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FINITE TEMPERATURE SU(2) GAUGE THEORY WITH EXTERNAL MAGNETIC FIELD *

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ABSTRACT

We employ the heat kernel method to evaluate the effective potential of the SU(2) gauge theory (with scalar triplet) when both external magnetic field and temperature are applied to it. The calculation is performed in high temperature approximation.

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I. INTRODUCTION

The finite temperature formalism which was developed by Linde [1] and others has been employed to study the phase transition at finite temperature. On the other hand, it has also been known that the external fields such as the external electric or magnetic field and the classical curved space-time background could cause the phase transition. And after Schwinger's classical work various people [2] have discussed the subject in various contexts.

In the present paper we make use of both formalisms to evaluate the one-loop effective potential at finite temperature with external magnetic field taken into account. As a physical system we consider for simplicity SU(2) gauge theory with a scalar triplet. To express the result in analytic form we will make use of the high temperature expansion.

The evaluation of the one-loop effective potential is essentially the summation of all the eigenvalues. Therefore we could adopt to this case what Ninomiya and Sakai [3] did for SU(2) QCD and start with the explicit expression for the eigenvalues. Instead we make use of the heat kernel method which has recently been discussed by Shore [2]. He calculated in the general gauge and at zero temperature the one-loop effective potential of the same physical system we will discuss. Thus our analysis is the extension of Shore's to finite temperature case. We limit our scope and consider only magnetic field case for which the result simplifies greatly. Also our calculation is done in high temperature approximation. The main reason is that only in this case we can separate out the imaginary part without ambiquity. (The situation may be improved in the future.)

In Sec.II we closely follow Shore's paper and derive the finite temperature effective potential. Sec.III is the presentation of the high temperature expansion.

II. DERIVATION OF THE FINITE TEMPERATURE Tr EFFECTIVE POTENTIAL WITH EXTERNAL MAGNETIC FIELD

In the heat kernel method [4] the Green's function G(x,y) of an operator $\mathcal{D}$ is given by

$$G(x,y) = \int_0^\infty dt\, \mathcal{G}(x,y:t) \tag{1}$$

where $\mathcal{G}$ is the heat kernel and satisfies

$$\mathcal{D}\,\mathcal{G}(x,y:t) = -\frac{\partial \mathcal{G}}{\partial t} \tag{2}$$

The model we consider contains gauge bosons, ghosts and a scalar-triplet. Their heat kernels have been evaluated in the general gauge;

$$g_{scalar}(x,y;t;M^2)$$

$$= \frac{1}{(4\pi)^{n/2}} t^{-\frac{n}{2}} \Phi(x,y) \exp\left\{-\frac{1}{4}(x-y)_\mu (gF\cot gFt)_{\mu\nu}(x-y)_\nu - \frac{1}{2}\,\mathrm{tr}\log\frac{\sin gFt}{gFt} - M^2 t\right\} \tag{3}$$

$$g_{ghost}(x,y;t)$$

$$= \frac{1}{(4\pi)^{n/2}}\left(\frac{t}{\alpha^{1/2}}\right)^{-\frac{n}{2}} \Phi(x,y) \exp\left\{-\frac{1}{4}(x-y)_\mu \left(gF\cot\frac{gFt}{\alpha^{1/2}}\right)_{\mu\nu}(x-y)_\nu - \frac{1}{2}\,\mathrm{tr}\log\frac{\sin gFt}{gFt\alpha^{-1/2}} - \alpha^{1/2} m^2 t\right\} \tag{4}$$

$$g_{gauge}(x,y;t;m^2)$$

$$= \bar{g}_{\mu\nu} + D_\mu D_\nu\left\{H_{\mu\nu}(t) - H_{\mu\nu}(t/\alpha)\right\} \tag{5}$$

where

$$\bar{g}_{\mu\nu} = \frac{1}{(4\pi)^{n/2}} t^{-n/2} \Phi(x,y) \exp\left\{-\frac{1}{4}(x-y)_\rho (gF\cot gFt)_{\rho\sigma}(x-y)_\sigma - \frac{1}{2}\,\mathrm{tr}\log\frac{\sin gFt}{gFt} - 2iFt\right\} \tag{6}$$

