Conf-810767--7

# Uncertainties in the Calculation of Solar-Neutrino Capture Rates


B. W. Filippone
Argonne National Laboratory, Bldg. 203
9700 S. Cass Avenue
Argonne, IL 60439 USA
and
The University of Chicago, Chicago, IL 60637



CONF-810767--7

DE83 009598



## Abstract

A detailed estimate is presented of the possible uncertainty range for the neutrino flux from a standard solar model. Using present estimated errors in the key input parameters, detailed solar models are calculated to give an uncertainty in the theoretical $\nu_e$ capture rate in both the on-going $^{37}$Cl experiment and the proposed experiment using $^{71}$Ga. The uncertainty in capture rate is investigated by considering individual parameter variations about a mean model, by simultaneously varying several key parameters to yield upper and lower limits, and by a Monte Carlo method.


## Introduction

The apparent discrepancy between the calculated and observed flux of $\nu_e$ from the presumed nuclear reactions in the solar interior has stimulated much discussion about our understanding of both stellar evolution and neutrino physics. The importance of the sun for these studies is twofold: one, because it is so near many of its properties are eaily measured and two, because it is so far it provides a long baseline to observe possible neutrino oscillations.

However because of the complications involved in the calculation of a model for the sun, as well as the large amount of external input data needed (both experimental and theoretical) a serious comparison between observations and predictions must include a detailed analysis of uncertainties in the predictions. It is therefore important to re-examine the theoretical predictions[1,2] (especially in light of recent changes in many of the solar model input parameters—in some cases outside of previously quoted errors) to determine if there does indeed exist a discrepancy between theory and experiment.

With this in mind we have performed extensive solar-model calculations, first as an independent check of the new value for the $^{37}$Cl capture rate, and second to push the input parameters of a standard model to the extremes of experimental and theoretical limits, to see what can be learned of stellar evolution or neutrino physics from solar neutrino experiments. The capture rates and uncertainties are calculated for both the on-line $^{37}Cl$ experiment[3] and the proposed $^{71}Ga$ experiment.[4]

The effects of uncertainties in the input parameters on the predicted capture rate in the two detectors is examined in three ways: (1) by calculating the effect of individual uncertainties on the mean standard model and summing these errors in quadrature, (2) by simultaneously varying several key parameters to yield upper and lower limits, (3) by generating a set of models whose parameters are determined by a Monte Carlo method.

This last procedure may prove the most illuminating for reasons that will be discussed below. A more detailed discussion of the calculation will be presented elsewhere.[5]

## Key Input Physics

Although there are many assumptions involved in constructing a solar model[6] we will concentrate on two of them:

1.) The rate of energy production is given by the reactions of the familiar p-p chain (with a few percent from the CNO cycle).

2.) Energy is transported to the surface by radiation and convection (which process dominates in a particular region depends on local conditions).

It is uncertainties in the input physics for these assumptions (nuclear cross sections for the first and radiative opacities for the second) that dominate the uncertainty in the prediction of solar neutrino capture rates.

The difficulty with the determination of reaction rates lies in the relatively (as far as nuclear reactions are concerned) low temperature in the solar core, i.e. $kT \approx 1.3$ keV. When this thermal distribution is averaged over the decreasing cross section due to the Coulomb barrier one finds most of the p-p reactions take place at a few tens of keV. The resulting cross sections at these energies are virtually inaccessible to laboratory measurement due to the high Coulomb barriers involved (on the order of 1 MeV).

However, below the Coulomb barrier, by factoring out the non-nuclear energy dependence in the cross section, the non-resonant p-p reactions may be parametrized by a nearly energy-independent variable, the so-called S factor[6,7]

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta}$$

where $\eta \equiv Z_1 Z_2 e^2/hv_{12}$, the 1/E factor is from the usual $\lambda^2$ geometrical term, and the exponential accounts for the penetration probability.

Of the experimentally accessible S factors the canonical uncertainties have been of the order of 10%, derived from extrapolation to low energy of the measured cross section. There is recent evidence though that two of the S factors - $S_{34}$ and $S_{11}$ - may differ considerably from their previously accepted values.

Two low-energy measurements[8] of the $^3He(\alpha,\gamma)^7Be$ reaction had yielded $S_{34}(0) = 0.61 \pm 0.06$ keV - barns from a polynomial fit to the combined data. However, new measurements by Rolfs et al.[9], made at even lower energy than the previous experiments indicate that $S_{34}(0)$ may be reduced by as much as 55%. It should be noted that the disagreement is with the normalization of the cross section as the shape appears now to be in agreement with previous experiments and theoretical calculations. Also a recent reanalysis of the old data by Parker[10] reduces $S_{34}(0)$ by ~ 15% from the old value.

