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ON THE PROPERTIES OF W- BOSON WITHIN THE SUBCONSTITUENT MODELS OF ELECTROWEAK INTERACTIONS.

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A B S T R A C T

We discuss some properties of the W- boson using spectral function sum rules within the framework of constituent models of quantum haplo-dynamics (QHD).
Contraints on the W- decay amplitude and so on its mass and total width have been derived. The results may give a test on possible aspects of this model of electroweak interactions.

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I. INTRODUCTION TO THE MODEL

Recent interests on models where leptons, quark and weak bosons are composite particles have been considered in the literature [see e.g ref.1 for a review]. In some of these models [the haplon model], the weak interactions can be interpreted as effective interactions of the Van Der Vaals type [2] like the nuclear forces in quantum chromodynamics (QCD). At the subconstituent level, the dynamic is described by the gauge symmetry group $U(1)_{EM} \times SU(3)_C \times G_H$ where G_H is the hypercolor group responsible for the binding of subconstituents in the fermions and weak bosons. For simplicity, G_H is taken to be $SU(N)_H$. Within the framework, the W- boson can be taken to be a bound state of elementary spin 1/2 fermions (α, β,...) where the latter are hypercolored N-plets and can be colored or color singlets. In the following, we shall consider the simplest case, where α, β,.... are color singlets and so the W-boson can be represented as :

$$\begin{aligned} W^+ &\equiv \alpha\bar{\beta} \\ W_3 &\equiv \frac{1}{\sqrt{2}}(\alpha\bar{\alpha} - \beta\bar{\beta}) \\ W^- &\equiv \bar{\alpha}\beta \end{aligned} \tag{1}$$

a representation similar to the case of the ρ-meson in the quark model. W_3 is the isovector component of the W- boson which is responsible of the most part of the observed neutral current. The massless photon and gluons are supposed to be elementary; so their interactions with the subconstituents occur in a point-like way.

In this model, the masses of the W- bosons are generated dynamically in the much same way the ρ-meson mass is generated in QCD. In the absence of electromagnetism the $W^\pm$ and W_3 masses are degenerated [weak isospin SU(2) symmetry] as in the case of the $\rho^\pm, \rho^0$, for the strong SU(2) symmetry. The breaking of the $SU(2)_W$ is, therefore, due the W_3-boson and the photon mixing via Fig.1, and leads to a non-diagonal W- mass matrix with the aigenvalues [1,2]

$$M_W \simeq g\ 123\ \text{GeV} \tag{2}$$

$$M_Z \simeq M_W (1-\lambda^2)^{-1/2} \tag{3}$$

representing the observed W-boson masses; g is the strength of the W-fermion vertex which is related to the Fermi coupling G_F as usual ($G_F/\sqrt{2} \equiv g^2/8M_W^2$); λ is the mixing parameter related to the effective value of the weak angle $\sin^2\theta_W \simeq 0.22$ via :

$$\lambda = g \sin^2\theta_W \frac{1}{e} \tag{4}$$

Using a W- dominance to the spectral function associated to the weak current $J^{\mu}_{W} \equiv \bar{\alpha}\, \gamma_{\mu} \frac{(1-\gamma_5)}{2} \beta$, one can introduce the decay amplitude F_W in the similar way as for the ρ - meson :

$$< o \; | J^{\mu}_{W} | W^{\pm} > \; = \; \varepsilon^{\mu} M_W F_W \quad . \tag{5}$$

Assuming the W dominance to the form factors of the weak-current, i.e to the matrix elements between lepton or quark states and using the normalization of the form factor at zero-momentum transfer, it can be easily shown that [1] :

$$\frac{F_W}{M_W} = \frac{1}{g} \quad . \tag{6}$$

However, the relation in eq (6) comes strongly from the above assumption fo the W- dominance.

A more weaker constraint can be obtained relating the W- decay amplitude to the mixing parameter λ. Following ref. 1a, one can deduce :

$$\lambda = e . \frac{F_W}{M_W} \quad . \tag{7}$$

Note that eqs (4) and (7) show that the result in eq (6) is the particular case where $g = e/\sin\theta_W$, i.e where the model reduces to the standard electroweak $SU(2)_L \times U(1)$ theory. The relation in eq (7) suggests that, once, one has an information on F_W, one can translate this information on other parameters of the model via λ and eqs (2) to (4).

