

DISSIPATIVE PHENOMENA : FUSION, FAST FISSION AND QUASIFISSION

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Abstract.- The last theoretical developments concerning the fusion process are reviewed. They concern the appearance of new dissipative mechanisms : fast fission, when the fission barrier of the compound nucleus vanishes due to angular momentum, and quasifission which takes place for heavy systems. The conditions under which these processes, as well as fusion, occur are discussed in details using the fast fission and the extra push models.

Over the last ten years, a large amount of experimental and theoretical studies have been devoted to dissipative phenomena in heavy ion reactions¹). In this respect, fusion, which is the most dissipative one has been investigated in great details. In this field of interest two important questions can be raised :

1. what happens when two heavy nuclei fuse together?
2. under which conditions can they fuse?.

Fifteen years back from now the situation was clear because the accelerators could only provide light projectiles at bombarding energies not too far above the Coulomb barrier. In this case, when two heavy ions merge, they form a compound nucleus. This is possible if the two ions can overcome the fusion barrier.

Little after the seventies two main problems were revealed by many experiments indicating that the situation is not as clear as it seemed to be before. First it was not possible to synthesize the superheavy element by fusion of two heavy nuclei. It was shown that the reason was not because the superheavy element could not exist, but because two very heavy ions cannot merge anymore²). The second experimental fact, although less spectacular, is nevertheless very important : for a given system, it turns out that when the bombarding energy is above the fusion barrier, we observe two regimes for the fusion cross section which are schematically pictured in fig. 1. The region just above the fusion threshold can be understood by looking if it is possible for the system, with a given impact parameter, to overcome the fusion barrier. This condition cannot be extrapolated at medium bombarding energies where it was shown that the fusion cross section decreases compared to what can be expected from the preceding consideration¹).

Two simple explanations have been given to interpret these surprising results. They are both based on potential energy considerations :

- the non fusion of very heavy systems can be simply explained by the Coulomb repulsion between the two heavy ions which become so strong that the nuclear attraction cannot counteract it anymore^{3,4}) ;
- for intermediate systems, at medium bombarding energies, it has been proposed to understand the experimental results, that passing the fusion barrier is a necessary but not a sufficient condition for fusion. To fuse, the system should

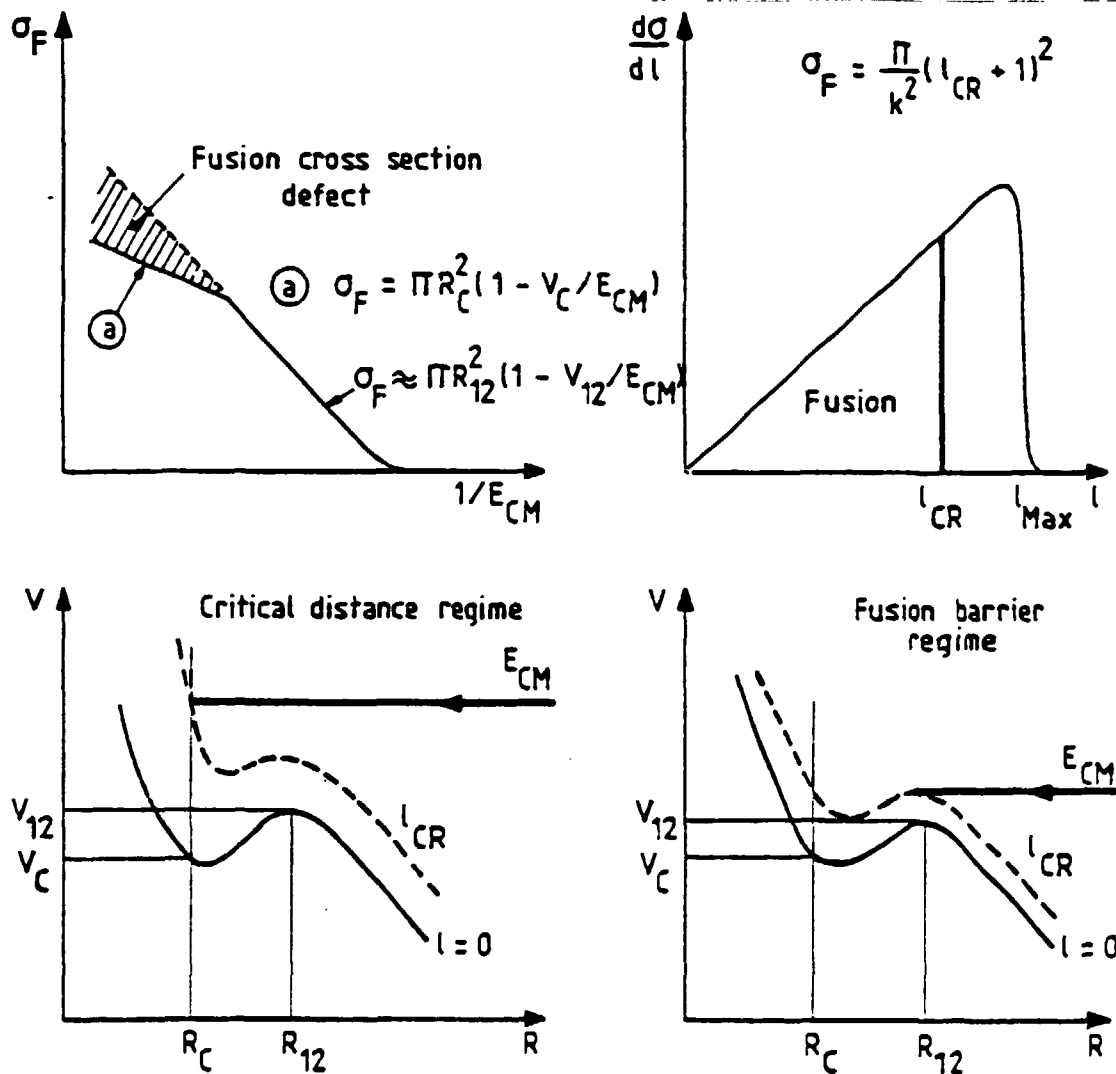


Fig. 1 - Schematic picture of our understanding of fusion in a static approach. On top left, is shown the fusion cross section σ_F (full line) as a function of E_{CM} the inverse of the center of mass bombarding energy. On bottom right is shown the total interaction potential V between the two ions, as a function of the distance R separating their center of mass. V_{12} is the height of the fusion barrier for a head on collision and R_{12} its location. The first regime of σ_F , just above V_{12} , can be understood in terms of passing the fusion barrier. The critical angular momentum, l_{CR} , defined on top right is the largest value of the orbital angular momentum, l , leading to fusion (k is the wave number). The second regime of σ_F , at higher bombarding energies, occurs if the critical distance R_C is reached (see bottom left). V_C is the value of the total interaction potential, for head on collision, at distance R_C .

reach a particular distance, called critical distance for fusion⁶). The notion of critical distance has up to now no deep theoretical justification and has to be understood as a simple way to parametrize the data.

During the recent years some progress has been done in understanding fusion and it is some of these advances which I could like to review with a special emphasis on the physical ideas which have emerged.

1. EXPERIMENTAL DEFINITION OF FUSION

When two heavy ions fuse they form either a compound nucleus, or something close to it, with some excitation energy and angular momentum. This system will deexcite by emitting light particles and γ rays leading to residual nuclei. If the fission barrier is small or reduced sufficiently by angular momentum, it will fission.

The fusion cross section, σ_F , is experimentally defined as the sum of two terms : the evaporation residue cross section, σ_{ER} corresponding to nuclei with a mass close to the one of the compound nucleus, and the fission cross section, σ_f , corresponding to products which have a symmetric mass distribution around a mean value about half the compound nucleus mass :

$$\sigma_F = \sigma_{ER} + \sigma_f. \quad (1)$$

When light compound nuclei are formed, evaporation residues are a large part of the fusion cross section. It is the contrary for heavy compound nuclei for which σ_f is almost identical to σ_F . For some particular asymmetries of the initial system, there can be ambiguities to define the experimental fusion cross section due to a difficult separation of evaporation residues, or fission fragments, from similar products having a different origin.

A critical angular momentum, l_{CR} , is usually deduced from σ_F with the following assumptions : it is assumed that the lowest l values, or impact parameters, contribute to fusion and that the sharp cut off approximation is valid (see fig. 1). Then the critical angular momentum is the largest l value which fuses. It is defined by the relation :

$$\sigma_F = \frac{\pi}{k^2} \sum_{l=0}^{l_{CR}} (2l+1) = \frac{\pi}{k^2} (l_{CR} + 1)^2 \quad (2)$$

where k is the wave number. The critical angular momentum depends on the system and on the bombarding energy. It is a property of the entrance channel but not of the compound nucleus⁷).

2. THEORETICAL DEFINITION OF FUSION

Dissipative heavy ion collisions are used to be described by means of classical models with friction forces acting in the relative motion, as well as on other collective degrees. The dynamical evolution of the two colliding nuclei is governed by potential, dissipative and inertial terms entering the equations of motion. Within this framework, fusion occurs when the initial system is trapped in the interaction region. For this to occur, the total interaction potential, including the centrifugal force, should have a pocket. The system can be trapped in this pocket if dissipation is large enough (see fig. 2). If not we have a deep inelastic reaction.

3. FUSION AND COMPOUND NUCLEUS FORMATION

Compound nuclei having a high fissility parameter have a large probability to decay by fission. This probability increases with angular momentum because the effective barrier against fission decreases when more and more angular momenta are brought in the compound nucleus⁸). For a certain value, denoted by l_{Bf} , the fission barrier will vanish. Since a compound nucleus cannot be formed with an angular momentum larger than l_{Bf} , if fusion could be identical to compound nucleus formation, the critical angular momentum for fusion, l_{CR} , should be always smaller than l_{Bf} . However several experiments performed with medium systems show that l_{CR} can be larger than l_{Bf} (see ref.⁹) for a compilation of several examples).