New experimental work has also affected $S_{11}$ which is used to calculate the rates of both $^1H(p,e^+\nu_e)^2H$ and $^1H(pe^-,\nu_e)^2H$. Since these reactions essentially convert a proton to a neutron, it is not surprising that $S_{11}$ depends sensitively on the neutron half-life.[11] Two recent measurements[12] of the half-life are in considerable disagreement (nearly three standard deviations between them-however one is in good agreement with an earlier measurement[13]). Since the $^{37}Cl$ capture rate $\propto S_{11}^{-2.4}$ this disagreement can result in a large (~ 20%) range in the capture rate.

The question of the uncertainty in the radiative opacities is a difficult one to address. Part of the problem is that there are no laboratory checks of the opacity in the density-temperature regime appropriate for the sun. In addition errors can arise due to experimental input data (e.g. atomic structure and metal abundances—where changes larger than quoted errors have occurred in the past) and due to the theoretical approximations used in the calculation.

Previous discussion of the uncertainty in the mean opacities[14,15] quoted errors of 10-20% in different regions of the $\rho$-T plane. Recently Bahcall et al.[1] have generated solar models with two opacity codes, one from Los Alamos National Laboratory and the other from the Lawrence Livermore Laboratory. He reports that the most recent opacities show an increase of 15% in the center from the old opacities with the Los Alamos code, while the Los Alamos and Livermore codes differ by 10-15% in different regions.

Although the new opacities from the two codes differ by only 10-15%

from each other, in view of the above discussion we have assigned 1$\sigma$ errors of ±15% to the opacity throughout the sun.

It should be noted that the major change between the previous models[16,17] yielding 4.7 SNU and the most recent one[1] yielding 7.8 SNU (for the Los Alamos opacities) is the assumed opacity which accounts for ~ 60% of the difference.

## Results

Using the mean values of the input parameters listed in Table I, neutrino capture rates were calculated at $4.6 \times 10^9$ years after contraction to zero-age main sequence. The results are 7.0 SNU (SNU $\equiv 10^{-36}$ captures/target atom-sec) for $^{37}$Cl and 111 SNU for $^{71}$Ga. The higher value for $^{71}$Ga is due

TABLE I

The effect on the mean model of individual parameter uncertainties. $\sigma_{^8B}$, $\sigma_{^7Be}$, and $\sigma_{pp}$ are the neutrino capture rate cross sections. OPI, OPII, and OPIII are surface, middle, and core opacities, respectively.

| Parameter | Adopted Error | $\Delta^{37}$Cl SNU (%) | $\Delta^{71}$Ga SNU (%) |
|---|---|---|---|
| $S_{11}$ | 5% | 0.76 (11%) | 2.0 (2%) |
| $S_{33}$ | 10% | 0.22 (3%) | 1.0 (1%) |
| $S_{34}$ | +17% −44% | 1.55 (22%) | 7.0 (6%) |
| $S_{17}$ | 10% | 0.52 (7%) | ~ 0 – |
| $S_{e7}$ | 5% | 0.26 (4%) | ~ 0 – |
| $\sigma_{^8B}$ | 10% | 0.52 (7%) | 0.1 (<1%) |
| $\sigma_{^7Be}$ | 10% | 0.12 (2%) | 3.1 (3%) |
| $\sigma_{pp}$ | 10% | ~ 0 – | 6.7 (6%) |
| OPI | 15% | 0.03 (<1%) | 0.5 (<1%) |
| OPII | 15% | 1.10 (16%) | 3.5 (3%) |
| OPIII | 15% | 1.39 (20%) | 3.5 (3%) |
| Z | 11% | 0.56 (8%) | 2.5 (2%) |
| Age | 2% | 0.30 (4%) | 1.2 (1%) |
| Luminosity | 0.7% | 0.30 (4%) | 1.6 (1%) |
| Total | – | 2.70 (39%) | 12 (11%) |

chiefly to its relatively large cross section for the high flux p-p neutrinos, which are below threshold in $^{37}$Cl. Models were also calculated with $S_{34}(0)$ = 0.29 keV-barns, the value suggested by Rolfs[9] which gives a total of 4.8 SNU for $^{37}$Cl and 101 SNU for $^{71}$Ga. The result for $^{37}$Cl is in good agreement with the Bahcall et al.[1] value of 7.8 using the LANL opacities, the difference being mainly due to slightly different parameter choices.

To investigate the uncertainties we will first follow previous workers[14,17] and calculate an uncertainty in the capture rate by constructing Table I where the uncertainties in the input parameters are inserted individually into the standard mean model to give an uncertainty in SNU associated with each parameter. The resulting errors (assumed independent) can then be added in quadrature to give an overall uncertainty. Carrying out this procedure with the present data gives ±39% or 7.0 ± 2.7 SNU uncertainty in the standard mean model for $^{37}$Cl and ±11% or 111 ± 12 SNU for $^{71}$Ga.