2. SPECTRAL FUNCTION SUM RULES FOR F_W

In the following, we shall use the idea of spectral function sum rules in order to derive bound on the W- decay amplitude in such a model. Our approach is similar to that used in QCD for light mesons system (π, ρ,...) and is based on the Laplace transform sum rule of SVZ [3] :

$$F(M^2) = \frac{1}{M^2} \int_0^{\infty} dt\, e^{-t/M^2} \frac{1}{\pi} \operatorname{Im} \Pi(t) \tag{8}$$

where Im $\Pi(t)$ is the spectral function associated to the two-point function :

$$\begin{aligned} \Pi^{\mu\nu}(-q^2 \equiv Q^2 > 0) &\equiv \int d^4x\, e^{iqx} \langle 0 | \mathrm{T}\, J^{\mu}_{W}(x) \left(J^{\nu}_{W}(0)\right)^{\dagger} | 0 \rangle \\ &= -(g^{\mu\nu} q^2 - q^{\mu} q^{\nu})\, \Pi(Q^2) + g^{\mu\nu} D(Q^2), \end{aligned} \tag{9}$$

with $J^{\mu}_{W} \equiv \bar{\alpha}\, \gamma_{\mu} \frac{(1-\gamma_5)}{2} \beta$, being the charged weak current; M is the Laplace transform sum rule scale.

In fact, the Laplace transform sum rule is the improved version of the dispersion relation :

$$\Pi(Q^2) = \int_0^{\infty} \frac{dt}{t + Q^2} \frac{1}{\pi} \operatorname{Im} \Pi(t) + \text{« subtractions »} \tag{10}$$

and is obtained after applying to both sides of eq. (10) the Laplace operator

$$\hat{L} \equiv \lim_{\substack{Q^2 \to \infty \\ N \to \infty \ \frac{Q^2}{N} \equiv M^2}} (-1)^N \frac{1}{(N-1)!} \left(\frac{\partial}{\partial Q^2} \right)^N . \tag{11}$$

The main attraction on the Laplace transform sum rule is the role of the exponential factor which improves the contribution of the low mass resonance to the spectral function for a reasonable value of M. In QCD, the optimal value of M^2 is around $1 \sim 2.$ GeV^2 where we have a balance between the non-perturbative contributions to the two-point function and the continuum contribution to the spectral function [3, 4]. On the other hand, because we work with various derivatives of the two-point function, we can avoid a possible dependence of the sum rule on the external subtraction point.

The left-hand side of the sum rule is determined by the hypercolor theory (QHD) and the lowest order contribution to $F(M^2)$ comes from the Feynman diagram in Fig. 2 [for $SU(N)_H \equiv G_H$] :

$$F(M^2) \simeq \frac{N_H}{24\pi^2} \left\{ 1 + \text{'corrections'} \right\} . \tag{12}$$

The " correction-terms" can be due to gluon exhange as well as to the possible contributions of operators of higher dimensions which have a non-zero vacuum expectation valves. These operators are mainly the haplon-condensate $m_\alpha \langle o | \bar{\beta} \beta | o \rangle$, $\langle o | \bar{\alpha} \Gamma \beta \bar{\beta} \Gamma \alpha | o \rangle$ (Γ is any Dirac matrices). The gluon condensate $\langle o | \alpha_s G^2 | o \rangle$ contribution is expected to be suppressed, from our knowledge of the QCD estimate of that quantity, at the energy scale where the sub-constituent nature of the W shows up.

The right-hand side of the sum rule is estimated using a narrow width approximation to the W- contribution to the spectral function and adding the "continuum" contribution :

$$\frac{1}{M^2} \int_0^\infty dt \, e^{-t/M^2} \frac{1}{\pi} \operatorname{Im} \Pi(t) \simeq \frac{1}{M^2} F_W^2 e^{-M_W^2/M^2} + \text{« continuum »} . \tag{13}$$

We use the positivity of the continuum contribution to the sum rule and we neglect to a first approximation the QHD corrections provided that $M^2 \gg \Lambda_H^2$ (Λ_H is the QHD scale analogous to Λ_C of QCD). Then eqs. (12) and (13) give :

$$F_W \lesssim \left(\frac{M}{M_W}\right) \left(\frac{N_H}{24\pi^2}\right)^{1/2} e^{M_W^2/2M^2} \quad M_W \qquad (14)$$

3. OPTIMIZATION OF THE SUM RULE AND THE NATURE OF THE W- BOSON

The optimization of the constraint in eq. (14) can be obtained using a direct analogy between QCD and QHD. In fact, as we mentioned earlier, the Laplace transform sum rules applied to light mesons system, bound states of light quarks, are optimized for $M^2 = 1 \sim 2.\ GeV^2$, i.e. for M of the order of $(8 \sim 10)\ \Lambda_C$ where Λ_C is the QCD scale which has a value of the order of m_π. Really, at this optimization point, we have a balance between the non-perturbative effects to the sum rule and the continuum one.