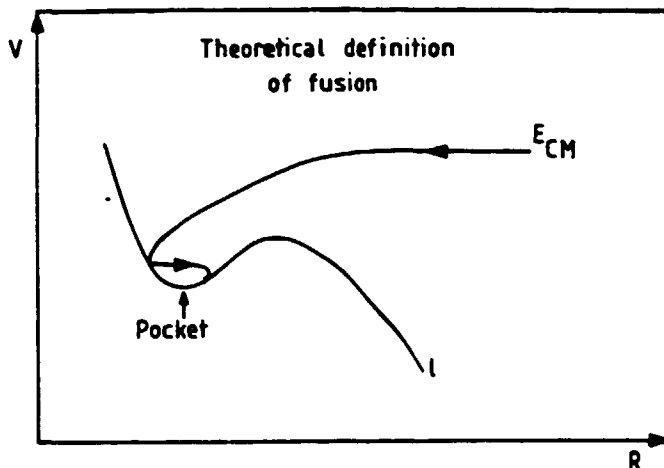
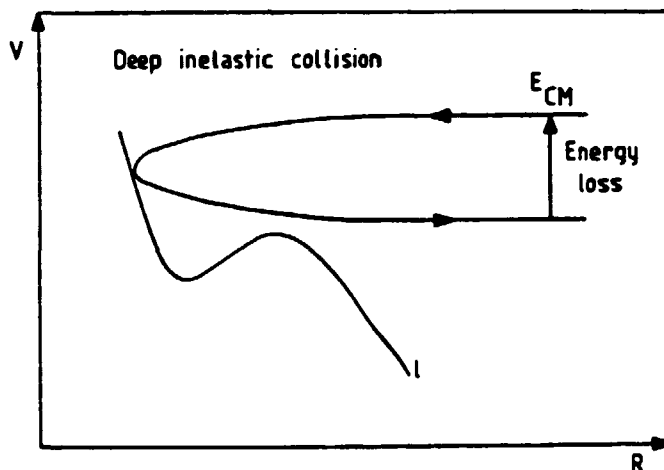


Fig. 2 - Schematic description of fusion and deep inelastic reactions. The total interaction potential V including the centrifugal energy (the orbital angular momentum is l) is plotted as a function of R , the distance separating the center of mass of the two ions. E_{CM} is the initial bombarding energy. A part of it is lost in the interaction region and the system could be trapped (top figure) leading to fusion or escape (bottom part of the figure) giving deep inelastic fragments.



This means that fusion cannot be identified with compound nucleus formation. Since l_{CR} can be larger than l_{B_f} , one of the question we have to address ourselves is the following : what happens for l values between l_{B_f} and l_{CR} for which we have fusion but not compound nucleus formation?

4. ENHANCED FUSION THRESHOLD

Very heavy systems, typically with a product $Z_1 Z_2$ of the two atomic numbers larger than about 2500-3000, do not fuse. The region of systems where fusion just disappears has been investigated in a systematic way by Bock et al.¹⁰⁾ at GSI. They found the following extremely interesting result : as one goes towards the limit where fusion disappears, the associated threshold becomes larger than expected from the systematic calculation of fusion barriers.

The above enhancement of the fusion threshold, together with the non identity between compound nucleus formation and fusion, deserves further studies and this will be the object of the following sections.

5. LIMITS OF FUSION

Sudden potentials, calculated assuming that the densities of the two heavy ions remain frozen during the collision, describe pretty well fusion which is a process mainly governed by the entrance channel. They usually exhibit a pocket where the two heavy ions have to be captured in order to fuse. This can occur because of dissipative forces which are acting in the interaction region. In fig. 3 we display, for a head on collision, an example of interaction potential calculated using the energy density formalism.

The pocket in the total interaction potential $V(R)$ can disappear because of two effects :

Fig. 3 - Total interaction potential $V(R)$, for a head on collision, as a function of R , the distance separating the center of mass of the two ions. V_N is the nuclear part and V_C the Coulomb one. This calculation has been performed using the energy density functional of ref.¹¹), for the Ar + U system. In this case $V(R)$ exhibits a pocket and we can have fusion.

a) Coulomb effects

The Coulomb interaction increases with the size of the two partners much more than the nuclear potential. Indeed the first one goes like $Z_1 Z_2$ whereas the second one goes only like $A_1^{1/3} A_2^{1/3} / (A_1^{1/3} + A_2^{1/3})$ [ref.^{4,12}]. As a matter of fact heavy systems will not have a pocket anymore and consequently fusion disappears. This is illustrated in fig. 4 for the Pb + U system.

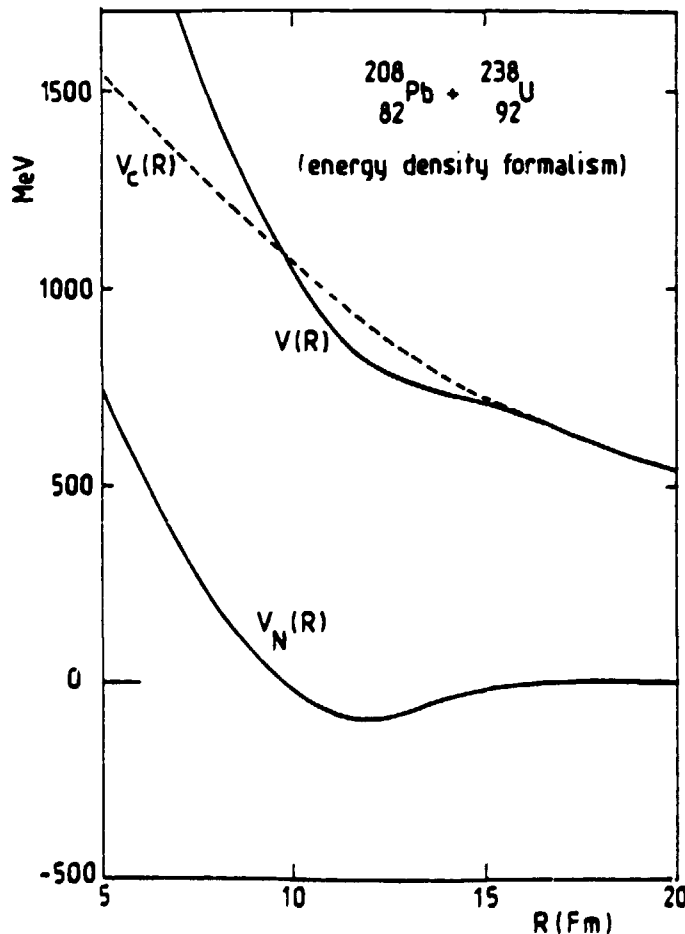
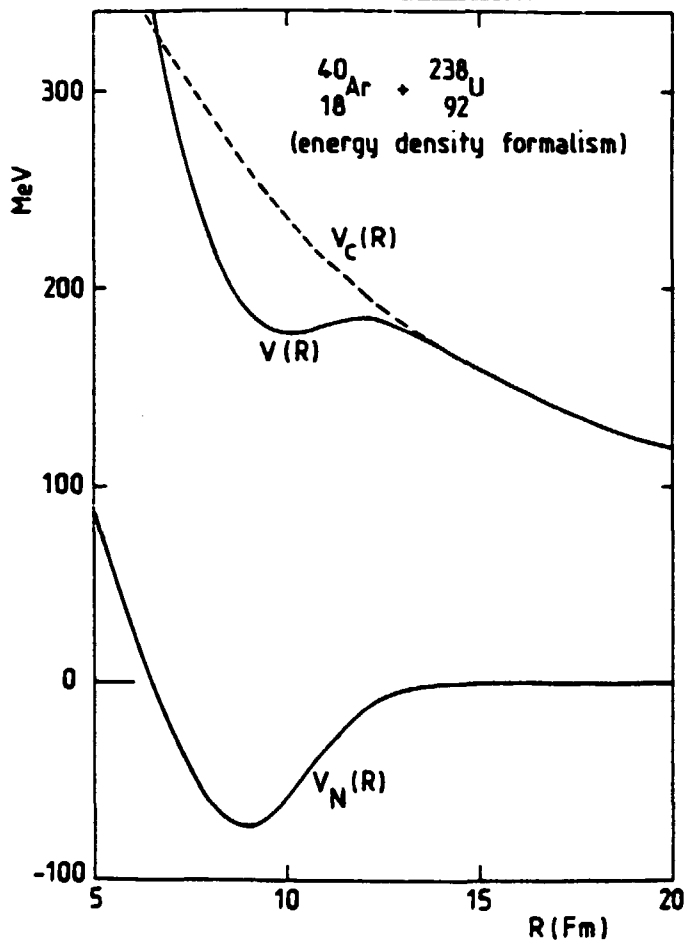


Fig. 4 - Same as fig. 3 for the Pb + U system. In this case $V(R)$ has no pocket and the system cannot fuse (calculation according to ref.¹¹)).

b) Angular momentum effects

For a given system the pocket of $V(R)$ can also disappear due to angular momentum, because the centrifugal force is repulsive. This is illustrated in fig. 5 for the Ar + U system.

Let us call l_I the value of the orbital angular momentum for which the pocket disappears. Then because of tangential friction the maximum l value for capture will be :

$$l_M = \frac{1}{f} l_I \quad (3)$$

where f is the fraction of orbital angular momentum kept in the relative motion :

$$f = \frac{5}{7} \quad \text{for rolling} \quad (4)$$

and

$$f = \frac{J}{J_1 + J_2 + J} \quad \text{for sticking} \quad (5)$$

J_1 , J_2 and J are the momenta of inertia of nuclei 1, 2 and in the relative motion. In many cases it has been found that $f = 5/7$ is the relevant value to be considered¹³⁾.

