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A LINEAR AND NON LINEAR ANALYSIS OF HIGH-POWER RF AMPLIFIERS*

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Summary

After a survey of the state variable analysis method the final amplifier for the CBA is analyzed taking into account the real beam waveshape.

An empirical method for checking the stability of a non linear system is also considered.

Introduction

The continuous demand for increasing the number of accelerated particles, both in linear and circular machines, results in the need of very sophisticated high power RF systems.

Power levels of several hundreds of kW per unit, stringent stability requirements, low output impedance, low noise, reliability, control capability, wide bandwidth etc... demand a very careful non linear analysis, which must include the effect of beam loading.

Beam induced voltages in accelerating cavities have been extensively studied in connection with beam instabilities, but very little attention has been paid to the effect of the beam on the cavity driving system.

To consider the beam as a simple "impedance," as has been generally done in the past, could be justified by assuming that the final amplifier is a linear device and that the beam current is negligible when compared with the tube current. With the demand for higher beam currents and large power amplifier, this simple assumption is no longer valid.[1]

In this paper an iterative method for calculating the beam loading effect is developed where the beam is schematized as an ideal current generator connected to the accelerating gap and the actual shape of the beam current is taken into account. This method of calculating is then applied to a network mesh representing the equivalent scheme of the whole circuit where the actual characteristics of the tube are to be taken into account.

The State Variable Approach

In principle any of the well known methods can be used for characterizing the behavior of an electrical network, but for non linear feedback systems the state variable method is preferable for the following reasons:

a. It is applicable to any electrical network without any restrictions.

b. The derivatives of the input stimuli (voltages and currents) do not appear in the final differential equations.

c. Both, voltage and current, generators can be simultaneously used in the same network without any special transformation.

d. Each of the equations that describes the behavior of the network can be reduced to the "normal form" as indicated.

$$\dot{q}_k = F_k(q_1 \ldots q_n, s_1 \ldots s_m, t), \qquad (1)$$

where q_k are the chosen state variables, s are the impressed stimuli (voltages and currents), t is the time, and, in general, F_k's are non linear functions of the previous variables.

The state variable method, extensively treated in many excellent books and articles,[2] can be summarized as follows:

1) The voltages across the capacitors and the current through the inductors are normally chosen as the state variables of the system; polygon of capacitors and stars of inductors, obviously, reduces the number of independent variables. Sometimes a different choice of the state variables might be more suitable; an example is given in the Appendix.

2) As many nodal and mesh equations are written in order to match the number of independent variables, and, it is clear, only the first derivative of the variables are implied.

3) The system obtained after the reduction to "normal form", of the various equations, can be integrated numerically with the standard methods now available.

Normally, the third point presents the most difficulty, but, fortunately, this is not our case. In dealing with the power RF amplifiers the non linearity is introduced by the power tube characteristics, including the occurrence of the cut-off. Recognizing that the tube current is controlled by the voltages developed across the inherent tube capacitances, the non linearity cannot, therefore, involve the derivatives of the state variables. This means that the reduction to "normal form" of the state equations can be obtained in a very straight-forward way with Kramer's rule.

A Typical Example

As an example, an analysis is made for a simplified model of the CBA accelerating final amplifier.

Due to the very special performance required, the accelerating cavity is driven with a cathod follower amplifier.[3] For the purpose of illustrating the procedure, a simplified equivalent circuit will be used. As it should be noted, the analysis is not limited by the circuit complexity. The following "reasonable" simplifications are introduced:

1) A parallel RLC circuit simulates the behavior of the cavity in the neighboring of the dominant mode.

2) The driving amplifier is schematized with a voltage generator and its own output resistance.

3) The biases are schematized with ideal batteries.

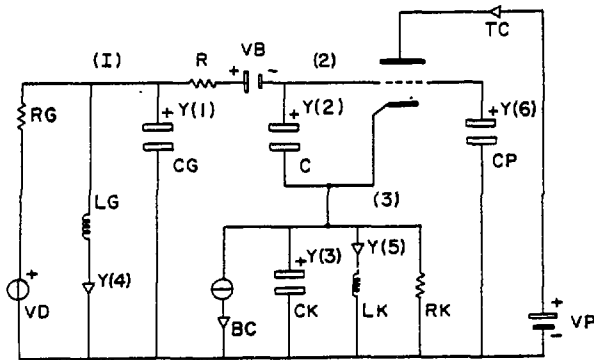With these assumptions the circuit to be analyzed is as shown in Fig. 1.