$$D_\mu D_\nu H_{\mu\nu}(t) = \frac{1}{(4\pi)^{n/2}}\int_0^\infty ds \int d^n z\, s^{-\frac{n}{2}} t^{-\frac{n}{2}} \Phi(x,z)\left\{M(s) - M^T(s)(x-z)(x-z)M(s)\right\} \times$$
$$\times \exp\left\{-\frac{1}{4}(x-z)A(s)(x-z) - \frac{1}{4}(y-z)A(t)(y-z) + C(s) + C(t)\right\} \times$$
$$\times \exp\left\{-2i(s+t)F\right\} \Phi(z,y) \tag{7}$$

$$M_{\mu\nu} \equiv -\frac{1}{2}A_{\mu\nu} - \frac{i}{2} g F_{\mu\nu} \tag{8}$$

$$A_{\mu\nu} \equiv (gF\cot gFt)_{\mu\nu} \tag{9}$$

$$C(t) \equiv -\frac{1}{2}\,\mathrm{tr}\log\frac{\sin gFt}{gFt} \tag{10}$$

$$F_{\mu\nu} = F^a_{\mu\nu} T^a \quad (T^a : \text{representation matrix}) \tag{11}$$

$$\Phi(x,y)\ (= \text{phase factor}) = e^{i\int_x^y F_{\mu\nu} dz_\nu} \tag{12}$$

$$(m^2)_{ab}\ (= \text{scalar mass matrix}) = M^2 \frac{\lambda}{2}\phi^2(1-P)_{ab} + \left(\frac{\lambda}{6} + \alpha g^2\right)\phi^2 P_{ab} \tag{13}$$

$$(m^2)_{ab}\ (= \text{gauge boson mass matrix}) = g^2 \phi_i T^a_{ik} T^b_{kj} \phi_j = g^2\phi^2 P_{ab} \tag{14}$$

$$P_{ab} = \delta_{ab} - \frac{\phi_a \phi_b}{\phi^2}$$

Tr and tr denote traces over group and Lorentz indices, respectively.

In the present paper we consider only magnetic field. The reason is two-fold. Firstly, the phase factor drops out while it does not in the electric field case. Secondly, the double parameter integral in $D_\mu D_\nu H(t)$ (Eq.(7)) can be carried out and the final expression becomes very simple while it seems impossible in the electric field case.

The one-loop effective action is given by

$$\Gamma = \int d^n x \left[-\frac{1}{4}F_{\mu\nu}^2 - \frac{\lambda}{4!}\phi^4\right]$$
$$- \frac{1}{2}\log\det g_{gauge} - \frac{1}{2}\log\det g_{scalar} + \log\det g_{ghost} \tag{15}$$

Among these terms the gauge boson contribution is most complicated. Thus we try a simplification of the second term of log det g_{gauge}. Note,

$$-\frac{1}{4}(x-z)A(s)(x-z) - \frac{1}{4}(z-y)A(t)(z-y)$$
$$= -\frac{1}{4}\left\{z - (xA(s) + yA(t))(A(s)+A(t))^{-1}\right\}(A(s)+A(t))\left\{z - (A(s)+A(t))^{-1}(A(s)x + A(t)y)\right\}$$
$$-\frac{1}{4}L(s,t). \tag{16}$$

where

$$-\frac{1}{4}L(s,t) = -\frac{1}{4}(x-y)\frac{A(s)A(t)}{A(s)+A(t)}(x-y) \tag{17}$$

Also note that we may write,

$$(x-z)(x-z) = (x-a+a-z)(x-a+a-z) \tag{18}$$

where

$$a = (x\,A(s) + y\,A(t))(A(s)+A(t))^{-1}$$

Thus upon z' integration ($z' \equiv z - a$) the cross terms in Eq.(18) drops out and we have,

$$\begin{aligned}\int d^n z\, &\{M(s) - M^T(s)(x-z)(x-z)M(s)\} \times \exp\{-\tfrac{1}{4}(x-z)A(s)(x-z) - \tfrac{1}{4}(z-y)A(t)(z-y) + \\ &+ C(s)+C(t)\} \times \exp\{-2ig\,F(s+t)\} \\ = (4\pi)^{n/2}&\{M(s) + (x-a)M(s)\,e^{-2igF(s+t)}M^T(s)(x-a) + 2\,\mathrm{tr}\,M(s)\,e^{-2igF(s+t)}M^T(s) \times \\ &\times (A(s)+A(t))^{-1}\} \times \exp\{-\tfrac{1}{2}\mathrm{tr}\log(A(s)+A(t)) + L(s,t) + C(s) + C(t)\}\end{aligned} \tag{19}$$