Fig. 5 - Total interaction potential $V_l(R)$ for the Ar + U system calculated for different values of the orbital angular momentum l (calculation according to ref.¹¹⁾).

6. QUANTITATIVE ESTIMATES

We can quantitatively estimate the limits of fusion using an analytical expression of the interaction potential between two heavy ions based on the energy density formalism of ref.¹¹⁾. With a simplified value of the radius parameter r_0 used to calculate nuclear radii it reads :

$$V(R) = V_C(R) + V_N(R) \quad (6)$$

$$V_C(R) = \frac{Z_1 Z_2 e^2}{R} \quad (7)$$

$$V_N(R) = \frac{C_1 C_2}{C_1 + C_2} \mathcal{U}_N(s) \quad (8)$$

where C_1 and C_2 are the central radii :

$$C_i = R_i \left(1 - \frac{1}{R_i^2} \right) \quad (9)$$

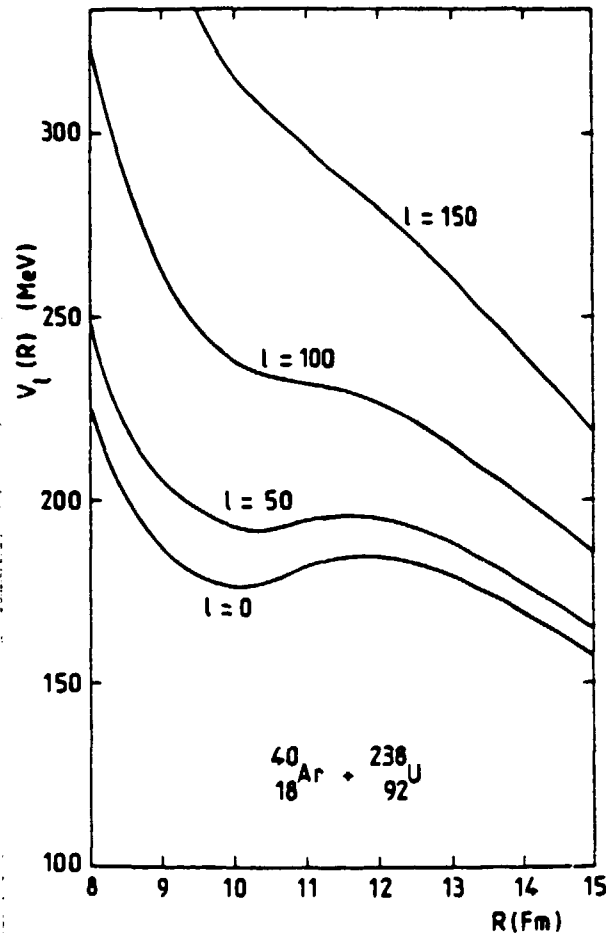
$$R_i = r_0 A_i^{1/3} \quad (r_0 = 1.16 \text{ fm}) \quad (10)$$

and

$$\mathcal{U}_N(s) = -34 \exp\left[-\frac{(s - s_0)^2}{5.4}\right] \text{ MeV} \quad (11)$$

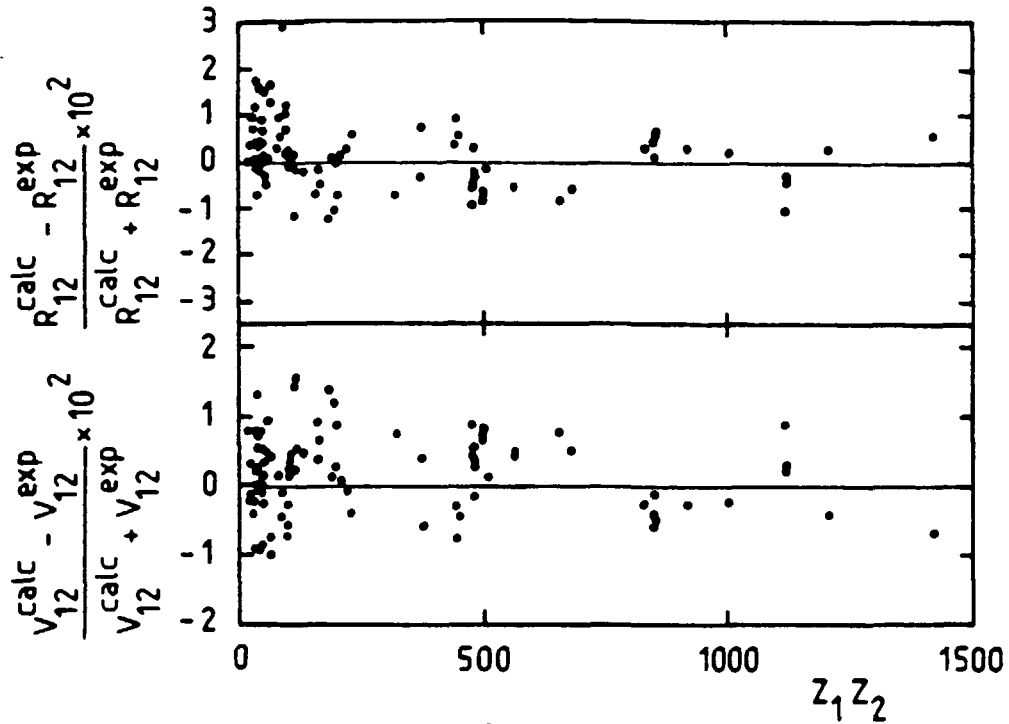
if $s > s_0 = -1.6 \text{ fm}$

$$\mathcal{U}_N(s) = -34 + 5.4 (s - s_0)^2 \quad \text{if } s < s_0. \quad (12)$$



Using expression (6) it is easy to calculate the fusion barrier which is defined as the outer maximum of $V(R)$. Since the maximum value of the nuclear force is obtained for $s=0$, the position of the fusion barrier, when it exists, is always

located at $s \geq 0$. The quality of the parametrization given by equations (6-12) is seen in fig. 6. It shows for several systems, a comparison between the calculated values and the experimental ones (from ref.¹⁴) for the fusion barrier height and for its location.



$$V(R) = Z_1 Z_2 e^2 / R + V_N(R)$$

$$V_N(R) = \bar{c} u_N(s) \quad \bar{c} = C_1 C_2 / (C_1 + C_2)$$

$$s = R - C_1 - C_2 \quad C_i = 1.16 A_i^{1/3}$$

$$u_N(s) = \begin{cases} -34 \exp\left(-\frac{(s+1.6)^2}{5.4}\right) & (s > -1.6 \text{ Fm}) \\ -34 + 5.4(s+1.6)^2 & (s < -1.6 \text{ Fm}) \end{cases}$$

Fig. 6 - Relative errors (in percent) for the height of the fusion barrier (bottom) and for its location (top), between the calculated values and those deduced from experiment using the analysis of ref.¹⁴). The comparison has been done as a function of $Z_1 Z_2$, the product of the atomic numbers of the two heavy ions for all the systems compiled in ref.¹⁴).

With the above parametrization the nuclear force is maximum for $s = 0$ and is equal to :

$$(F_N)_{\max} \approx 12.54 \frac{C_1 C_2}{C_1 + C_2}. \quad (13)$$

According to the proximity approach¹³) it is also equal to :

$$(F_N)_{\max} = 4\pi \gamma \frac{C_1 C_2}{C_1 + C_2} \quad (14)$$

Comparing eqs.(13) and (14) we see that the surface tension $\gamma \approx 1 \text{ MeV/fm}^2$. For a head on collision, the pocket will disappear if the modulus of the Coulomb force, at $s = 0$, is larger than the modulus of the nuclear force :

$$4\pi \gamma \frac{C_1 C_2}{C_1 + C_2} > \frac{Z_1 Z_2 e^2}{(C_1 + C_2)^2} . \quad (15)$$

This gives the following condition for a system to fuse :

$$\frac{Z_1 Z_2}{C_1 C_2 (C_1 + C_2)} < 8.7 \quad (16)$$

which can be expressed in a simpler way introducing the effective fissility parameter of Bass⁴⁾ (also used extensively later by Swiatecki¹⁶⁾) :

$$\left(\frac{Z^2}{A}\right)_{\text{eff}} = \frac{4 Z_1 Z_2}{A_1^{1/3} A_2^{1/3} (A_1^{1/3} + A_2^{1/3})} \leq 48 \quad (17)$$

the above limit of fusion has been calculated from eq.(16) approximating the central radii along the beta stability line.

For a given system we can investigate in a similar way the disappearance of the pocket due to angular momentum. The condition reads :

$$\frac{Z_1 Z_2 e^2}{C_1 C_2 (C_1 + C_2)} + \frac{f^2 \lambda^2 \hbar^2}{\mu C_1 C_2 (C_1 + C_2)^2} < 4\pi \gamma . \quad (18)$$

To solve this equation for a given system, we need the factor f representing the proportion of orbital angular momentum remaining in the relative motion. The choice of this factor is however not obvious and might depend upon the system under consideration (see below). Nevertheless eq.(18) tells us that the critical angular momentum for fusion is bounded at high bombarding energies.

The above approach is a static one since it is based only on potential energy considerations. We will see later on how the dynamics influence the preceding conclusions. However we shall first briefly describe two dynamical models allowing a better understanding of the fusion process itself.

