SU8303037

ITEP - 80

INSTITUTE OF THEORETICAL
AND EXPERIMENTAL PHYSICS

A.D.Dolgov

ON A POSSIBLE MECHANISM OF COSMOLOGICAL TERM CANCELLATION

MOSCOW 1982

УДК 530.145:538.3 М-16

A b s t r a c t

A model is proposed in which cosmological constant is cancelled automatically because of the back reaction on it of a scalar field φ coupled to curvature R. The cancellation takes place if two conditions are imposed on the theory, namely vanishing of mass and selfinteraction of field φ.

The vacuum energy in the frameworks of the model considered does not dissapear completely but only up to the order of $\rho_c \simeq m_\varphi^2/t^2$. This leads to some modification of the standard expansion scenario.

The Einstein equations as is well known permit the generalization by adding a cosmological term:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi M_p^{-2} T_{\mu\nu} + \lambda g_{\mu\nu} \qquad (1)$$

where $M_p \approx 10^{19}$ GeV is the Planch mass, $T_{\mu\nu}$ is the energy-momentum tensor of matter and λ corresponds to gravitational interaction of vacuum. It is convenient to define the vacuum energy density which is connected with the last term in eq.(1): $\rho_{vac} = -\lambda M_p^2 / 8\pi$. Modern astronomical data show that ρ_{vac}, if nonvanishing, is extremely small:

$$|\rho_{vac}| < 10^{-46} m_N^4 \qquad (2)$$

Any energetic scale in elementary particle physics is enormous in comparison with this number.

The first question which arises is why vacuum does not gravitate? Why does not gravitate the zero point energy [1]? There are infinite contributions from any particle species into this energy. Probably the infinities in the total sum of these contributions are cancelled out as a consequence of supersymmetry. In reality however supersymmetry is broken so the finite part in ρ_{vac} should generally survive and be of the order of m^4, where m is a characteristic mass scale. By an unknown reason this energy does not influence the evolution of the universe.

The situation is even more weird in gauge theories with spontaneous symmetry breaking. The point is that a vacuum expectation value of Higgs field χ necessarily leads to the nonzero cosmological term, $T_{\mu\nu}(\chi) \sim g_{\mu\nu} \langle\chi\rangle^2 m_\chi^2$. The corresponding vacuum energy is about $10^8\, m_N^4$ for the Weinberg - Salam model and is of the order of $10^{60}\, m_N^4$ for Grand Unification Theories. As the condensates of the Higgs fields develop after sufficient expansion and cooling of the universe [2], the initial vacuum energy should be nonzero and its value have to coincide with that of the later developed condensates with the accuracy, which is better than a hundred orders of magnitude. One can hardly believe in such a superfine tuning and some attempts [3, 4] have been made to find a more natural explanation of the smallness of the cosmological constant.

This paper presents an attempt to find a model of self-cancellation of cosmological term. It is assumed that there exists a field φ which interacts with vacuum energy in such a way that the amplitude of the field rises so that vacuum condensate of φ is produced. The energy of this condensate should compensate the vacuum energy which initialy was the source of the condensate developement. Naturally the rate of this process should be of the order of H^{-1}, where H is the Hubble constant, and the expected value of the resulting cosmological term is about H^2 which is close to the existing bound.

As the first and let admit, unsatisfactory possibility we will consider a massless scalar field which has no interactions but gravitional one. Some features of this model can be of interest for what follows. Let assume for simplicity that the universe is spatially flat ($k=0$):

$$ds^2 = dt^2 - a^2(t)\, d\vec{r}^{\,2} \qquad (2)$$

In this metric φ satisfies the equation

$$\varphi^{;\lambda}{}_{;\lambda} + \xi R\varphi = \partial_t^2\varphi + 3H\partial_t\varphi + \frac{1}{a^2}\partial_i^2\varphi + \xi R\varphi = 0 \qquad (4)$$

where $H = \dot{a}/a$ is the Hubble constant; ξ is a numerical constant (for conformally invariant case ξ = 1/6). Scalar curvature R can be expressed, using the Einstein equations, through the trace of the energy-momentum tensor

$$R = 8\pi M^2 \left(4\rho_{vac} + T^{\mu}{}_{\mu} \right) \qquad (5)$$

where M does not coincide with M_{φ} because of the corrections due to nonzero φ (see below).