To derive Eq.(19) use has been made of

$$\int d^n z\, \exp\{-\tfrac{1}{4}z_\mu A_{\mu\nu} z_\nu\} = (4\pi)^{\frac{n}{2}}\exp\{-\tfrac{1}{4}\mathrm{tr}\log A\} \tag{20}$$

$$\begin{aligned}\int d^n z\, &\exp\{-\tfrac{1}{4}z_\mu A_{\mu\nu} z_\nu\} z_\lambda B_{\lambda\alpha} z_\alpha \\ &= 2(4\pi)^{n/2}\exp\{-\tfrac{1}{2}\mathrm{tr}\log A\}\,\mathrm{tr}\,A^{-1}B\end{aligned} \tag{21}$$

The result can further be simplified by the use of formulae

$$\begin{aligned}&\mathrm{tr}\,M(s) + 2\,\mathrm{tr}\,M(s)\,e^{-2igF(s+t)}M^T(s)\,(A(s)+A(t))^{-1} \\ &= -\tfrac{1}{2}\,\mathrm{tr}\,gF\cot gF(s+t)\end{aligned} \tag{22}$$

and

$$\begin{aligned}&-\tfrac{1}{2}\,\mathrm{tr}\log\{A(s)+A(t)+C(s)+C(t)\} \\ &= -\tfrac{1}{2}\,\mathrm{tr}\log(gFst)^{-1}\sin gF(s+t)\end{aligned} \tag{23}$$

Therefore

$$\begin{aligned}&\log\det \mathcal{J}_{gauge} \\ &= -\frac{1}{(4\pi)^{n/2}}\int d^n x\int dt\; t^{-1-\frac{n}{2}}\,\mathrm{Tr}\,(\mathrm{tr}\cos 2gFt)\exp\Big(-\tfrac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt} - m^2 t - \\ &\quad -\tfrac{1}{4}(x-y)\,gF\cot(gFt)\,(x-y)\Big) \\ &\quad -\frac{1}{(4\pi)^{n/2}}\int d^n x\int dt\int ds\; s^{-\frac{n}{2}}t^{-1-\frac{n}{2}}\,\mathrm{Tr}\,\{-\tfrac{1}{2}\mathrm{tr}\,gF\cot gF(s+t) + \mathrm{tr}\,M^T(s)(x-a) \times \\ &\quad \times (x-a)M(s)\,e^{-2igF(s+t)}\} \times \exp\{-\tfrac{1}{2}\mathrm{tr}\log(gFst)^{-1}\sin gF(s+t) + L(s,t)\} \times \\ &\quad \times \{e^{-m^2 t} - e^{-\alpha m^2 t}\}\end{aligned} \tag{24}$$

In the first term of Eq.(24) exp(-2igFt) is replaced by cos2gFt noting the fact that trace of the odd number of F's is vanishing.

To perform the intergation over s we note

$$\begin{aligned}&\{-\tfrac{1}{2}\,\mathrm{tr}\,gF\cot gF(s+t) + \mathrm{tr}\,M^T(s)(x-a)(x-a)M(s)\} \times \\ &\times \exp\{-\tfrac{1}{2}\mathrm{tr}\log(gF)^{-1}\sin gF(s+t) + L(s,t)\} \\ &= \frac{\partial}{\partial s}\exp\{-\tfrac{1}{2}\mathrm{tr}\log(gF)^{-1}\sin gF(s+t) + L(s,t)\}\end{aligned} \tag{25}$$

since

$$\frac{\partial}{\partial s}\exp\{-\tfrac{1}{2}\mathrm{tr}\log(gF)^{-1}\sin gF(s+t)\} = -\tfrac{1}{2}\mathrm{tr}\log(gF)^{-1}\sin gF(s+t) \tag{26}$$

and

$$\frac{\partial}{\partial s} L(s,t) = -\frac{1}{4}(x-y)\left(\frac{\partial}{\partial s}A(s)\right)\frac{A^2(t)}{(A(s)+A(t))^2}(x-y) \tag{27}$$

and

$$\begin{aligned}
&\mathrm{tr}\, M^T(s)(x-a)(x-a)M(s) \\
&= \frac{1}{4}\mathrm{tr}\,(A(s)-igF)\frac{A(t)}{A(s)+A(t)}(x-y)(x-y)\frac{A(t)}{A(s)+A(t)}(A(s)+igF) \\
&= \frac{1}{4}(x-y)\left(A^2(s)+g^2F^2\right)\frac{A^2(t)}{(A(s)+A(t))^2}(x-y) \\
&= -\frac{1}{4}(x-y)\left(\frac{\partial A(s)}{\partial s}\right)\frac{A^2(t)}{(A(s)+A(t))^2}(x-y)
\end{aligned} \tag{28}$$