7. THE FAST FISSION MODEL

A large part of the results to be discussed in this paper are based on a dynamical model which was developed in ref.¹⁷⁻²⁰). The collision of two heavy ions is described by means of a few collective degrees of freedom which are treated explicitly : two describing the relative motion of the 2 ions (R, θ the usual polar coordinates), one describing the mass asymmetry of the system, and the last one associated to the neutron excess in one of the fragments. The deformation degrees of freedom, which play an important role in the collision, are treated implicitly by allowing a dynamical transition between a sudden and an adiabatic potential. This method is similar to the one proposed by Nörenberg and Riedel²¹⁾ who make a dynamical transition between a diabatic and an adiabatic potential.

The sudden potential, calculated assuming that the densities of the two ions are frozen, provides a good description of the entrance channel (fusion valley). At variance, the adiabatic potential (taken from ref.²²⁾) which is obtained by minimizing the potential energy of the system for a given elongation, gives a good description of the fission valley.

If the overlap and the contact time between the two ions is sufficient, there is a complete transition from the sudden, to the adiabatic potential. If not, only a partial transition occurs (for instance, for quasielastic interactions there is no transition at all).

The dynamical evolution of the system is followed by means of a transport equation which was derived by Hofmann and Siemens using linear response theory²³). Its solution gives the distribution function of the system in collective phase space at each step of the collision. The evolution of the colliding system is entirely determined by potential, friction and inertia terms.

This model allows to describe deep inelastic reactions but its main interest concerns fusion where new features appear.

a) Fast fission

The first feature is the appearance, if certain conditions are fulfilled, of a mechanisms intermediate between deep inelastic and compound nucleus formation. This is illustrated in fig. 7 for the 340 MeV Ar + Ho system. Three typical trajectories are displayed in the plane mass asymmetry-radial distance :

- for $\lambda=195$ the interaction between the two nuclei is weak and the time of contact is short. There is almost no mass exchanged between the two nuclei and a small energy transfer between them. We have to deal with a quasielastic reaction ;
- for $\lambda=138$ the interaction between the two heavy ions is stronger. Some mass is exchanged and, for this particular λ value, the kinetic energy in the relative motion is completely damped. We are faced with a typical deep inelastic collision ;
- for $\lambda=75$ the system is trapped in the pocket of the sudden potential. Then mass asymmetry relaxes to equilibrium, which in this case corresponds to a symmetric composite system. Simultaneously the potential landscape changes from sudden to adiabatic. However, for this particular system the value of the angular momentum for which the fission barrier of the compound nucleus vanishes, λ_{B_f} , is equal to 72 [ref. 22)]. Therefore the system escapes again by fissioning in two fragments. This kind of trajectory corresponds to fast fission phenomenon and appears naturally in the model. The mass and the kinetic energy distributions of the products will be practically identical to those of fission fragments following compound nucleus formation. The interaction time of such a process ranges from 10^{-21} to 10^{-20} s, which is larger than the one of a deep inelastic collision, but shorter than for compound nucleus formation.
- For λ smaller than $\lambda_{B_f} = 72$, a real compound nucleus is formed since it has a non vanishing fission barrier. The fast fission model cannot however describe the future evolution of the captured system.

From the 340 MeV Ar + Ho system illustrated in fig. 7 it emerges the following picture for heavy ion reactions : compound nucleus formation occurs for $0 \leq \lambda \leq \lambda_{B_f}$. Then we observe fast fission when $\lambda_{B_f} \leq \lambda \leq \lambda_{CR}$. For $\lambda_{CR} < \lambda < \lambda_{max}$ we have deep inelastic collisions and then quasielastic reactions close to λ_{max} .

It should be noted that the existence of long life time trajectories like the one illustrated by $\lambda=75$ in fig. 7 have also been obtained by other authors : Nörenberg and Riedel for heavy systems²¹) and Broglia, Dasso and Winther²⁴). These last authors were probably the firsts who have quantitatively obtained fast fission trajectories using a dynamical model which includes explicitly the deformation degrees of freedom of the two incident nuclei. However the range of λ values associated to this mechanism is different from the one obtained here. If λ_{CR} exceeds λ_{B_f} they have the following picture: compound nucleus formation for $\lambda < \lambda_{B_f}$, then, as λ increases, deep inelastic collisions, fast fission, deep inelastic again and finally quasielastic reactions. Therefore fast fission occurs in a λ window in between λ values associated to deep inelastic collisions.

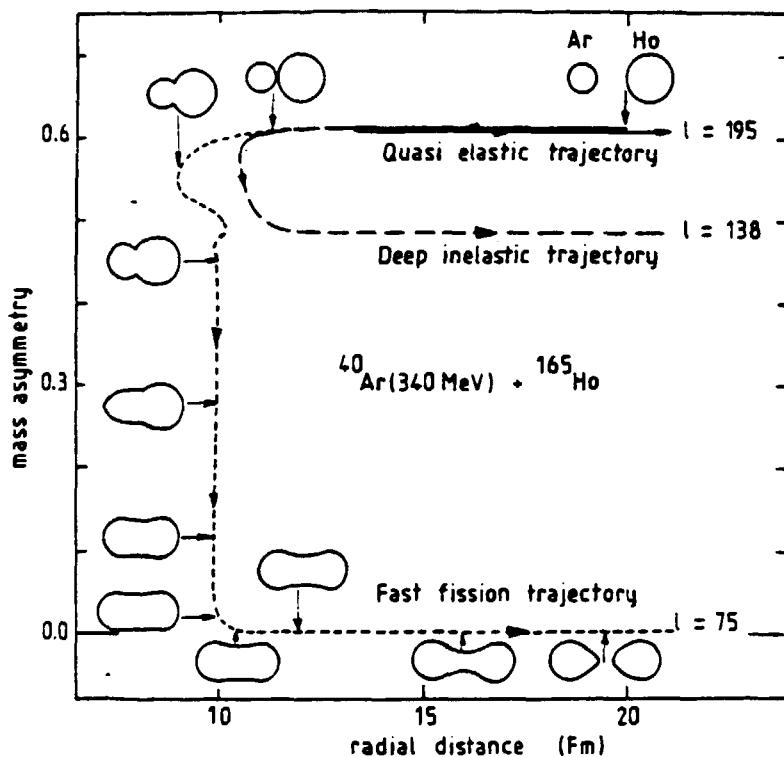


Fig. 7 - Few mean trajectories for various initial values of the orbital angular momentum, l , plotted in the plane radial distance-mass asymmetry. Three kinds of mechanism are illustrated in this plot : 1) quasi-elastic process for $l=195$, 2) deep inelastic collision for $l=138$ and 3) fast fission phenomenon for $l=75$. For $l < l_{B_f} = 72$, a compound nucleus is formed. This figure has been extracted from ref.¹⁷.

b) Fast fission or quasi-fission

For the Ar + Ho system discussed above, compound nucleus formation occurs

when $l < l_{B_f}$. It is so because the system remains trapped when the potential energy surface becomes adiabatic. This is due to the fact that the saddle configuration of the compound nucleus is less compact than the pocket configuration. This condition is fulfilled if the fissility parameter $\eta = Z^2/A$ of the compound nucleus is not too large (see section 11). However, for heavier compound nuclei, the saddle configuration can be more compact than the pocket configuration. In this case, even if the compound nucleus has a fission barrier, the system can escape because it is located outside the saddle configuration. As a consequence we get fast fission also for $l < l_{B_f}$. For this special situation, it has been suggested by Swiatecki to call it quasifission¹⁶).

c) Fission like mass distributions

In ref.^{25,26}), the fission like mass distribution of the Ar + Ho system at several bombarding energies have been investigated in great details. It has been found an unusual broadening of these mass distributions when the critical angular momentum exceeds l_{B_f} (see fig. 8). It has been suggested²⁵⁻²⁷) that this could be an indication that a new mechanism occurs when l values larger than l_{B_f} are involved.

Since the fast fission model is based on a transport equation, we are able to calculate, for each l value, the width of the fast fission mass distribution. For compound nucleus fission the width of the fission fragments mass distributions are taken from ref.²³). At a given bombarding energy, summing up all contributions from $l=0$ to l_{CR} , we can get the total mass distribution of the fission like products. A comparison with the experimental results is shown in fig. 8 for the Ar + Ho system. The agreement between experiment and theory is pretty good. However, it should be noted that, with the fast fission model, we can only calculate the mass distribution of the products if the system is not too asymmetric. The reason is that the transport equation is solved by moments expansion. For more asymmetric systems such a method is inapplicable because along the mass asymmetry coordinate the system is injected in a region where the distribution function splits in two parts : one going to more asymmetric configurations, the other one going to more symmetric ones.

Fig. 8 - Full width half maximum, Γ , of the fission like mass distribution, as a function of the excitation energy of the fused system, for Ar + Ho. The dots are the experimental points in refs. ^{25, 26}). The full curve is the results of the calculation of ref. ²⁰).

For the same system it is also interesting to plot the excitation functions for compound nucleus formation, fast fission and fusion (which is the sum of both preceding ones) and compare the result of the calculation with the experimental data. This is shown in fig. 9 and we see a rather good agreement between experiment and theory.

