boundary condition. II. A more difficult question is: should not the free fractional charges have given a signal in the infinite volume expectations if we had not restricted ourselves to local quantities? One conjectures a positive answer because the operator creating a free quark eigenstate mus be such a non-local quantity. III. The third problem is that we have not shown - although it should be true - that the phase structure determined by this criterion is the same as that which follows from the bulk properties of the system (i.e. from the singularities of the infinite volume expectations).

As far as the finite temperature phenomena are concerned, equations (3.15) and (3.16) - since they contain the temperature $1/\tau$ - can be considered formally as the definition of the finite temperature deconfining phase transition in the presence of dynamic quarks. However, there are two (perhaps weak) arguments for not to do so. 1) We have shown at the end of section 3 that (3.15) is satisfied in the strong coupling, small K region for any temperature $1/\tau$. That would mean, that there is no deconfining phase even at very high temperature in this region of the couplings, what contradicts to the usual expectations. 2) In deriving the criteria (3.15) and (3.16) we used a physical picture which we feel not to be compatible with the existence of a heath reservoir.

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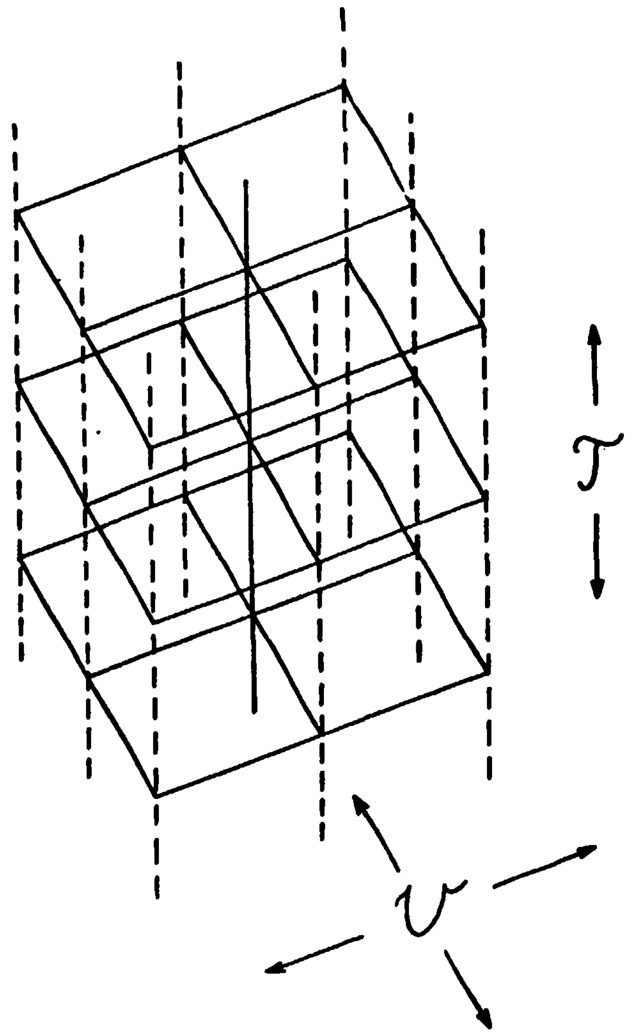

Fig. 1. The boundary condition for the 2+1 -dimensional lattice of size 2x2x3: the links with dashed lines are the frozen links

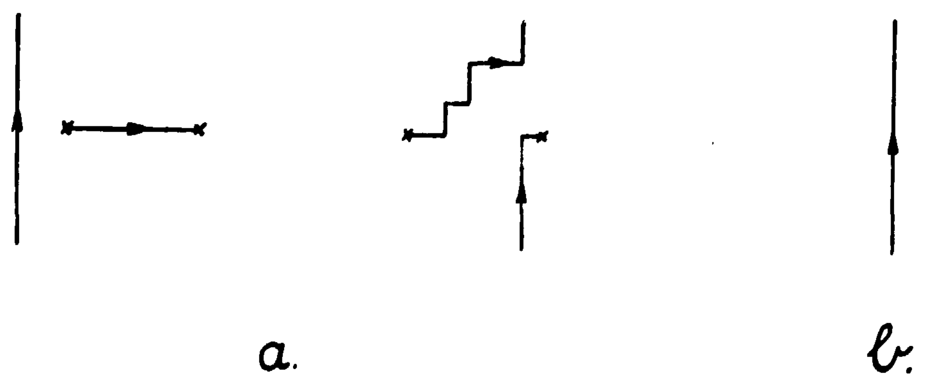

Fig. 2. Lowest order diagrams for a/ the denumerator, b/ the denominator of /5.1/

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