For comformal fields T^{μ}_{μ} = 0 if quantum corrections are neglected (i.e. in the tree approximation).

We will consider spatially homogeneous solutions of eq.(4) $\varphi = \varphi(t)$. If $\xi R < 0$, there exist solutions which rise with time. In particular when the energy-momentum tensor is dominated by the vacuum term, $H = (8\pi\rho_{vac}/3M^2)^{1/2}$, $R = 12H^2$, and eq.(4) has the solutions

$$\varphi = \exp\left\{ \left(-\frac{3}{2} \pm \sqrt{\frac{9}{4} - 12\xi}\right) Ht \right\} \qquad (6)$$

For $\xi < 0$ one of these solutions increases with t and consequently the contribution of φ into energy-momentum tensor should not be neglected:

$$T_{\mu\nu}(\varphi) = \varphi_{;\mu}\varphi_{;\nu} - \frac{1}{2} g_{\mu\nu}\varphi_{;\lambda}\varphi^{;\lambda} - \xi\varphi^2\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R\right) - \xi(\varphi^2)_{;\mu;\nu} + \xi g_{\mu\nu}(\varphi^2)^{;\lambda}{}_{;\lambda} \tag{7}$$

Correspondingly the scalar curvature is equal to

$$R = \frac{8\pi\left[4\rho_{vac} + (6\xi - 1)\varphi^2\right]}{M^2 + 8\pi\xi(6\xi - 1)\varphi^2} \tag{8}$$

and H is defined by the equation

$$H^2 = \frac{8\pi}{3} \frac{\rho_{vac} + \frac{1}{2}\dot{\varphi}^2 + 6H\xi\varphi\dot{\varphi}}{M^2 - 8\pi\xi\varphi^2} \tag{9}$$

The dependence of R and H on φ being taken into account, the exponential rise of solution (6) becomes linear one such that $\dot{\varphi}^2 \to 4\rho_{vac}/(1-6\xi)$ when $t \to \infty$. Consequently R quickly vanishes, $R \ll t^{-2}$. Unfortunately it is not only R that vanishes but also the gravitation itself because the effective gravitational constant tends to zero:

$$G_{eff} = \left[M^2 + 8\pi\xi(6\xi - 1)\varphi^2\right]^{-1} \sim t^{-2} \tag{10}$$

Before discussing a better model let us comment on the spatial dependence of unstable solution of eq.(4). If the scalar curvature is not spatially constant it will drive the formation of the condensate of φ only in the case of rather large characteristic scale of the matter inhomogeneity

$$\ell > M / \sqrt{8\pi \rho \xi} \qquad (11)$$

Foe example, for $\rho = 1$ g/cm^3 the φ-condensate would be formed if the matter distribution is homogeneous up to distances 10^{14} cm/ $(8\pi\xi)^{1/2}$.

Apart of killing the gravition the model discussed above does not lead to compensation of the cosmological term because $T_{\mu\nu}(\varphi) \not\sim g_{\mu\nu}$, the scalar curvature being compensated however. Moreover, infinitely rising field $\varphi(t) \sim t$ does not look very appealing. It is desirable to find a model where $\varphi \to \varphi_0$ = const as $t \to \infty$ and $T_{\mu\nu}(\varphi_0) \sim g_{\mu\nu}$.

The simplest way to realize this idea by introducing a nonzero mass of the field φ or its selfcoupling $\lambda\varphi^4$ immediately encounters difficulties. In this ca'e extra terms appear in eq.(4):

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi + \lambda\varphi^3 + \xi R\varphi = 0 \qquad (12)$$

As above R can be expressed through the trace of $T_{\mu\nu}$ in which now new terms appear which do not vanish for φ=const:

$$\delta T^{\mu}_{\mu} = m^2\varphi^2 + \beta_\lambda \varphi^4 \qquad (13)$$

The last term in the r.h.s. of this equation is the quantum conformal anomaly [5, 6] produced by loops of selfinteracting φ. One should remember that in matrix elements of T_μ^μ the anomalous term survives only for momenta exceeding m.