Fig. 1. Simplified scheme of the accelerating amplifier.

The network considered has four capacitors and two inductors; therefore, there are six state variables. The variables are designated by Y(j). For the particular case in Fig. 1, we have that Y(2) + Y(3) = Y(6) and the number of independent variables are then reduced to 5. BC indicates the beam current (that is assigned). TC indicates the tube current that depends upon the voltages applied to the tube which are a consequence of the driver, bias voltages and beam current.

The "state equations", three for the nodes and two for the meshes, are written as follows:

$(Y(1)-VD)/RG+Y(4)+CG*\dot{Y}(1)+(Y(1)-(VB+Y(2)+Y(3)))/R=0$

$(Y(1)-(VB+Y(2)+Y(3)))/R=CP*(\dot{Y}(2)+\dot{Y}(3))+C*\dot{Y}(2)$

$Y(3)/RK+Y(5)+CK*\dot{Y}(3)=C*\dot{Y}(2)+TC-BC$

$LG*\dot{Y}(4)=Y(1)$

$LK*\dot{Y}(5)=Y(3)$.

Simplifying the notations we set:

$F1=(Y(1)-(VB+Y(2)+Y(3)))/R$, $F2=TC-BC-Y(3)/RK-Y(5)$

$F3=(Y(1)-VD)/RG+Y(4)$ and $F4=(CP+C)*CK+CP*C$.

Substituting and reducing to "normal" form we obtain:

$$\dot{Y}(1)=-(F1+F3)/CG$$

$$\dot{Y}(2)+(CK*F1-CP*F2)/F4$$

$$\dot{Y}(3)=((CP+C)*F2+C*F1)/F4 \qquad (2)$$

$$\dot{Y}(4)=Y(1)/LG$$

$$\dot{Y}(5)=Y(3)/LK$$

The system can be solved numerically as soon as the beam and tube current are specified. With a high degree of accuracy we can assume that

$$BC=4*BCP*(\frac{t-t0}{d}) * (1-\frac{t-t0}{d}),$$

$$BC(t)=BC(t+T), \text{ and } BC \geq 0.0$$

Where T is the RF period, d is the beam duration, t indicates the time (starting from zero at the beginning of each cycle), t0 is the delay of the beam at the beginning of the RF cycle and BCP is the beam peak current.

The tube current characteristics could be stored in the computer and read at each step of the integration. In many cases a very good approximation can be reached using the formula

$$TC=G*(VP-Y(3)+\mu*Y(2))^a=G*VT^a \text{ and}$$

$$TC=0.0, \text{ for } VT \leq 0.0 \text{ and for } VP-Y(3) \leq Y(2),$$

where G and a are constants that depend upon the tube.

The system (2) was integrated with a fourth order Runge-Kutta method on a CDC-7600 computer for the following value of the parameters:

CK=12E-9, LK=3.82E-5, RK=6000, C=250E-12, CP=250E-12,

R=500, RG=700. LG=2.29E-4, CG=2E-9, VP=12000,

VB=350, G=8.21E-5, μ=32.7, a=1.5,

T=4.25E-6, t0=d=T/3, BCP=36, VD=1E4*SIN(2π t/T).
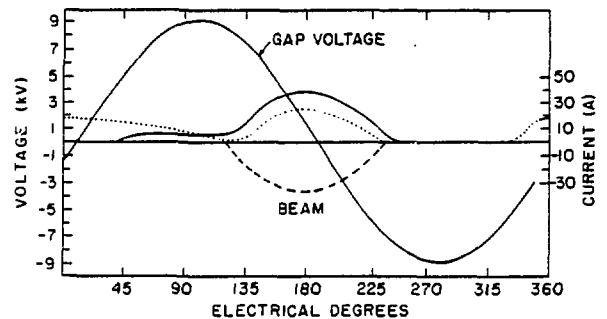
The results are summarized in Fig. 2.