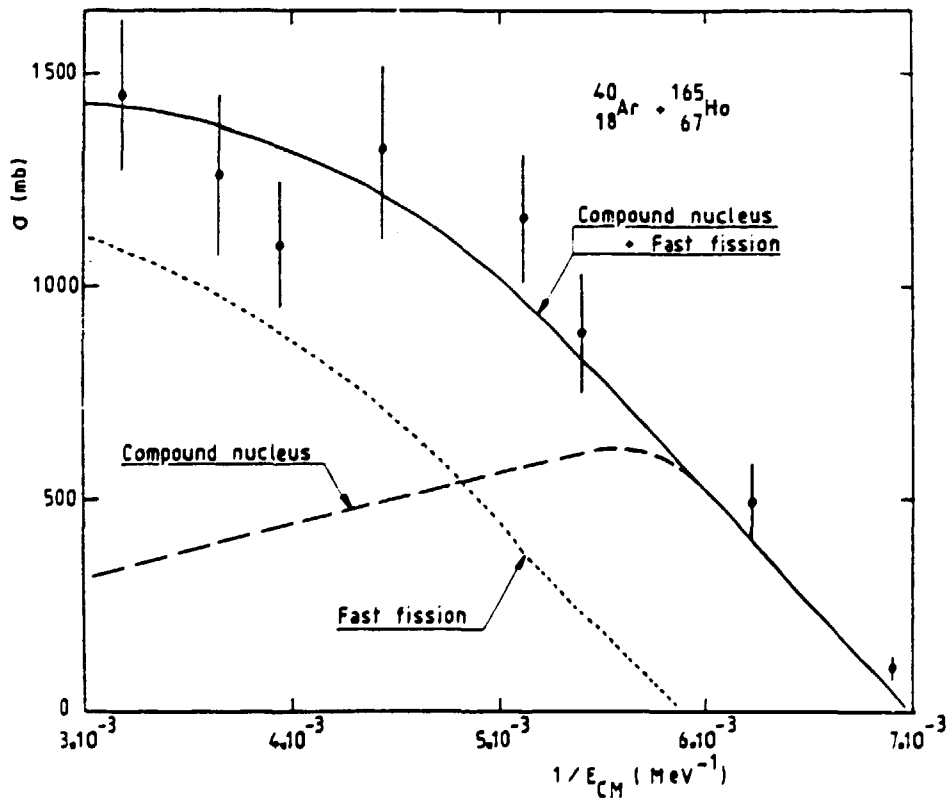
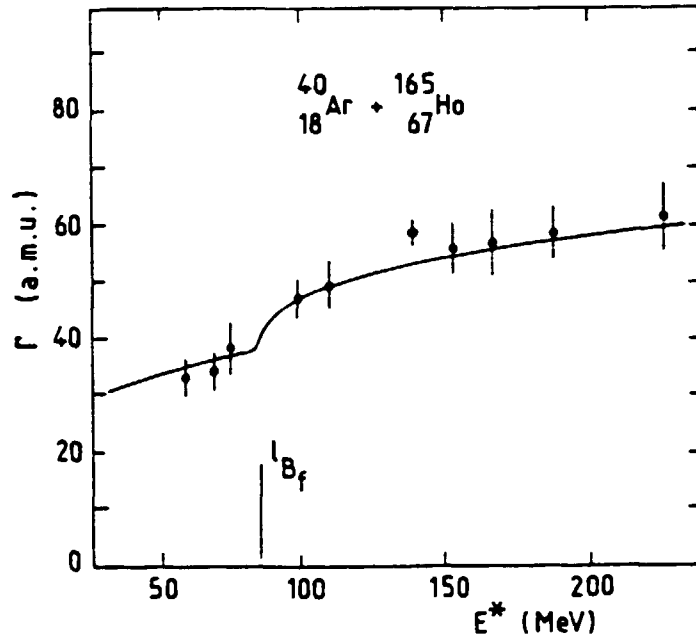


Fig. 9 - Experimental fusion cross section (dots) from ref. ^{25, 26}) plotted as a function of $1/E_{CM}$, the inverse of the center of mass bombarding energy. It is compared with the calculated fusion cross section of ref. ¹⁷) (full curve). The fusion cross section is the sum of the compound nucleus and of the fast fission cross sections. Their corresponding excitation functions are also shown in the figure. This figure is extracted from ref. ¹⁷).

d) The four classes of dissipative collisions

In fig. 10 we summarize, in a schematic manner, the four classes of dissipative heavy ion collisions which appear in the fast fission model : deep inelastic,

fast fission, quasifission and compound nucleus formation. We have represented the sudden and the adiabatic potential as a function of R , as well as a trajectory. This one dimensional representation is just to have a physical feeling of what is going on but it should be stressed that, in the fast fission model, the collision is described on a four dimensional potential energy surface.

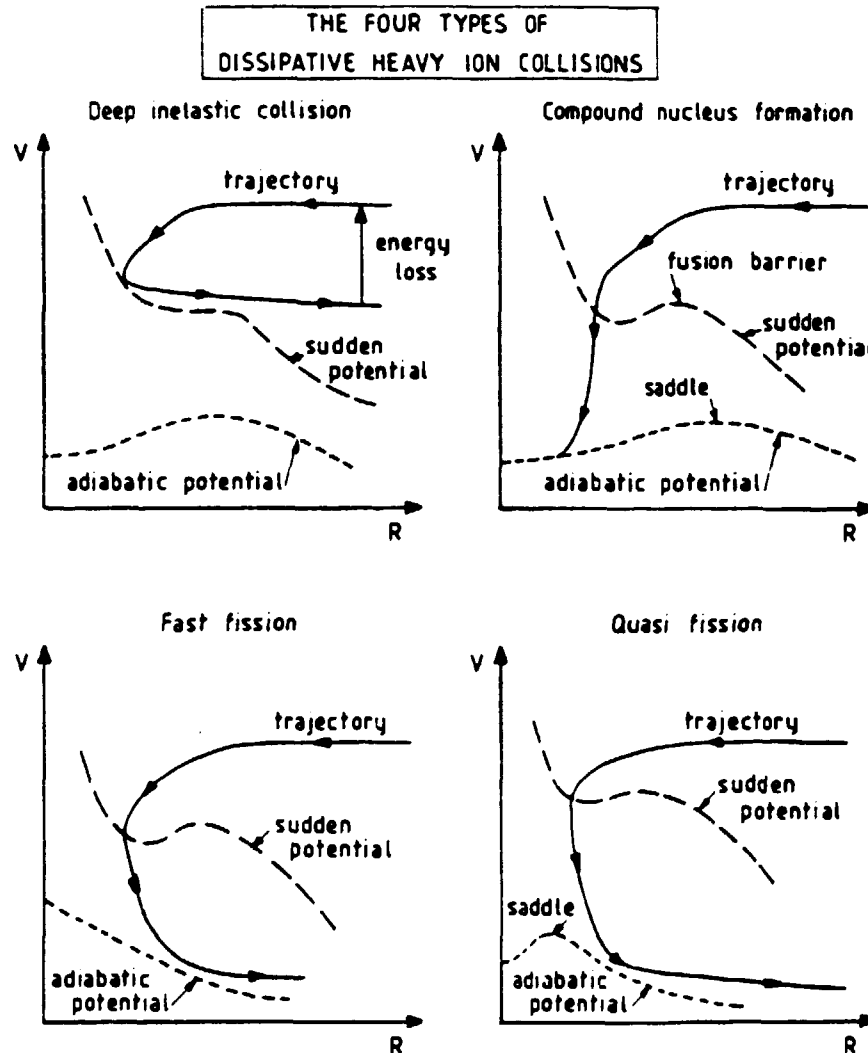


Fig. 10 - Typical illustration of the four dissipative mechanisms occurring in a heavy ion reaction : Top left : the system is not trapped but it loses a lot of kinetic energy in the relative motion : we have a deep inelastic collision. Top right : the system is trapped in the entrance channel. The sudden potential goes to the adiabatic one but the saddle configuration is elongated enough to keep the system trapped : we have compound nucleus formation. Bottom left : the system is trapped but the fission barrier of the compound nucleus has vanished due to angular momentum. Therefore it desintegrates in two almost equal fragments because mass asymmetry had time to reach equilibrium : we have fast fission. Bottom right : the compound nucleus has a fission barrier but the saddle configuration is too compact to keep the trapped system : we have also fast fission or quasi-fission.

8. THE EXTRAPUSH MODEL

In ref.^{15,29}) Swiatecki has developed a dynamical model for head on collisions. He assumes that it is possible to describe the evolution of the two colliding nuclei by a sequence of shapes consisting of two spheres connected by a conical neck. If it is so, three collective variables are enough to describe the

dynamics : one connected to the distance separating both fragments, one associated to mass asymmetry and one related to the neck connecting the two pieces. Only the mean values of the collective variables are followed as a function of time by means of Newton equations with friction forces given by the one body approach³⁰). Except for the motion governing the distance between the two nuclei, all the collective motions are assumed to be overdamped in the sense of Kramers³¹).

They are three key configurations : the first one corresponds to the contact of the two nuclei supposed to be represented by liquid drops. This contact configuration is close to the one associated to the interaction barrier. It is at this point that the neck degree of freedom is unfrozen. The second one is the conditional saddle configuration which is a maximum of the potential energy under the constraint that mass asymmetry remains frozen to its initial value. The third one is the usual saddle point which is associated to the compound nucleus. It corresponds to a splitting in two symmetric fragments. For a symmetric system, conditional and unconditional saddles are the same.

To each of the preceding configurations are associated three thresholds and three kinds of mechanism. This is summarized in fig. 11 taken from ref.³²). We see that fusion is obtained only if the conditional saddle point is reached. For light systems this configuration is less compact than the contact one. Consequently the extra push is zero. For heavier systems it can be the contrary and it is necessary to bring the system from the contact point to the saddle configuration by giving it some extra energy : the extra push.