With the appropriate choice of the parameter sings eq.(10) has solutions which tend to a nonzero constant ($\varphi \to \mathrm{const} \neq 0$) as $t \to \infty$. This solutions however leads to the condition $m^2 + \lambda \varphi^2 + \xi R = 0$ but not to $R = 0$. If we put $m = \lambda = 0$ then simultaneously dissapears the anomalous term in eq.(13) and we return to the first model.

Probably the massless scalar field φ with the Yukawa coupling $g\varphi\bar{\psi}\psi$ to the originally massless spinor field ψ (a l/or $g\varphi\bar{\psi}\psi$ to a vector field) will serve our purpose better. Anomalous term $\delta T_\mu^\mu = \beta_\gamma \varphi^4$ can appear because of the coupling between φ and ψ. If in addition the theory can be formulated in such a way that neither mass nor an effective selfinteraction of φ (for constant φ) arise, the gravitating vacuum energy proves to be tending to zero with time.

Such conditions imposed on the theory is difficult to call natural. Probably the model could be changed so that some symmetry arguments forbid nonvanishing effective selfinteraction for constant φ. No such model was found yet but at least it should be checked whether the model under discussion is not selfcontradictory.

Thus we assume that the classical homogeneous field satisfies the equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \xi R\varphi = g\bar{\psi}\psi \qquad (14)$$

and that for $\varphi \to$ const

$$R \to \frac{8\pi\,[4\rho_{vac} + \beta\varphi^4]}{M^2 + 8\pi\xi(6\xi-1)\varphi^2} \tag{15}$$

One can see that $\varphi = \varphi_0 = (4\rho_{vac}/|\beta|)^{1/4}$ is the solution of this equation. In what follows we argue that φ indeed tends to φ_0 and that this classical field indeed eats up the cosmological term. In this model the gravitational constant is changing with time from its initial value $G_i = M^{-2}$ to its asymptotic value $G_0^{-1} = M^2 + 8\pi\xi(6\xi-1)\varphi_0^2 = M_g^2$. This variation however is not in contradiction with astronomy. The analogous changing of the gravitational constant due to the classical scalar field was discussed in ref. [7].

If $\xi R < 0$, eq.(14) has unstable solutions leading to vacuum condensate of φ. As a result the spinor field becomes massive, $m_\psi = g\varphi_0$. It is noteworthy that despite the obtained mass of ψ the energy-momentum tensor remains traseless in the tree approximation if $\xi = 1/6$. (If one considers a massive from the beginning spinor field interacting with a scalar field, the condition $T^\mu_\mu = 0$ can be also fullfilled. To this end a total derivative $(1/6)(g_{\mu\nu}\partial_\alpha^2 - \partial_\mu\partial_\nu)(\varphi + g^{-1}m_\psi)^2$ should be added to $T_{\mu\nu}$ in the flat space-time.) If $\xi \neq 1/6$ an extra term proportional to $g\varphi\bar\psi\psi$ appears in T^μ_μ. We will not take it into account however because it proves to be small. The main contribution into T^μ_μ, which compensates ρ_{vac}, is given by the anomalous term $\beta\varphi^4$.

Already at this step the inconsistency of the approach is very well seen. Eq.(14) considered as the equation for matrix elements of φ is valid in the tree approximation whereas the anomalous term in $T_{\mu\nu}$ is absent in this approximation. In one loop approximation the effective potential receives the contribution of the type [8]

$$\delta U = \varphi^4\left(\gamma_6 \ln \frac{\Lambda^2}{\varphi^2+\varkappa_6 R} + \gamma_f \ln \frac{\Lambda^2}{\varphi^2+\varkappa_f R}\right) + \lambda\varphi^4 \tag{16}$$

For $\varphi^2 \gg \varkappa R$ the first term leads to renormalization of ξ. The second term is to be exterminate "by hand", i.e. by imposing the normalization condition $\lambda = 0$. It is also assumed that no ultraviolet cutoff Λ enters the calculation due to (supposed) cancellation between fermion and boson contribution to the effective potential $\gamma_6 = -\gamma_f$ (note that their contribution to the conformal anomaly are of the same sign). This is evidently the weakest point of the construction. As for many loops, their contribution into the effective potential is unknown. If the cutoff Λ is also cancelled for many loops, their contribution is of the form $\delta U = \varphi^4 F(R/\varphi^2)$ and the single condition which should be imposed on the theory is $F(0) = 0$.