Utilizing the above results, we may rewrite Eq.(24) as follows:

$$\begin{aligned}
\log\det \mathcal{G}_{\text{gauge}} &= -\frac{1}{(4\pi)^{n/2}}\int d^nx\int dt\; t^{-1-\frac{n}{2}}\,\mathrm{Tr}\left(\mathrm{tr}\cos 2gFt\right)\exp\Big\{-\frac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt} \\
&\quad -m^2t-\frac{1}{4}(x-y)\,gF(\cot gFt)(x-y)\Big\} \\
&\quad -\frac{1}{(4\pi)^{n/2}}\int d^nx\int dt\int ds\; t^{-1}\,\mathrm{Tr}\frac{\partial}{\partial s}\exp\Big\{-\frac{1}{2}\mathrm{tr}\log gF\sin gF(s+t)-\frac{1}{4}(x-y)\frac{A(s)A(t)}{A(s)+A(t)}(x-y)\Big\}\times \\
&\quad \times\left\{e^{-m^2t}-e^{-\alpha m^2t}\right\} \\
&\quad -\frac{1}{(4\pi)^{n/2}}\int d^nx\int dt\int ds\; t^{-1}\,\mathrm{Tr}\;\mathrm{tr}\,M^T(s)(x-a)(x-a)M(s)\left(e^{-2igF(s+t)}-1\right)\times \\
&\quad \times\exp\Big\{-\frac{1}{2}\mathrm{tr}\log gF\sin gF(s+t)-\frac{1}{4}(x-y)\frac{A(s)A(t)}{A(s)+A(t)}(x-y)\Big\} \\
&\quad \times\left\{e^{-m^2t}-e^{-\alpha m^2t}\right\}
\end{aligned} \tag{29}$$

And then we find that in the magnetic field case where $F_{0i} = 0$ $(i = 1,2,3)$ the last term drops out upon taking the trace over x_i and y_i since $M_{0i} = (e^{-2igF(s+t)} - 1)_{00} = 0$. Therefore s integration can be done. Noting further that $(gF\cot gFt)_{00} = \frac{1}{t}$ and $\left.\left[\frac{A(s)\,A(t)}{A(s)+A(t)}\right]_{00}\right|_{s=0} = \frac{1}{t}$, we end up with

$$\begin{aligned}
&\log\det \mathcal{G}_{\text{gauge}} \\
&= \frac{1}{(4\pi)^{n/2}}\int d^nx\int dt\; t^{-1-\frac{n}{2}}\,\mathrm{Tr}\left(\mathrm{tr}\cos 2gFt\right)\exp\left\{-\frac{1}{4t}(x_0-y_0)^2-\frac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt}-m^2t\right\} \\
&\quad -\frac{1}{(4\pi)^{n/2}}\int d^nx\int dt\; t^{-1}\,\mathrm{Tr}\exp\left\{-\frac{1}{2}\mathrm{tr}\log gF\sin gFt-\frac{1}{4t}(x_0-y_0)^2\right\} \\
&\qquad\qquad \times\left\{e^{-m^2t}-e^{-\alpha m^2t}\right\}
\end{aligned} \tag{30}$$

Now we take the standard procedure to go to the finite temperature case, i.e. Fourier transform from x_0-y_0 to k_0 and replacement of $\int dk_0$ with $\frac{1}{\beta}\sum_{k=-\infty}^{+\infty}$ (with $k_0 = \frac{2\pi k}{\beta}$), to find

$$\begin{aligned}
&\log\det \mathcal{G}_{\text{gauge}} \\
&= -\int d^nx\int dt\;\frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}}\frac{1}{\beta}\sum_{k=-\infty}^{\infty}\mathrm{Tr}\left\{\mathrm{tr}\cos 2gFt\right\}\exp\left\{-\frac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt}-m^2t-\frac{4\pi^2k^2}{\beta^2}t\right\} \\
&\quad +\int d^nx\int dt\;\frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}}\frac{1}{\beta}\sum_{k=-\infty}^{+\infty}\mathrm{Tr}\exp\left\{-\frac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt}-\frac{4\pi^2k^2}{\beta^2}t\right\}\times\left\{e^{-m^2t}-e^{-\alpha m^2t}\right\}
\end{aligned} \tag{31}$$