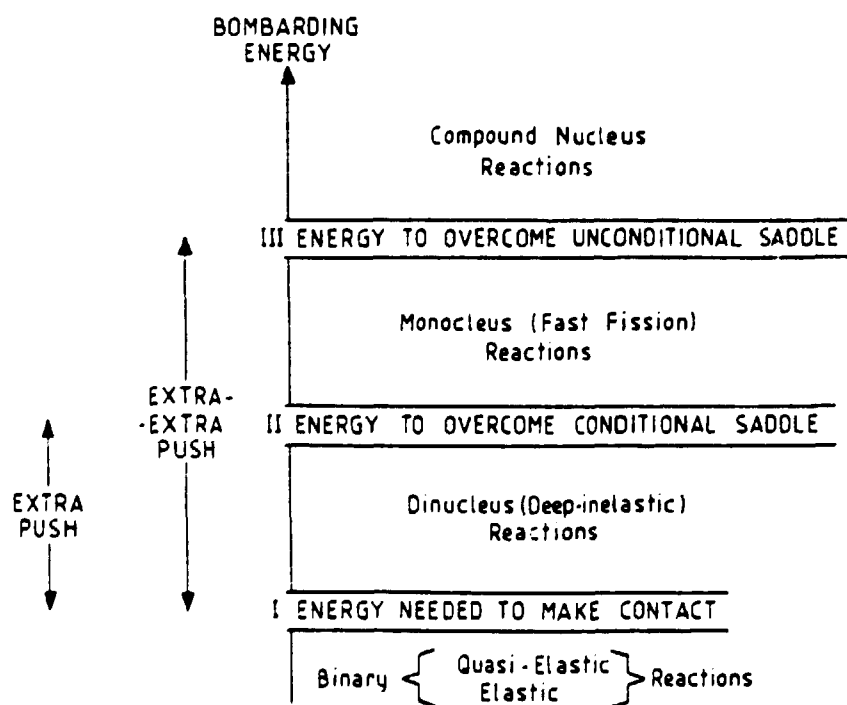


Fig. 11 - Schematic illustration of the different mechanisms which are obtained in the dynamical model of Swiatecki³³) for head collisions. If the bombarding energy is not sufficient to reach the contact configuration (I) we get elastic or quasi elastic processes. If the energy is sufficient to make contact but not to reach the conditional saddle configuration we have to deal with deep inelastic reactions. To reach the conditional saddle from the contact configuration an extra energy is needed : it is the extrapush. If this point is reached there is fusion but not necessarily compound nucleus formation. To reach this latter situation more extra energy is needed (extra-extrapush) otherwise there is quasi-fission (fast fission). This figure is taken from ref.³²).

9. BRIEF COMPARISON BETWEEN THE FAST FISSION AND THE EXTRAPUSH MODELS

The fast fission and the extrapush models have been initially developed in order to explain different experimental data. They present a lot of similarities but also differences. However they both give a very close description of the fusion process. We shall briefly recall the advantages as well as the drawbacks of each of them.

Both are able to describe the dynamical evolution of the colliding system. The fast fission model takes into account the orbital angular momentum explicitly and is able to describe the fluctuations of the macroscopic variables around their mean values. These points are disregarded in the extra push model which is only devoted to head on collisions. However the later approach has the advantage of treating explicitly, although in a simplified manner, the deformations of the two ions. This is not the case for the fast fission model where they are only simulated. As far as the frictional forces are concerned, the extra push model heavily rely on the overdamped approximation. This is not the case of the fast fission model. This simplification avoids the choice of the inertial parameters which are taken, in the fast fission model, sometimes without a deep theoretical justification.

There is nevertheless one basic difference in both models. In the extra push description the neck is unfrozen after the contact configuration has been reached. It is at contact that the transition between a sudden and an adiabatic potential occurs. According to the results of ref.¹⁶⁾ this happens very fast : in a few 10^{-22} s. It is necessary to have an extra kinetic energy at the contact point to be able to reach the conditional saddle and to be captured. For the fast fission model the sudden to adiabatic transition occurs earlier but is slower : of the order of 10^{-21} s [ref.¹⁷⁾]. An extra kinetic energy is needed in order to overcome the fusion barrier and to fuse. This is illustrated in fig. 12.

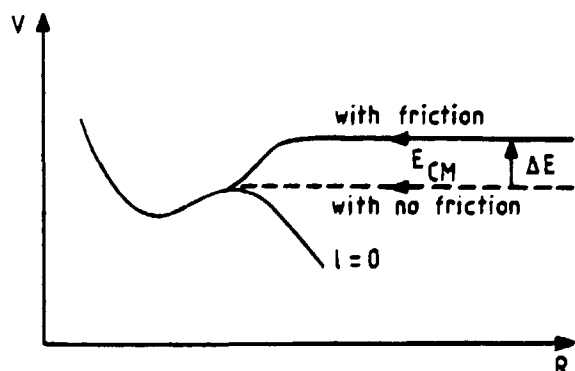
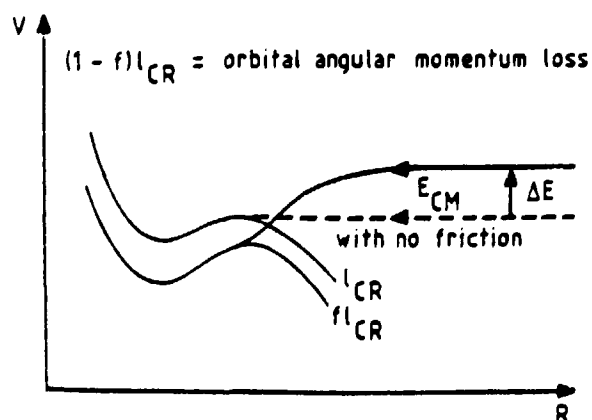


Fig. 12 - Schematic illustration of the fact that same extra kinetic energy is needed to overcome the fusion barrier in the fast fission model : on top is the case of a head on collision. In the bottom when the orbital angular momentum is equal to l_{CR} . In this later case the total interaction potential including centrifugal energy changes due to angular momentum loss.



10. THE STATIC EXTRA PUSH MODEL

With what has been learned using the dynamical extra push model, Swiatecki^{15, 23, 32, 33)} has developed a simple static approach where many of the experimental data can be described in terms of simple formulas. This concerns what can be called the static extra push model.

For a head on collision the extra push energy, E_x , can be parametrized in terms of the effective fissility parameter, $(Z^2/A)_{eff}$, defined in eq. (17) :

$$E_x = \begin{cases} 0 & \text{for } \left(\frac{Z^2}{A}\right)_{\text{eff}} < \left(\frac{Z^2}{A}\right)_{\text{eff}}^{\text{thr}} \\ a^2 C_0 \left[\left(\frac{Z^2}{A}\right)_{\text{eff}} - \left(\frac{Z^2}{A}\right)_{\text{eff}}^{\text{thr}} \right]^2 & \text{for } \left(\frac{Z^2}{A}\right)_{\text{eff}} > \left(\frac{Z^2}{A}\right)_{\text{eff}}^{\text{thr}} \end{cases} \quad (19)$$

where a is a slope factor and C_0 a dimensional constant defined in ref.^{16,33}. From eq.(19) we see that the extra push is different from zero if $(Z^2/A)_{\text{eff}} > (Z^2/A)_{\text{eff}}^{\text{thr}}$, a threshold value of the effective fissility parameter. The schematic model of ref.¹⁰) gives for the values of the parameters :

$$a \approx 5 \quad \text{and} \quad \left(\frac{Z^2}{A}\right)_{\text{eff}}^{\text{thr}} \approx 26-27.$$

The above description can be extended to non central collisions if the centrifugal force is simulated by an increase of the Coulomb force. In this way one has to introduce a λ dependent effective fissility parameter defined as :

$$\left(\frac{Z^2}{A}\right)_{\text{eff}}(\lambda) = \left(\frac{Z}{A}\right)_{\text{eff}}(\lambda=0) + g(f\lambda)^2 \frac{A_1 + A_2}{A_1^{4/3} A_2^{4/3} (A_1^{1/3} + A_2^{1/3})}. \quad (20)$$

To a constant factor, $(Z^2/A)_{\text{eff}}(\lambda)$ is the Coulomb plus centrifugal forces divided by the nuclear force evaluated at the point where this later quantity is maximum. We see that there is some ambiguity (already discussed in section 6) for the value of f which corresponds to the proportion of orbital angular momentum kept in relative motion. An analysis of the experimental data obtained in ref.¹⁰) leads to the following values of the parameters^{10,33}) :

$$a \approx 10, \quad \left(\frac{Z^2}{A}\right)_{\text{eff}}^{\text{thr}} \approx 32.5 \quad f \approx \frac{5}{7} \text{ (rolling)}$$

These values are rather different from those obtained in the schematic approach of ref.³⁵). As far as the factor f is concerned, the rolling value can be reproduced by the simple model of ref.³⁴).

It should be noted that the above simulation of angular momentum is probably a very rough description of reality. In particular we know that angular momentum will change the saddle properties of the compound nucleus and of the composite system³). This is not taken into account by just modifying the value of $(Z^2/A)_{\text{eff}}$.

At the same level of simplicity, a parametrization of the extra extrapush energy can be obtained in term of $(Z^2/A)_{\text{eff}}$, and of a parameter which is the geometric mean of the normal and effective fissility parameters^{32,35}. We shall not go into the details of this parametrization and refer the reader to ref.³²) for more details. We just would like to point out a few important things which are coming out of this parametrization :

the extra extrapush energy, E_{xx} , is the important quantity to be considered for compound nucleus formation. In contrast to E_x , which increases smoothly with the excess of $(Z^2/A)_{\text{eff}}$ over $(Z^2/A)_{\text{eff}}^{\text{thr}}$, E_{xx} looks more as a step function at a threshold (cliff) which depends critically upon the mass asymmetry of the initial system. Indeed for symmetric systems $E_x = E_{xx}$ but the extra extrapush becomes larger than E_x with increasing asymmetry. An important outcome of the model is that the threshold associated to the extra extrapush corresponds to a si-

tuation where the fission barrier of the compound nucleus has not always vanished : consequently this approach predicts that we can get fast fission even for λ values smaller than λ_{B_f} . This point is in contradiction with the fast fission model.