Maybe some other models of this type, but based on gauge or chiral fields, will be more natural. The gauge or chiral invariance could be violated because of interaction with R. Unfortunately no satisfactory model was yet found. It seems however that independently of concrete datails the phenomenological consequences of these model should be qualitatively the same.

Thus, with all the weak points kept in mind, the proposed feed-back mechanism enforces the scalar curvature to zero when φ tends to a constant value. We will argue in what follows that not only R tends to zero but also all the cosmological term $\rho_{vac} g_{\mu\nu}$. In other words the energy momentum tensor of the classical field φ_0 is proportional to $g_{\mu\nu}$. First of all let note that $T_{\mu\nu}(\varphi)$ does not vanish when φ = const because $T^{\mu}_{\mu} \sim \varphi^4$. $T_{\mu\nu}(\varphi)$ has a rather complicated nonlocal structure but when $\varphi \to$ const only terms proportional to $g_{\mu\nu}$ should survive. To see this let consider the equation

$$\frac{\dot{\rho}}{\rho + p} = -3H \tag{17}$$

where ρ and p are respectively energy and pressure density of fields φ and ψ in spatially homogeneous case (i.e. $\varphi = \varphi(t)$ and $\psi = \psi(t)$). This equation is equivalent to the sum of Dirac equation for ψ and Klein-Gordon equation for φ. In the tree approximation $\dot{\rho} = \dot{\varphi}\ddot{\varphi}$ + (terms not containing $\ddot{\varphi}$) and $p + \rho = \dot{\varphi}^2 + \ldots$. If higher orders of perturbation theory (loops) led to $p + \rho \neq 0$ for φ = const , it would drastically change the wave equation for φ :

$$\ddot{\varphi} + 3H\left[\dot{\varphi} + \frac{A(\varphi)}{\dot{\varphi}}\right] + \ldots = 0 \tag{18}$$

where $A(\varphi) = p + \rho$ at $\varphi \to$ const . This seems improbable and one would expect $p + \rho \to 0$ and consequently $T_{\mu\nu} \sim g_{\mu\nu}$ as $\varphi \to$ const.

Returning to eq.(15) let us note that the condition $\dot{\rho}=0$ in the flat space-time is fulfilled separately in each order of perturbation theory.

A detailed investigation of solution of eq.(12) is impossoble because the dependence of R in derivatives of φ is unknown (see eq.(15)). Moreover the production of quanta of φ-field by variating classical field $\varphi(t)$ is not taken into account. The latter is proportional to $\dot{\varphi}(t)$ to a power and is equivalent to a friction force.

We will assume that at the "beginning" the cosmological term was nonzero, $\rho_{vac} \lesssim m_g$, $\rho_{vac} > 0$. The following behaviour of $\varphi(t)$ is expected in this case. When $T_{\mu\nu}$ is dominated by the vacuum energy, φ rises exponentially in accordance with eq.(6). It is convenient to consider the difference $\chi = (\varphi_0 - \varphi)/\varphi_0$, where $\varphi_0 = (4\rho_{vac}/|\beta|)^{1/4}$ ($\beta < 0$) is the limiting value of φ as $t \to \infty$. In neglect of derivative terms in R and H, $\chi \sim t^{-1}$ when $\rho_{vac}\chi < \rho_{matter}$ where ρ_{matter} is the energy density of the usual matter present in the universe. The damping of χ being determined by the term $3\dot{\chi}\,(8\pi\rho_{vac}/3M^2)^{1/2}(\rho_m/\rho_{vac}+\chi)^{1/2}$, χ should decrease as t^{-2}. (The substitution of $\chi \sim t^{-1}$ into $(\rho_m/\rho_{vac}+\chi)^{1/2}$ leads to the solution $\chi \sim exp(-\sqrt{t})$.)

The important point here is the necessity of a large "friction" term, which is not explicitely written down in the equation, so that the difference $(\rho_m/\rho_{vac}+\chi)$ keeps to be positive. Moreover φ should tend to φ_0 slow enough in such a way that a fast change of ρ_{vac} because of phase transitions mentioned above leaves $\rho_{vac} > 0$. Otherwise the described mechanism with the chosen signs of parameters becomes nonoperative.