It is interesting to use this observation for the optimization of the QHD sum rule. In fact, if the analogy idea between QHD and QCD is correct, we expect to have a replica of the light quark condensate $m_u < o|\ \bar{d}\ d\ |o>$ of QCD in the theory of composite models. As from current algebra $m_u < o|\ \bar{d}\ d\ |o> \simeq -\frac{1}{2}\ f_\pi^2\ m_\pi^2 \simeq -\frac{1}{2}\ \Lambda_C^4$, we expect that the value of $m_\alpha < o|\ \bar{\beta}\ \beta\ |o>$ is of the order of $-\frac{1}{2} \cdot \Lambda_H^4$ in QHD. So, because of the simple structure of the weak charged current, we can have a rough order of magnitude of the contribution of the haplon condensate to the QHD sum rule translating the QCD result of the contribution of these non-perturbative terms to the sum rule associated to the vector and axial-vector current of light quarks [3]. As the optimal M- value is a partial manifestation of the non-perturbative contributions, we expect to optimize the QHD sum rule at the M- value :

$$M \simeq (8 \sim 10)\ \Lambda_H \qquad (15)$$

as the QCD sum rules are optimized at $M = (8 \sim 10)\ \Lambda_C$. In fact, at this value of M, the corrections terms to the leading order one, are less than 20% in QCD and the continuum contribution to the sum rule is less than 30% of the resonance one if the QCD model is used for the continuum . We expect to have a similar situation in QHD for the choice of the M- value in eq. (15). Also, as in QCD, the optimal M- value is of the order of the typical hadronic scale 1 GeV, i.e. of the order of the ρ - meson mass, we can identify the value of M in eq. (15) as the typical scale of QHD. Then, we can relate, to the M- value in eq. (15), the nature of the W- boson. In fact, we have to understand if the size of the W- boson is analogous to the ρ - meson or to the π- meson? As we shall see, in the following, the two possibilities for the size of the W- boson will lead to two completely different physical situations :

a) The structure of the W- boson is similar to the ρ- one.

In that case, the idea of analogy between QCD and QHD suggests that the sum rule in eq. (13) is optimized for M around the W- mass. In order that the QHD result remains valid, the corrections term in eq. (12) have to be small and so a low value of the QHD scale Λ_H becomes necessary as is suggested by eq. (15). Within such a condition, one deduces from eq. (14) :

$$F_W \lesssim \left(\frac{N_H}{3}\right)^{1/2} 0.18\, M_W \quad . \tag{16}$$

This result combined whith eqs. (4) and (7) gives :

$$g \lesssim \left(\frac{N_H}{3}\right)^{1/2} 0.08 \quad . \tag{17}$$

i.e.

$$M_W \lesssim \left(\frac{N_H}{3}\right)^{1/2} 9 \text{ GeV} \tag{18}$$

Eq. (18) shows that if the W has the same structure as the ρ- meson, its mass is relatively light unless N_H is anomalously large. Also eq. (15) would suggest that the QHD scale Λ_H is relatively small and so, the subconstituent structure of the W can show up at ordinary energies. The above scenario seems to be impossible for the present knowledge of low energy data. Alternative approach based on the value of the W- wave function at the origin does not also favour the possibility of analogy in size between the W and the ρ [2] .

b) The other alternative is to assume that the W- boson and the pion have the analogous structure.

In that case, the value of the QHD scale can be of the order of M_W as the QCD scale Λ_C is of the order of m_π. We expect the QHD sum rule, to be optimized for (*) :

$$M \simeq (8 \sim 10)\, M_W \tag{19}$$

The fact, that the W- contribution to the sum rule is optimized at a higher value of M can also suggest, that the possible radial excitations of the W (analogous to π'....) stand at the M value bigger than $(8 \sim 10)\, M_W$. Using our optimal value of *M in eq. (19), we get from eq. (14) :*

$$F_W \lesssim (8 \sim 10) \left(\frac{N_H}{24\pi^2}\right)^{1/2} M_W \tag{20}$$

(*) Note that for the M- value of the order of $(8 \sim 10)\, m_\pi$, the QCD sum rules applied to the π- meson reproduces with a good accuracy the pion decay amplitude f_π [4a]. We expect that a similar accuracy can be obtained for the W- decay amplitude F_W, using QHD sum rules.

This constraint in eq. (20) together with eqs. (7) and (2) leads to the bound :

$$g \lesssim \left(\frac{N_H}{3}\right)^{1/2} (0.4 \sim 0.5) \quad . \qquad (21)$$

Then, the W- mass is constrained to be :

$$M_W \lesssim \left(\frac{N_H}{3}\right)^{1/2} (45 \sim 56) \text{ GeV} \quad , \qquad (22)$$

i.e. for a small value of $N_H \simeq 3,4$ the model discussed previously in its simplest version predicts a lower value of the W- mass than usually expected from the standard $SU(2)_L \times U(1)$ electroweak theory of Salam and Weinberg. Again for such a low value of N_H, it can be deduced from eq. (3), the bound on the Z- mass :

$$M_Z \lesssim (47 \sim 60) \text{ GeV} \qquad (23)$$

The prediction of the Z- mass in the minimal haplon model is lower than the one of $SU(2)_L \times U(1)$. However, it seems that the present data on the analysis of frontback asymetry from e^+e^- is not accurate enough to rule out the above possibility [5].