The effect of a particular parameter-$S_{34}$-on the $^{37}$Cl capture rate is shown in Fig. 1. Included are the 1 σ uncertainties from the other parameters of Table I and the 1 σ limits from the experiment of Davis.[3]. For a value of $S_{34}$ ≈ 0.30 keV-b, the disagreement between theory and experiment is just over one standard deviation.

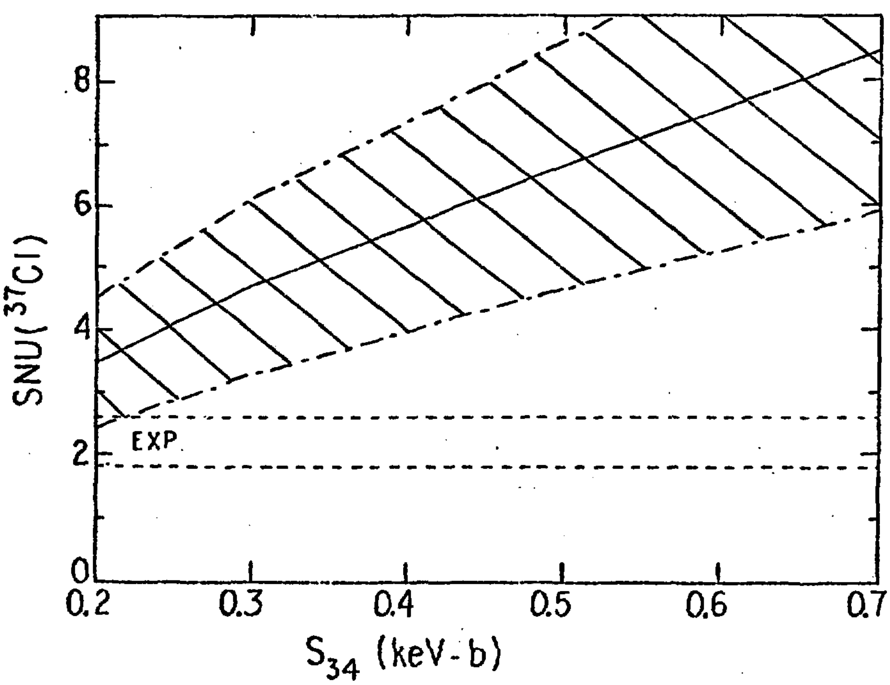


Fig. 1. Dependence of the $^{37}$Cl capture rate on $S_{34}$ (solid line). Also shown are the 1σ limits in both the experiment of Davis and the present calculation.

The problem with determining an overall uncertainty by summing individual errors in quadrature (as in Table I) is that many of the parameters enter in a highly non-linear manner. In other words since the $^{37}Cl$ capture rate is such a strong function of temperature ($\lesssim T^{14}$) the effect of a particular parameter could change significantly if one deviates slightly from the mean model-a point first brought up by Rood.[18] Therefore we have also generated a set of models where the parameters are allowed to vary simultaneously in a Gaussian distribution, about their mean values. The means and standard deviations of the parameters are taken from Table I.

The results, in the form of histograms, are shown in Figs. 2 and 3. The $^{37}Cl$ distribution shows a peak between 5 and 6 SNU which is probably due to the asymmetric error in $S_{34}$. In addition, the fact that values less than zero SNU are impossible may cause the skewness in the distribution with its high SNU tail.

Although the distribution for $^{37}Cl$ is clearly not Gaussian, a standard deviation can be extracted. For $^{37}Cl$ the result is ±3.0 SNU (43%). For $^{71}Ga$ the result is ±13 SNU (12%). The much smaller error for $^{71}Ga$ is, of course, due to the sensitivity of $^{71}Ga$ to neutrinos from the $H(p,e^+\nu_e)^2H$ reaction (below threshold for $^{37}Cl$) whose flux is effectively determined only by the solar luminosity.