In this qualitative discussion of the behaviour of $\varphi(t)$ we did not yet take into account the r.h.s. of the eq.(14). It seems natural to estimate $\bar{\Psi}\psi$ as the number density of massive spinor particles ($m_\psi = g\varphi_0$). Usually this number density falls as t^{-2} and in this sence there is no contradiction with our claim that $R \sim (\rho_{vac} - |\beta|\varphi^4| \sim \gamma \sim t^{-2}$. However in this case the energy density of ψ -particles proves to be too large. Probably the detailed calculations show no contradiction with the data (e.g. because of initial conditions). If it does not help, there is another way to mend the leakage by inventing a gauge vector field interacting with ψ . In this case the interaction between ψ's becomes long range and leads to an increase of their annihilation (see ref. [9] for the discussion of the burning out of magnetic monopoles in early universe). As a result the energy density of ψ can be sufficiently small.

Thus with some efforts the cosmological term might be made small. The standard scenario of the universe evolution is changed in this case. First of all the standard expression for the universe age

$$t_u = \frac{1}{H_0}\,\frac{1}{1+\frac{1}{2}\sqrt{\Omega}} \qquad (.9)$$

becomes invalid. If $\rho_{vac} > 0$ the contradiction between the modern value of the Hubble constant $H_0 = 100$ km/sec Mps and $t_u = (15 - 17)\cdot 10^9\,y$ could be avoided. The bound on the number of neutrino species [10], which follows from the data on cosmological abundances of He^4 and H^2, becomes less restrictive. If the cosmological constant always satisfies inequality (2), it would not influence the primordial nucleosynthesis. In the discussed model however ρ_{vac} (as the Hubble constant) is not constant in time but goes like M_φ^2/t^2. That is why its role could be noticeable during all the history of the universe. It is interesting also to consider the spatial dependence of the classical field φ. As was noted above its variation is possible on scales restricted by condition (11). For objects which size is larger than ℓ a drastic change of gravitational forces could be possible. The detailed discussion seems however premature until a more realistic and more definite model is found. One could hope nevertheless that the principal features of this approach remain the same in a better model.

I would like to thank A.D.Linde and Ya.B.Zeldovich for discussions and critisism.

References

1. Ya.B.Zeldovich. Uspekhi Fiz.Nauk 95 (1968) 209.
2. D.A.Kirznits. JETP Lett. 15 (1972) 529.
   D.A.Kirznits and A.D.Linde. Phys.Lett. 42B (1972) 471.
   A.D.Linde. JETP Lett. 19 (1974) 183.
3. A.M.Polyakov. Uspekhi Fiz.Nauk 136 (1982) No 3.
4. S.Hawking. Cambridge University Preprint, 1982.
5. R.J.Crewther. Phys.Rev.Lett. 28 (1972) 142.
6. M.Chanowitz, J.Ellis. Phys.Lett 40B (1972) 397.
7. L.Smolin. Nucl.Phys. B160 (1979) 253.
   F.Englert, C.Truffin, and R.Gastmans. Nucl.Phys. B117 (1976) 407.
   A.Zee. Phys.Rev.Lett. 42 (1979) 417.
   A.D.Linde. Phys.Lett. 93B (1980) 394.
8. S.Coleman and E.Weinberg. Phys.Rev. D7 (1973) 1888.
   A.V.Veryaskin, V.G.Lapchinsky, and V.A.Rubakov. Theor. and Math.Phys. 45 (1980) 807.
9. A.D.Dolgov and Ya.B.Zeldovich. Rev.Mod.Phys. 53 (1981) 1.
10. V.F.Schwartzman. JETP Lett. 9 (1969) 315.
    G.Steigman and D.N.Schramm, J.E.Gunn. Phys.Lett. 66B (1977) 202.

А.Д.Долгов
О возможном механизме сокращения космологического члена
Работа поступила в ОНТИ 7.06.82

Подписано к печати I4.06.82 TI209I Формат 60x90 I/I6
Офсетн.печ. Усл.-печ.л.0,75 Уч.-изд.л.0,5 Тираж 290 экз.
Заказ 80 Индекс 3624 Цена 7 коп.

Отпечатано в ИТЭФ,II7259, Москва, Б.Черемушкинская,25

ИНДЕКС 3624

М.,Препринт ИТЭФ, 1982, № 80, с.1-12