In fact, in order that the haplon model reproduces correctly the masses of the W and Z of $SU(2)_L \times U(1)$, or in other word, the so called "unification condition" ($\lambda^2 \simeq \sin^2 \theta_W$ [1,2]) to be satisfied, one needs for color singlet haplons to be at least a 8 - plet of hypercolor :

$$N_H \gtrsim 8 \quad . \qquad (24)$$

However, the haplon models with color triplet subconstituents can reproduce for $N_H \simeq 3, 4$, the $SU(2)_L \times U(1)$ prediction of the W and Z masses. In this case, our result in eq. (12) has to be multiplied by the color factor $n_c = 3$. So, the bounds in eqs. (22) and (23) become $\sqrt{n_c}$ times weaker.

Finally, let's mention that in these composite models, the W and Z can decay into multifermion pairs, multiphotons and multigluons. However, these anomalous decay modes seem to be of the order of the correction terms to the conventional decay modes, i.e. the decay of W and Z into pair of fermion - anti-fermions [6]. For instance, we expect, to a first approximation, that the total width of the W is given by the standard electroweak relation [7] :

$$\Gamma_{W \to all} \simeq \frac{1}{48\pi} \, g^2 M_W \, (4 N_G) \qquad (25)$$

in the case where fermions are much lighter than the W and where N_G is the total number of fermion generations ($N_G = 3$ for the usual electroweak scheme).

Then, with the help of our results in eqs. (21) and (22), we deduce from eq. (25) the upper bound :

$$\Gamma_{W \to all} \lesssim N_G \left(\frac{N_H}{3}\right)^{3/2} (0.2 \sim 0.4) \ \text{GeV} \qquad (26)$$

Clearly, the total width of the W can be smaller than the one in the standard electroweak model if the number of hypercolor is taken to be small. Again, we can recover the electroweak prediction for $N_H \geq 8$ or working with a haplon-model where the haplons are color triplet and having a hypercolor number N_H of the order 3 to 4. Note that eq. (26) suggests that treating the W within a narrow width approximation is a good approximation.

4. CONCLUSION

We have extended the idea of the Laplace transform sum rules, which have been successful to describe within quantum chromodynamics (QCD) the properties of light mesons system [3, 4], into the theory where the W- boson of weak interactions is a composite object. We have, therefore, derived an upper bound on the W- decay amplitude and so on other parameters of the model. We have seen that the assumption where the W has the same structure as the ρ - meson seems to be impossible for the present knowledge of the low-energy data. The other assumption where the W has the same structure as the pion is more attractive. In that case, and provided that the idea of analogy between QCD hadrons and QHD bosons is valid, the QHD model in its simplest version, (W- boson is a bound state of spin 1/2 fermion with color singlet and hypercolor triplet or quartet), predicts a lower value of the W and Z masses and total width than the standard electroweak $SU(2)_L \times U(1)$ theory. We have also shown that the so-called "unification condition" ($\lambda^2 = \sin^2 \theta_W = (e/g)^2$) i.e. the prediction of $SU(2)_L \times U(1)$ can be recovered by the haplon-models provided that the haplons are color singlet and have a hypercolor number N_H at least equal to 8, or the haplons are color triplet and have a hypercolor number N_H around 3 to 4.

It is interesting to have experimental measurement of the masses and width of these bosons in the future generation of accelerators (LEP, Isabelle,....). That can help for a better understanding of the nature of the W and will clarify our confusion on the present models of electroweak interactions.

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FIGURES CAPTIONS

Fig. 1 $\gamma - W_3$ mixing amplitude in composite models

Fig. 2 Two-point function for the weak current $J^\mu_W \equiv \bar{\alpha}\, \gamma^\mu \frac{(1-\gamma_5)}{2}\, \beta$.
(X) denotes the weak current insertion.

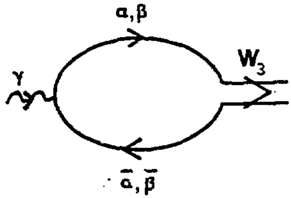

Fig.1

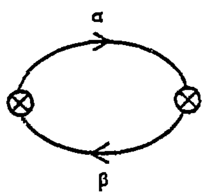

Fig.2