11. THE STATIC FAST FISSION MODEL

As for the extrapush approach, it is interesting to parametrize some of the results of the fast fission dynamical model by simple analytic formulas. In section 6 we already started to derive the condition under which fusion is possible. However, for a system where it is so, we do not know if fusion will be easily realized or not. Indeed, a dynamical approach, including dissipative forces, shows that some extra kinetic energy above the static fusion threshold is needed to overcome the fusion barrier (see fig. 12 for a schematic illustration). We shall now try to estimate roughly the value of this extra energy using recent experimental results.

By definition, λ_{CR} is the largest λ value which is able to pass the fusion barrier. It should satisfy the following equation :

$$E = V(R_{f\lambda_{CR}}) + \frac{f^2 \lambda_{CR}^2 \hbar^2}{2\mu R_{f\lambda_{CR}}^2} + \Delta E_R + \Delta E_t \quad (22)$$

where E is the center of mass bombarding energy, $R_{f\lambda_{CR}}$ is the position of the fusion barrier for $\lambda = \lambda_{CR}$, and μ is the reduced mass. ΔE_R and ΔE_t are respectively the energy loss in the radial and tangential motions when the system reaches $R_{f\lambda_{CR}}$. If we assume that the energy loss in the tangential motion occurs at almost constant distance, it can be shown³⁶⁾ that ΔE_t is just equal to the loss of centrifugal energy due to the decrease of orbital angular momentum from λ_{CR} to $f\lambda_{CR}$. Equation (22) can be rewritten as :

$$E = V(R_{f\lambda_{CR}}) + \frac{\lambda^2 \hbar^2}{2\mu R_{f\lambda_{CR}}^2} + \Delta E \quad (23)$$

where ΔE is the extra kinetic energy which we have to provide above the static fusion threshold, $V(R_{f\lambda_{CR}})$, in order to fuse. The fusion cross section, σ_F , can then be written as :

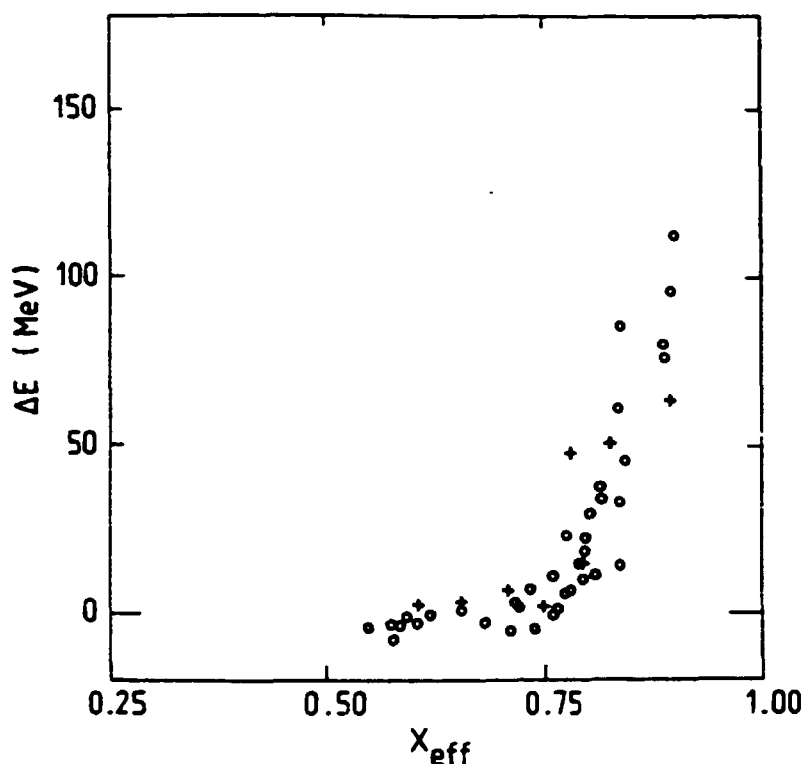
$$\sigma_F = \pi R_{f\lambda_{CR}}^2 \left[1 - \frac{V(R_{f\lambda_{CR}}) + \Delta E}{E} \right] \quad (24)$$

For the cases investigated in ref.¹⁰⁾ we have used the experimental value of λ_{CR} to calculate $R_{f\lambda_{CR}}$ and $V(R_{f\lambda_{CR}})$ with the interaction potential defined in section 6. We assumed the rolling value for f ($f=5/7$). ΔE was deduced from eq.(24) using the experimental value of the fusion cross section. The extra energy, ΔE , obtained in this way, is plotted in fig. 13 (circles) as a function of the parameter, X_{eff} , defined by :

$$X_{eff} = \frac{1}{4\pi\gamma} \left\{ \frac{Z_1 Z_1 e^2}{C_1 C_2 (C_1 + C_2)} + \frac{f^2 \lambda^2 \hbar^2}{m} \frac{A_1 + A_2}{A_1 A_2} \frac{1}{C_1 C_2 (C_1 + C_2)^2} \right\} \quad (25)$$

where m is the nucleon mass and $4\pi\gamma = 12.54$ MeV/fm² for the interaction potential used in section 6. In the same figure we have also plotted (crosses) the values of ΔE deduced for the Ar + Ho system investigated in refs.^{25,26)}. However,

Fig. 13 - The extra energy, ΔE , needed to pass the fusion barrier is plotted as a function of X_{eff} defined by eq.(25). The circles are associated to the systems investigated in refs.¹⁰⁾ with $f=5/7$ (rolling). The crosses correspond to the Ar + Ho system of ref.^{25, 26)} with f given by eq.(5) (sticking).



for this system, we have used for f the sticking value (eq.(5)), otherwise the crosses would not have followed the general trend of the systems measured in ref.¹⁰⁾. This problem, related to the choice of the factor f , also exists for the static extra push model. It tells us that simulating angular momentum by an increase of the Coulomb force is only a rough description of the real situation.

The general behaviour of ΔE can be parametrized by the following expression :

$$\Delta E \approx 2000 \left(X_{\text{eff}} - 0.68 \right)^2 . \quad (26)$$

The next quantity which is interesting to parametrize is the limit where quasifission appears. According to the fast fission model this happens when the saddle configuration becomes more compact than the pocket one. In fig. 14 we have plotted the distance corresponding to the pocket configuration and to the compound nucleus saddle point (deduced from ref.³⁷⁾), as a function of the fissility parameter X defined by³⁹⁾ :

$$X = \frac{Z^2/A}{50.88 (1 - 1.7826 I^2)} \quad (27)$$

and

$$I = \frac{N-Z}{A} \quad (28)$$

we see that the pocket becomes less compact than the saddle configuration when :

$$X \geq 0.83 . \quad (29)$$

Quasifission occurs when the above inequality is satisfied. For compound nuclei along the beta stability line, $X \approx 0.83$ corresponds approximately to :

$$n = \frac{Z^2}{A} \approx 38.5 . \quad (30)$$

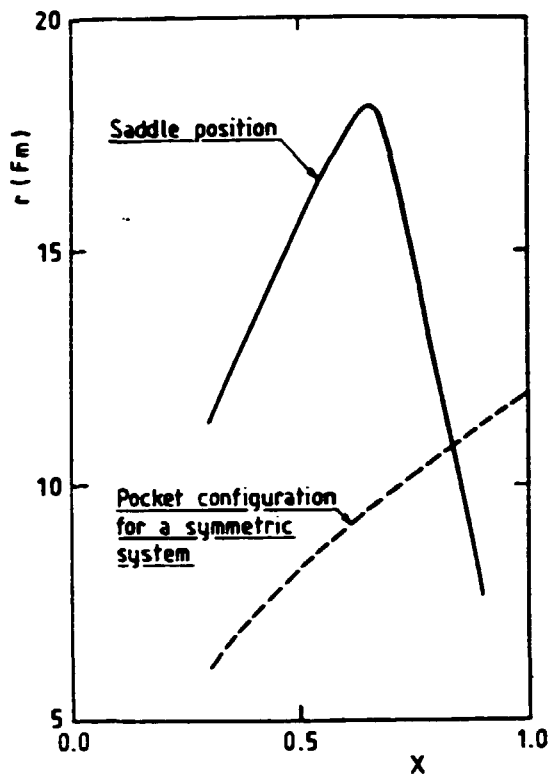


Fig. 14 - Distance corresponding to the pocket configuration of a symmetric system (calculated using the interaction potential of section 5), and to the saddle position of the compound nucleus (from ref. ³⁸), as a function of the fissility parameter X (eq. (27)).

The conditions under which compound nucleus, fast fission and quasifission can be obtained are summarized in fig. 15. It should be noted that fast fission and quasifission can only be observed if the initial system is not too asymmetric. Typically, the mass asymmetry variable x , defined in fig. 15, should be smaller than about 0.7, but this value may depend upon the system and the bombarding energy. The reason is that fast fission or quasifission occur only if we form a symmetric two-center composite system. For that, the driving force along the mass asymmetry coordinate should let the system evolve in this direction. If the initial system is too asymmetric, this is not

the case, and a one-center configuration will be formed which will deexcites almost like a compound nucleus.

Finally, in fig. 16, we summarize, in a schematic way, the range of λ values which are to be associated to the mechanisms following fusion.

12. CONCLUSION

Since a few years a lot of progress have been done in the understanding of fusion. New mechanisms : fast fission and quasifission, appear naturally in the theoretical models described in this review. They help to understand a lot of experimental data which were hard to fit in our old understanding of fusion. However, up to now, there is no direct experimental evidence of these mechanisms but only some indications that they could be there. Therefore a large amount of experiments remain to be done in the near future. The simplification of the dynamical extrapush and fast fission models, to a static description of fusion, are very helpful but still unprecise. In particular a better treatment and understanding of angular momentum is urgently needed. Finally it is worth to note that the idea of critical distance, proposed in refs. ^{1,6}), could probably find a theoretical justification with the extrapush or the fast fission models.