Scalar and ghost contributions are easy to evaluate. We obtain

$$\begin{aligned}
&\log\det \mathcal{G}_{\text{scalar}} \\
&= -\int d^nx\int dt\;\frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}}\frac{1}{\beta}\sum_{k=-\infty}^{+\infty}\mathrm{Tr}\exp\left\{-\frac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt}-M^2t-\frac{4\pi^2k^2}{\beta^2}t\right\}
\end{aligned} \tag{32}$$

$$\begin{aligned}
&\log\det \mathcal{G}_{\text{ghost}} \\
&= -\int d^nx\int dt\;\frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}}\frac{1}{\beta}\sum_{k=-\infty}^{+\infty}\mathrm{Tr}\exp\left\{-\frac{1}{2}\mathrm{tr}\log\frac{\sin gFt}{gFt}-\alpha m^2t-\frac{4\pi^2k^2}{\beta^2}t\right\}
\end{aligned} \tag{33}$$

Collecting all contributions we obtain an expression of the effective action for the magnetic field case

$$
\begin{aligned}
\Gamma = & \int d^n x \left[-\frac{1}{4} F_{\mu\nu}^2 - \frac{\lambda}{4!} \phi^4 \right] \\
& + \frac{1}{2} \int d^n x \int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \sum_{k=-\infty}^{+\infty} \mathrm{Tr} \left\{ \mathrm{tr} \cos 2gFt \right\} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} - m^2 t - \frac{4\pi^2 k^2}{\beta^2} t \right\} \\
& - \frac{1}{2} \int d^n x \int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \sum_{k=-\infty}^{+\infty} \mathrm{Tr} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} - \frac{4\pi^2 k^2}{\beta^2} t \right\} \left\{ e^{-m^2 t} - e^{-\alpha m^2 t} \right\} \\
& + \frac{1}{2} \int d^n x \int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \sum_{k=-\infty}^{+\infty} \mathrm{Tr} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} - M^2 t - \frac{4\pi^2 k^2}{\beta^2} t \right\} \\
& - \int d^n x \int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \sum_{k=-\infty}^{+\infty} \mathrm{Tr} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} - \alpha m^2 t - \frac{4\pi^2 k^2}{\beta^2} t \right\} \qquad (34)
\end{aligned}
$$

The expression obtained is a very simple one. In fact it is the same as in the zero temperature case so far as the parts which involve group traces are concerned. Shore has already done the group traces. Therefore we may take his result to end up with.

$$
\mathrm{Tr} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} \right\} e^{-m^2 t} = 1 + 2 e^{\mathrm{tr}\, C} e^{-g^2 \phi^2 t} \qquad (35)
$$

$$
\begin{aligned}
& \mathrm{Tr} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} \right\} \mathrm{tr} \cos 2gFt \, e^{-m^2 t} \\
& = n + 2 \, \mathrm{tr} \cos 2gEt \, e^{\mathrm{tr}\, C} e^{-g^2 \phi^2 t}
\end{aligned} \qquad (36)
$$

$$
\begin{aligned}
& \mathrm{Tr} \exp \left\{ -\frac{1}{2} \mathrm{tr} \log \frac{\sin gFt}{gFt} \right\} e^{-M^2 t} \\
& = - \exp \left(-\frac{\lambda}{2} \phi^2 t \right) + 2 e^{\mathrm{tr}\, C} e^{-\left(\frac{\lambda}{6} + \alpha g^2 \right) \phi^2 t}
\end{aligned} \qquad (37)
$$