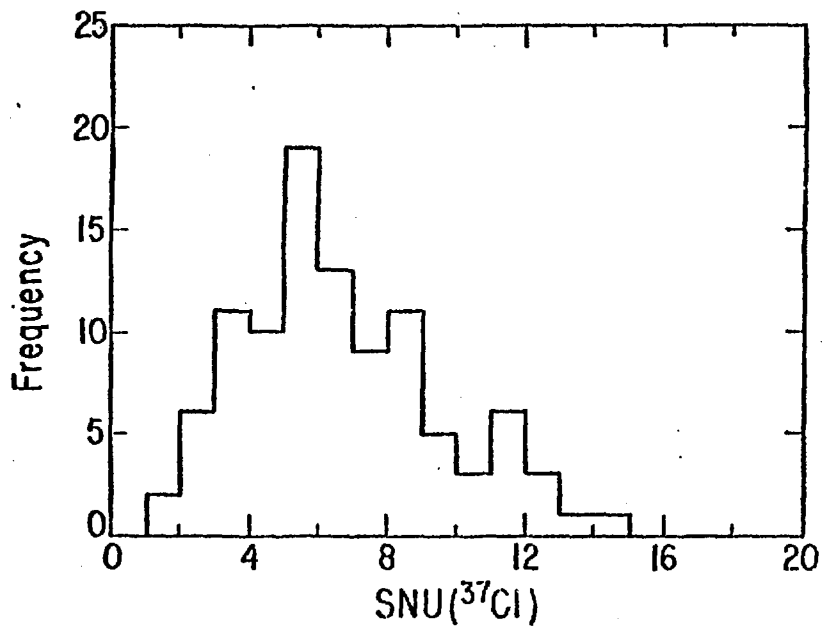


Fig. 2. Distribution of the $^{37}Cl$ capture rate from the Monte Carlo calculation.

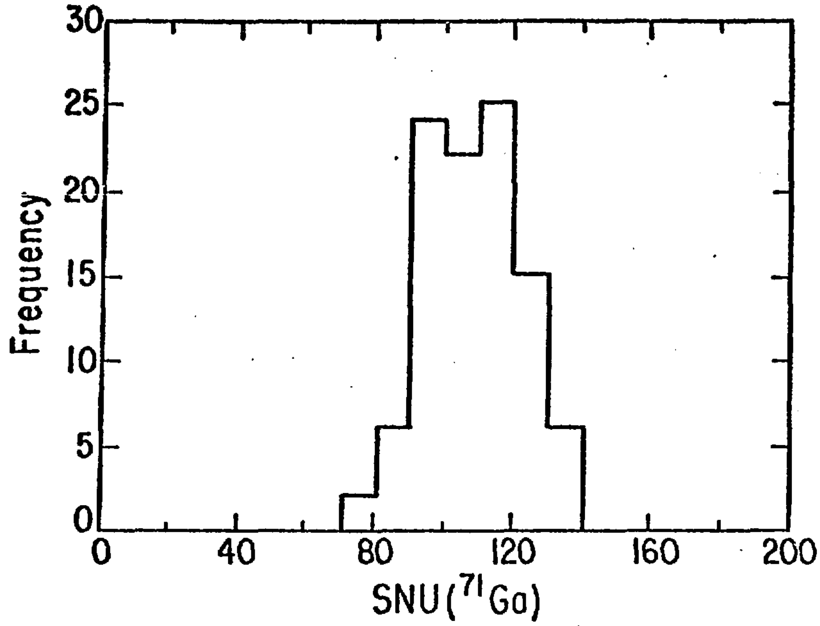


Fig. 3. Distribution of the $^{71}Ga$ capture rate from the Monte Carlo calculation.

## Conclusions

It must be pointed out that while the uncertainties used in these calculations may be in a few cases somewhat larger than the popularly quoted values, all calculations were done within the standard model of the sun. The possibility of non-standard models affecting the solar neutrino flux could introducd significant changes and uncertainties when attempts are made to quantify the amount of departure from the standard assumptions.

It would appear though, that even within the standard model, uncertainties in the basic input parameters indicate only weak evidence for phenomena such as neutrino oscillations or exotic stellar evolution in the "solar neutrino problem". Better measurements of the neutron $t_{1/2}$ and the cross section for $^{3}He(\alpha,\gamma)^{7}Be$ will certainly reduce the error in the predictions, but uncertainties in the opacity and heavy element abundance appear very difficult to reduce. Given all of these uncertainties and their sensitive effects on the $^{37}Cl$ experiment, it is clear that we may not have a resolution to the "solar neutrino problem" or even know if there really is a problem until an experiment sensitive to p-p neutrinos like the $^{71}Ga$ experiment is carried out.

## Acknowledgments

I would like to especially thank D. Schramm for many useful discussions. In addition I would like to acknowledge helpful discussion with

R. Rood, W. Fowler, L. Aller, C. Rolfs, P. Parker, and R. Davis, Jr.; with special thanks to A. Cox and the LANL opacity group for providing the opacities. This work was supported in part by the U. S. Department of Energy at Argonne under Contract W-31-109-Eng-38 and NSF grant AST 78-20402 and DOE grant DE-AC02-80ER10773 at Chicago.

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