13. ACKNOWLEDGEMENTS

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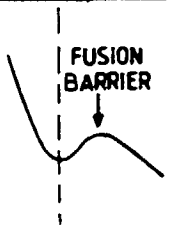
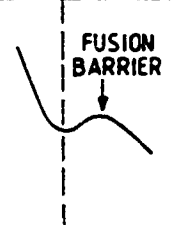
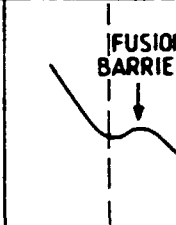
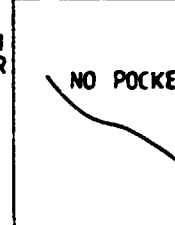
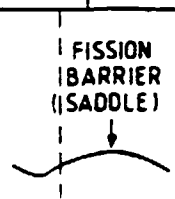

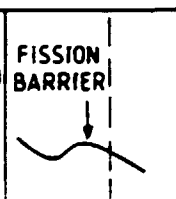
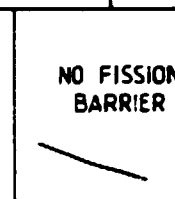
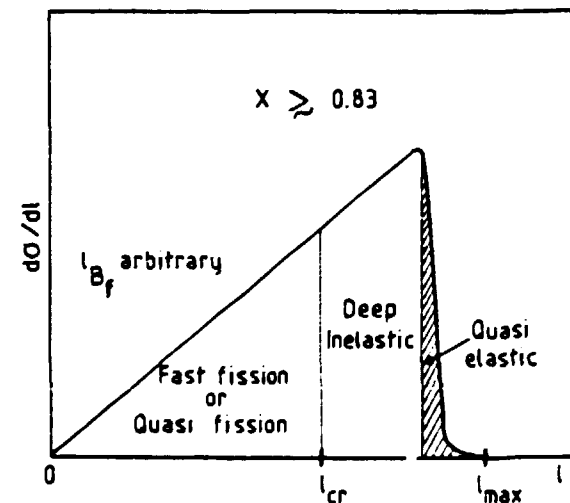
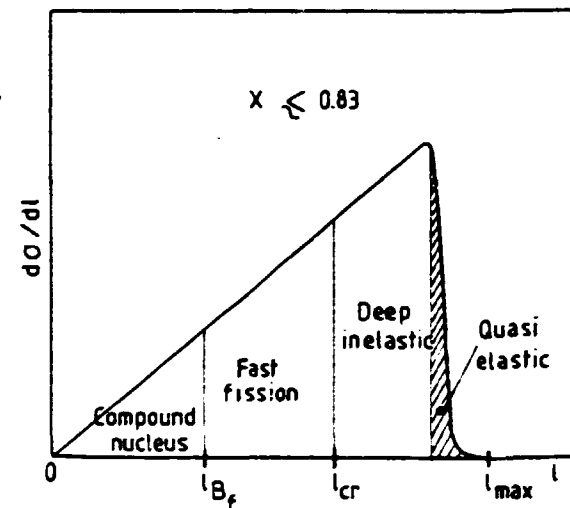
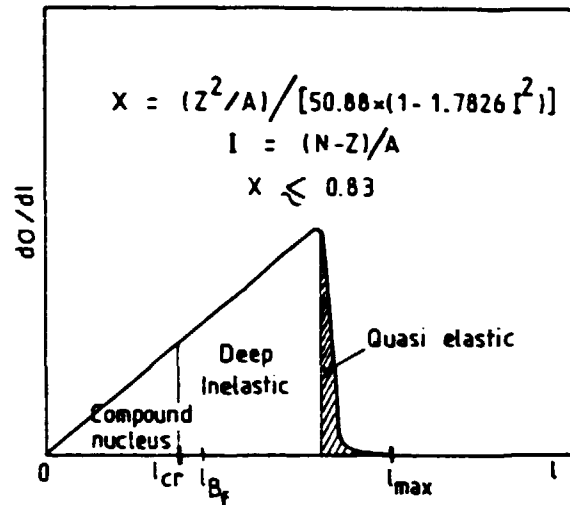
APPROXIMATE CONDITIONS			
$\xi \lesssim 48$	$\xi \lesssim 48$	$\xi \lesssim 48$	$\xi \gtrsim 48$
$\forall x$	$x \lesssim 0.7$ very roughly	$x \lesssim 0.7$ very roughly	$\forall x$
$\eta \lesssim 38.5$	$\eta \lesssim 38.5$	$\eta \gtrsim 38.5$	$\forall \eta$
$l < l_{Bf}$	$l \geq l_{Bf}$	$\forall l$	$\forall l$
SUDDEN POTENTIAL			
			
ADIABATIC POTENTIAL			
			
COMPOUND NUCLEUS FORMATION	FAST FISSION	FAST FISSION (QUASIFISSION)	NO COMPOUND NUCLEUS FORMATION NOR FAST FISSION
$x = \frac{A_2 - A_1}{A_1 + A_2}$	$\eta = \frac{Z^2}{A}$	$\xi = \frac{4Z_1Z_2}{A_1^{1/3}A_2^{1/3}(A_1 + A_2)}$	

Fig. 16 - Schematic summary of the different mechanisms following fusion and their domains of occurrence.

Fig. 16 - Schematic representation of the different ranges of l values associated to the four dissipative mechanisms which can be observed in heavy ion reactions.

REFERENCES

- 1) For reviews see for instance :
W. Schröder and J. Huizenga, *Ann. Rev. Nucl. Sci.* **27** (1977) 465.
M. Lefort and C. Ngô, *Ann. Phys. (Paris)*, **3** (1978) 5.
C. Ngô, *Proc. of the International School on critical phenomena in heavy ion physics, Poiana Brasov* (1980) p. 395.
- 2) M. Lefort et al. *Nucl. Phys.* **A216** (1973) 166.
- 3) For a review see for instance :
M. Lefort, *European Conference on nuclear physics with heavy ions, Caen* (1976) *J. Phys.* **C5** (1976) 57.
- 4) R. Bass, *Nucl. Phys.* **A231** (1974) 45.
- 5) C. Ngô et al., *Nucl. Phys.* **A240** (1975) 353.
- 6) J. Galin et al., *Phys. Rev.* **C9** (1974) 1018.
- 7) A.M. Zebelman and J.M. Miller, *Phys. Rev. Lett.* **30** (1973) 27.
- 8) S. Cohen et al., *Ann. of Phys.* **82** (1974) 557.
- 9) C. Ngô, *Proc. of the International Summer School at La Rabida (Spain)* (1982). *Lect. Notes in physics* **168** (1982) 185.
- 10) R. Bock et al., *Nucl. Phys.* **A388** (1982) 334.
- 11) H. Ngô and C. Ngô, *Nucl. Phys.* **A348** (1980) 140.
- 12) C. Ngô et al., *Nucl. Phys.* **A252** (1975) 237.
- 13) J.R. Birkelund et al., *Phys. Rep.* **56** (1979) 107.
- 14) L.C. Vaz et al., *Phys. Rep.* **5** (1981) 373.
- 15) J. Blocki et al., *Ann. Phys.* **105** (1977) 427.
- 16) W.J. Swiatecki, *Phys. Script.* **24** (1981) 113.
- 17) C. Grégoire et al., *Phys. Lett.* **99B** (1981) 17 and *Nucl. Phys.* **A383** (1982) 392.
- 18) C. Ngô et al., *2nd Europhysics Study Conf. on the dynamics of heavy ion collisions, Hvar, 1981* (North Holland) p. 211.



- 19) C. Grégoire et al., Int. Conf. on selected aspects of heavy ion reactions, Saclay 1982, Nucl. Phys. A387 (1982) 37.
- 20) C. Ngô et al., Nucl. Phys. A400 (1983) 259.
- 21) W. Nörenberg and C. Riedel, Z. Phys. A290 (1979) 335.
- 22) T. Ledergerber and H.C. Pauli, Nucl. Phys. A207 (1973) 1.
- 23) H. Hofmann and P.J. Siemens, Nucl. Phys. A275 (1977) 467.
- 24) R.A. Broglia et al., Phys. Lett. 61B (1976) 113 and Proc. of the International School of physics "Enrico Fermi", Varenna, 1979 (North Holland).
- 25) C. Lebrun et al., Nucl. Phys. A321 (1979) 207.
- 26) B. Borderie et al., Z. Phys. A299 (1981) 263.
- 27) C. Grégoire et al., Nucl. Phys. A361 (1981) 443.
- 28) C. Grégoire and F. Scheuter, Z. Phys. A303 (1981) 337.
- 29) W.J. Swiatecki, Prog. Part. Nucl. Phys. 4 (1980) 383.
- 30) J. Blocki et al., Ann. Phys. 113 (1978) 330.
- 31) H. Kramers, Physica VII (1940) 284.
- 32) S. Bjornholm and W.J. Swiatecki, Nucl. Phys. A391 (1982) 471.
- 33) W.J. Swiatecki, Nucl. Phys. A376 (1982) 275.
- 34) G. Fai, Preprint LBL-14413 (1982).
- 35) S. Bjornholm, International Conference on selected aspects of heavy ion reactions, Saclay (1982), Nucl. Phys. A387 (1982) 51.
- 36) U. Mosel, 2nd Europhysics study Conf. on the dynamics of heavy ion collisions, Hvar, 1981 (North Holland) p. 1.
- 37) J. Blocki and W.J. Swiatecki, Preprint LBL 12811-UC-34d (1982).
- 38) W.D. Myers and W.J. Swiatecki, Ark. Fys. 36 (1967) 342.