In Eqs.(35), (36) and (37)

$$
C \equiv -\frac{1}{2} \log \frac{\sin gEt}{gEt} \qquad (38)
$$

and

$$
E = \left(F^a_{\mu\lambda} F^a_{\lambda\nu} \right)^{1/2} \qquad (39)
$$

Inserting (35), (36) and (37) into (34) we obtain

$$
\begin{aligned}
\Gamma = & \int d^n x \left[-\frac{1}{4} F_{\mu\nu}^2 - \frac{\lambda}{4!} \phi^4 \right] \\
& + \frac{1}{2} \int d^n x \, \frac{1}{\beta} \sum_{k=-\infty}^{+\infty} e^{-\frac{4\pi^2 k^2 t}{\beta^2}} \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n}{2}-1}} \\
& \times \left[n - 2 + e^{-\frac{\lambda}{2} \phi^2 t} + 2 e^{\mathrm{tr}\, C} \left\{ \left(\mathrm{tr} \cos 2gEt - 1 \right) e^{-g^2 \phi^2 t} \right. \right. \\
& \left. \left. - 2 e^{-\alpha g^2 \phi^2 t} + 2 e^{-\left(\frac{\lambda}{6} + \alpha g^2 \right) \phi^2 t} \right\} \right]
\end{aligned} \qquad (40)
$$

The result is general gauge dependent. However, in the standard approximation. namely $\lambda \sim g^4$, the gauge dependence drops out, the same consequence as in zero temperature case.

It is also known (G. Shore [2])

$$
\exp \mathrm{tr}\, C = \frac{g^2 t^2 (f^+)^{1/2} (f^-)^{1/2}}{\sin g (f^+)^{1/2} t \, \sin g (f^-)^{1/2} t} \qquad (41)
$$

$$
\mathrm{tr} \cos 2gtE = 2 \left\{ \cos 2g (f^+)^{1/2} t + \cos 2g (f^-)^{1/2} t \right\} \qquad (42)
$$

where

$$
(f^\pm)^{1/2} = \frac{1}{\sqrt{2}} \left[(F_1 + iF_2)^{1/2} \pm (F_1 - iF_2)^{1/2} \right] \qquad (43)
$$

and

$$
\begin{aligned}
F_1 &= \frac{1}{2} (H^2 - E^2) \\
F_2 &= E \cdot H
\end{aligned} \qquad (44)
$$

In our case $E = 0$ and thus $F_1 = \frac{1}{2} H^2$ and $F_2 = 0$. Substituting these into (41) and (42) we have,

$$\exp \mathrm{Tr}\, C = \frac{gHt}{\sinh gHt} \tag{45}$$

$$\mathrm{tr} \cos 2gFt = 2(\cosh 2gHt + 1) \tag{46}$$

Substituting (45) and (46) back into (40),

$$\begin{aligned}\Gamma = & \int d^n x \left[-\frac{1}{4} F_{\mu\nu}^2 - \frac{\lambda}{4!}\phi^4 \right] \\ & + \frac{1}{2}\int d^n x \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \sum_{\ell=-\infty}^{+\infty} e^{-\frac{4\pi^2 \ell^2}{\beta^2}} \times \\ & \times \left[n-1 + 2\frac{gHt}{\sinh gHt} \cdot (4\sinh^2 gHt + 3) e^{-g^2\phi^2 t} \right]\end{aligned} \tag{47}$$

Here we note that an approximation, $\lambda \sim g^4$, has been made in Eq.(40). This is the final expression of Γ.

To derive the effective potential ($\equiv V$) we only have to drop $\int d^n x$ and change $-\frac{\lambda}{4!}\phi^4$ into $\frac{\lambda}{4!}\phi^4$. Therefore we have

$$\begin{aligned} V = & \frac{1}{2}H^2 + \frac{\lambda}{4!}\phi^4 \\ & + \frac{1}{2}\int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \sum_{\ell=-\infty}^{\infty} e^{-\frac{4\pi^2\ell^2}{\beta^2}t} \times (n-1) \qquad (\equiv I_a) \\ & + (gH)^{\frac{n-1}{2}} \int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{1}{\beta} \frac{t}{\sinh t} (4\sinh^2 t + 3) e^{-\frac{m^2}{gH}t} \qquad (\equiv I_{b-1}) \\ & + (gH)^{\frac{n-1}{2}} \int dt \, \frac{t^{-\frac{1+n}{2}}}{(4\pi)^{\frac{n-1}{2}}} \frac{2}{\beta} \sum_{\ell=1}^{\infty} e^{-\frac{4\pi^2\ell^2 t}{\beta^2 gH}} \frac{t}{\sinh t} (4\sinh^2 t + 3) e^{-\frac{m^2}{gH}t} \qquad (\equiv I_{b-2}) \end{aligned} \tag{48}$$