Fig. 2. Gap voltage, beam and tube currents after 99 period of RF and beam exitation.

The dashed line represent the beam current, reversed in sign for clarification. The solid line indicates the current in the tube when the cavity is tuned to its resonant frequency. The dotted line indicates the current that would flow in the tube if the cavity were tune to a lower frequency in order to compensate for the quadrature component of beam current. It is evident that there is a reduction in the peak tube current while the average tube current is only slightly affected.

We note that the gap voltage is essentially sinusoidal (due to filtering action of the cavity), while the tube current is badly distorted. This clearly demonstrates that the beam cannot be considered as a "simple" impedance, since the voltage across the gap has essentially no higher order Fourier frequency components, but the same cannot be said about the tube current.

The resistor R (Fig. 1) provides the damping that prevent the tube from oscillating. A larger part of the phase shift between the input and output voltage is the result of this resistor and the capacitance from grid to ground.

Very often the beam dynamic requirements dictates an overall maximum value for the output impedance of the amplifier-cavity system. For non linear systems both the definition of output impedance and the use of the superposition principle become meaningless. In this case the designer should interpret the given "output impedance" as a parameter indicating the order of magnitude of the beam induced voltage that is tolerable and only the fourier analysis of

the resulting waveform will tell if the beam dynamic requirements are met. For this case a correct operation of the cathode follower at higher frequencies is guaranteed by the capacitor CP. This capacitance is made equal to the grid cathode tube capacitance to insure that the feedback ratio β is greater than 0.5.

An Empirical Stability Criterion

It is well known that a linear time invariant system is stable if, and only if, all the eigen value of the state variable matrix have negative real parts. On the other hand, to determine whether or not a non linear system is stable might lead to an almost intractable mathematical problem.

For the designer it is important to know if a system is stable over a band width equal to a few order of magnitude of the operating frequency for which the system is designed to work. If in the actual system an oscillation were to occur at a frequency far removed from the operating frequency, it would be a relatively simple matter to damp this oscillation without effecting the fundamental characteristics of the design. Accordingly the procedure to check empirically the stability, using the previously derived network equation could be as follows:

1. The driving voltage (or current) and the beam current are set equal to zero.

2. Some reactive elements (usually capacitors) are given initial conditions with values very much larger than those normally reached under the design working conditions. Moreover, those elements should be chosen in such a way, as to be able to exite all the modes of the network to be examined.

3. The integration of the system is performed with a step (fixed or variable) that is correlated with the frequency range to be examined. Then if the computer answer goes to zero after a reasonable time, the system can be considered as "practically" stable.

It is obvious that the above test is only empirical, and that, moreover, it's accuracy depends upon the integration technique.

Appendix

For analyzing the circuit indicated in Fig. 3, we could assume that the currents I1 and I2 in the inductors, and the voltage V across the capacitor are the state variable characterizing the system.
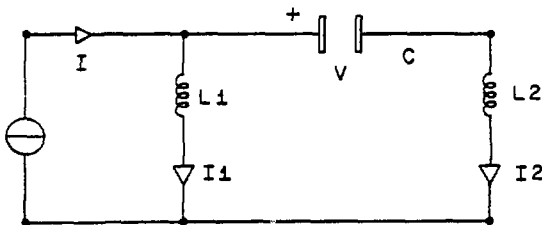

Fig. 3

With this choice the solving system cannot be written in "normal form" because the derivative of the input current appears explicitly as follows:

$$\dot{I1} = A1 \cdot \dot{I} + B \cdot V$$

$$\dot{I2} = A2 \cdot \dot{I} - B \cdot V$$

$$\dot{V} = (I - I1)/C,$$

where $A1 = L2/(L1+L2)$; $A2 = L1/(L1+L2)$ and $B = 1./(L1+L2)$.

The introduction of the first derivative of the input could be easily avoided with a different choice for the state variables as follows:

If we put $q1 = I1 - A1 \cdot I$ and $q2 = I2 - A2 \cdot I$ then the above systems becomes,

$$\dot{q1} = B \cdot V$$

$$\dot{q2} = -B \cdot V$$

$$\dot{V} = (A2 \cdot I - q1)/C,$$

where the first derivative of the input does not appear explicitly.

References

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