where

$$m^2 \equiv g^2\phi^2 \tag{49}$$

in

In the last two terms of Eq.(48) a change of variable, $gHt \to t$, has been made, which accounts for the factor, $(gH)^{n-1/2}$. This is the one-loop corrected effective potential obtained with an approximation $\lambda \sim g^4$. It incorporates the effects of both temperature and external magnetic field and is gauge independent. In fact it assumes a very simple and compact form. This compactness, however, is not realizable in the case of electric field, so far as we could see. We also mention that our analysis does not seem to go through even forgetting about temperature if scalar belong to other representations than the adjoint representation. The reason is technical. We have introduced a projection operator P_{ab}, which facilities taking the group trace. For other representations we would have to engineer a new method to keep the whole expression in a compact form.

What we would like to do next is to carry out t-integration. This is a rather difficult task. To perform the integral for arbitrary H and β seems to be impossible. All we have been able to manage is to resort to high temperature expansion. For completeness sake, we present our results in the following section.

III. HIGH TEMPERATURE EXPANSION IN THE MAGNETIC FIELD CASE

In this section we present the evaluation of the integrals in Eq.(48).

2.1 I_a integral

$$\begin{aligned} I_a & \equiv \frac{1}{2}\frac{1}{\beta}\frac{1}{(4\pi)^{\frac{n-1}{2}}} \sum_{\ell=-\infty}^{+\infty} \int_0^\infty dt \, t^{-\frac{1+n}{2}} (n-1) e^{-\frac{4\pi^2\ell^2}{\beta^2}t} \\ & = \frac{\pi}{30\beta^4} \end{aligned} \tag{50}$$

where use was made of a formula

$$\sum_{\ell=-\infty}^{+\infty} e^{-\frac{4\pi^2\ell^2}{\beta^2}t} = \left(\frac{\beta^2}{4\pi t}\right)^{1/2} + 2\sum_{k=1}^{\infty} e^{-\frac{\beta^2 k^2}{4t}} \times \left(\frac{\beta^2}{4\pi t}\right)^{1/2} \tag{51}$$

and

$$\int_0^{\infty} dt\ t^{-1-\frac{m}{2}} = 0 \tag{52}$$

2.2 $\underline{I_{b-1}\text{ integral}}$

$$I_{b-1} = \frac{1}{\beta}\frac{1}{(4\pi)^{\frac{m-1}{2}}}(gH)^{\frac{m-1}{2}}\int_0^{\infty} dt\ t^{-\frac{1+m}{2}}\frac{4t}{\sinh t}\left(\sinh^2 t + \frac{3}{4}\right)e^{-\frac{m^2}{gH}t} \tag{53}$$

We may use two formulae

$$\int_0^{\infty} dt\ t^{\mu-1}e^{-\beta t}\sinh \gamma t = \frac{1}{2}\Gamma(\mu)\left\{(\beta-\gamma)^{-\mu} - (\beta+\gamma)^{-\mu}\right\} \tag{54}$$

$$\int_0^{\infty} dt\ t^{\mu-1}e^{-\beta t}\frac{1}{\sinh \gamma t} = 2^{1-\mu}\Gamma(\mu)\,\zeta\left(\mu, \tfrac{1}{2}(\beta+\gamma)\right) \tag{55}$$

to end up with

$$\begin{aligned} I_{b-1} &= \frac{1}{\beta}\frac{1}{(4\pi)^{\frac{m-1}{2}}}(gH)^{\frac{m-1}{2}}\left\{-4\pi^{\frac{1}{2}}\left[\left(\frac{m^2}{gH}-1\right)^{-\frac{3-m}{2}} - \left(\frac{m^2}{gH}+1\right)^{-\frac{3-m}{2}}\right] + \right. \\ &\qquad \left. + 3\times 2^{\frac{m-1}{2}}\times\Gamma\left(\frac{3-m}{2}\right)\times\zeta\left(\frac{3-m}{2}, \frac{1}{2}\left(\frac{m^2}{gH}+1\right)\right)\right\} \\ &= \frac{(gH)^{3/2}}{\pi\beta}\times\left\{-\frac{1}{2}\left(\frac{m^2}{gH}-1\right)^{1/2} + \frac{1}{2}\left(\frac{m^2}{gH}+1\right)^{1/2} - \frac{3\sqrt{2}}{2}\zeta\left(-\frac{1}{2}, \frac{1}{2}\left(\frac{m^2}{gH}+1\right)\right)\right\} \end{aligned} \tag{56}$$

as $m \to 4$.

2.3 $\underline{I_{b-2}\text{ integral}}$

$$\begin{aligned} I_{b-2} \equiv\ & 8(gH)^{\frac{m-1}{2}}\frac{1}{(4\pi)^{\frac{m-1}{2}}}\frac{1}{\beta}\sum_{k=1}^{\infty}\int dt\ t^{-\frac{1+m}{2}}\frac{t}{\sinh t}\left(\sinh^2 t + \frac{3}{4}\right)\times \\ & \times e^{-\frac{m^2}{gH}t}\times e^{-\frac{4\pi^2 k^2}{\beta^2 gH}t} \end{aligned} \tag{57}$$

Firstly we rewrite

$$\frac{t}{\sinh t}\left(\sinh^2 t + \frac{3}{4}\right)e^{-\frac{m^2}{gH}t} = -\frac{1}{2}\left\{\frac{e^{-\left(\frac{m^2}{gH}-1\right)t}}{e^{-2t}-1} + \frac{e^{-\left(\frac{m^2}{gH}+3\right)t}}{e^{-2t}-1} + \frac{e^{-\left(\frac{m^2}{gH}+1\right)t}}{e^{-2t}-1}\right\} \tag{58}$$

[Those who have an experience in summing up the eigenvalue would recognize that the first, second and third term above represent respectively spin-parallel, anti-parallel vector boson contribution and the charged scalar contribution.

In this case we have,

$$E\ (= \text{energy eigenvalue})^{[5]} = (-1)^{2S}\frac{VeH}{4\pi^2}\int_{-\infty}^{+\infty} dk_3 \sum_{n=0}^{\infty}\sum_{S_3}\sqrt{2eH\left(n+\tfrac{1}{2}\right)+k_3^2-2eHS_3}$$

where V and S_3 denote respectively volume and the eigenvalue of spin along the direction of magnetic field.]

Then we make use of a formula,

$$\frac{z e^{xz}}{e^z - 1} = \sum_{l=0}^{\infty}\frac{B_l(x)}{l!}z^l \tag{59}$$

where $B_l(x)$ is Bernoulli polynomial,

$$B_0(x) = 1, \quad B_1(x) = x - \frac{1}{2}, \quad B_2(x) = x^2 - x + \frac{1}{6}. \tag{60}$$

and arrive at

$$\begin{aligned} I_{b-2} = 8\left(\frac{gH}{4\pi}\right)^{\frac{m-1}{2}}\frac{1}{\beta}\sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\frac{1}{4}\int dt\ t^{-\frac{1+m}{2}}e^{-\frac{4\pi^2 k^2}{\beta^2}t}\times \\ \times\left\{\frac{B_l\left(\frac{m^2-1}{2}\right)}{l!}(-2t)^l + \frac{B_l\left(\frac{m^2+3}{2}\right)}{l!}(-2t)^l + \frac{B_l\left(\frac{m^2+1}{2}\right)}{l!}(-2t)^l\right\} \end{aligned} \tag{61}$$

In this form the integrals are tractable. By the use of a formula

$$\int dt\ t^{-\frac{1+m}{2}}e^{-st} = \Gamma\left(\frac{1-m}{2}\right)s^{-\frac{1-m}{2}} \tag{62}$$

we obtain

$$I_{b-2} = \frac{\pi^2}{15\beta^4} - \frac{1}{12}\frac{gH}{\beta^2} + \frac{1}{16}\left(3\frac{m^4}{g^2H^2}+7\right)\times\pi^{\frac{m-8}{2}}\times\beta^{4-m}\times(gH)^2\times\zeta(5-m) \tag{63}$$

The integral is divergent. We perform the minimal subtraction and the final result is

$$I_{b-2} = \frac{\pi^2}{15\beta^4} - \frac{1}{4}\frac{m^2}{\beta^2} + \frac{1}{32\pi^2}\left(\log\beta^2\mu^2 - \log 4\pi + \gamma\right)\left(3m^4 + 7g^2H^2\right) + O(\beta) \tag{64}$$

where μ is the subtraction point.

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