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FRICTION FINITE ELEMENTS FOR NUMERICAL ANALYSIS OF PIPING SYSTEMS : THEORY AND APPLICATIONS

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ABSTRACT

Contact problems are often encounteted in civil engineering applications and specially in nuclear piping systems : pipe whip, impact on supports in case of seismic analysis, non linear calculations of thermal expansions, etc...

Most of the time, the geometrical non linearity of the contact problems is coupled with a material non linearity due to the nature of the contact associated with the state of the surfaces in contact.

For the sake of simplicity, the classical Coulomb friction model has been assumed, which leads to frictional sliding when the tangential force modulus reaches a fraction of the normal compression force.

In order to account for such a model in a beam type finite element program, a special element has been developped. It is formulated in terms of generalized stresses : axial forces and shear forces in two orthogonal directions. Below the sliding threshold, the element has only axial and shear elastic rigidities in compression. When the sliding threshold is reached, a perfectly plastic behavior is assumed.

The element can be used together with unilateral conditions for example in order to calculate the pipe whip impact on a structure.

This element has been implemented in the piping analysis TEDEL program of the CEASEMT finite element system.

In the second part of this paper, simple tests are presented.

1. INTRODUCTION

The design of complete piping systems is often made on the basis of elastic behavior hypothesis. However, it can be necessary to perform nonlinear analysis either due to the material behavior (plastic or creep regime) or to the geometrical features of the line. For example such problems are encountered in case of a pipe whip, since the pipe undergoes large displacements and may impact a mechanical stop unit. A similar problem comes from the locking of snubbers in case of seismic analysis.

Most of the time, material and geometrical non linearities must be simultaneously accounted for, as it is the case in contact problems, since the unilateral constraints are coupled with the material non linear properties associated with the nature of the contact.

For these reasons, a special friction element has been developped and implemented in the nonlinear finite element program TEDEL of the CEASEMT system. [9][10]

The main features of this program is to work with beam type elements and to use a global plasticity approach [1][2][3].

In the present paper, the theoretical formulation of the element is first described ; then application examples are given.

2. THEORY

2.1 - Element description in elasticity

The element is a two nodes straight element, with six degrees of freedom per node. An example of typical lay-out is shown on Figure 1. The element axis defines the vector normal to the sliding plane. The element possesses only axial and shear stiffness and has no bending and torsional stiffnesses. Thus, in local axis, the state of generalized stresses is defined by :

- the axial force N
- the shear forces T_y, T_z.

(see Figure 2).

The corresponding strains are :

- the longitudinal strain ε
- the shear strain γ_y, γ_z

They are computed by means of the local displacements u, v, w (see Figure 3) which are assumed to vary linearly over the element :

$$\varepsilon = \frac{du}{dx} = \frac{u_2 - u_1}{l}$$

$$\gamma_y = \frac{dv}{dx} = \frac{v_2 - v_1}{l}$$

$$\gamma_z = \frac{dw}{dx} = \frac{w_2 - w_1}{l}$$

where v_i, w_i are local displacements at note i, and l is the lenght of the element.

Two remarks can be made at this level :

- Torsion has not been accounted for in the element, while it can be easily introduced by means of the torsional moment G and the corresponding strain $\gamma_x = \frac{d\psi_x}{dx}$ where ψ_x is the axial relation.
- A more sophisticated element accounting for the end rotations can be made by using the following shear strains :

$$\gamma_y = \frac{dv}{dx} - \psi_z$$

.../...

$$\gamma_z = \frac{dw}{dx} + \psi_y$$

where ψ_y, ψ_z are local rotations.

The elastic stress-strain law of the element is therefore :

$$\{\sigma\} = \begin{vmatrix} N \\ T_y \\ T_z \end{vmatrix} = D\{\varepsilon\} = \begin{vmatrix} ES & o & o \\ o & GS_y & o \\ o & o & GS_z \end{vmatrix} \times \begin{pmatrix} \varepsilon_{xx} \\ \gamma_y \\ \gamma_z \end{pmatrix}$$

Where S is the element cross-section and S_y, S_z are the shear reduced areas. G is the shear modulus.

2.2 - Friction properties

A simple Coulomb friction law has been assumed for the element, that is sliding occurs in the y - z plane when the shear force modulus reaches the product

$f \,.\, |N|$ where f is the friction coefficient

i.e. $\sqrt{T_y^2 + T_z^2} \leq f\,|N|$

with $N < 0$

This is similar to the behavior of an elastic perfectly plastic material, with a yield stress R depending of the parameter N.

2.3 - Unilateral contact condition

When the friction element is connected directly to the structure or when there is a gap as shown on Figure 4, in both cases, the element has no stiffness when submitted to traction forces.

This has been first accounted for in the element by setting N, T_y, T_z to zero and treating the prescribed element forces as residual forces for the next equilibrium iteration. However such a method may show bad convergence properties according to the ratio between the friction element and the structure stiffnesses. This is the reason why it is more advisable to use a unilateral constraint technic [5][6] which ensures the friction element to be only compressed.

3. APPLICATION

3.1 - Stactic example

This first example consists of a slab of length . Eight units, of unit thickness, made on an elastic material with a Young's modulus E = 1 000 units of force per unit area and a Poisson's ratio ν = 0.3 ([7]). The slab is simply supported on one part of its boundary and subjected to a uniform pressure p acting through a frictional surface with a friction coefficient f = 0.3 and also subjected to a prescribed compressive force at one of its ends as illustrated on Figure 5. The analysis was done using 16 beam elements. The computed frictionnal forces for GS = 10^6 units and GS = 10^8 units are shown in the Figure 6. The rate of convergence can be small for high value of GS.

The discrepency between the present results and the reference ones can be due on one hand to the numerical algorithm (use of elastic stiffness versus penalty method) and on the other hand on the difference of the shear modelization. (discrete versus continuous).

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3.2 - Dynamic example

This example deals with the free vibration of a spring with a Coulomb damping. The friction element is assumed to have the same stiffness as the spring. The dry friction force F_d has the same direction as the spring and is proportional to a constant weight N as shown on Figure 7.

The friction force is opposite in direction to the velocity and remains constant until the velocity sign changes. There is then an elastic unloading followed by the sliding motion.

The elastic phasis is governed by the equation :

$$m \ddot{x} + (k_1 + k_2)\ x = k_2 x_p$$

where x_p is the plastic displacement of the mass when the velocity sign changes. Then, the sliding phasis is governed by the equation :

$$m \ddot{x} + k_1\ x = - \text{sign}\ (\dot{x})\ F_d$$

In our calculations, the motion starts from an initial position $x(0) = x_o$ with a non zero x_p.

The analytical solution, with $k_2 = \infty$, is given in [8], and shown on Figure 8 in dashed lines. Here, the solution is slightly different since the cycle period as well as the decay by cycle depends on k_2. Moreover, the movement ends by a sinusoïdal displacement of frequency $f = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$, while in the case $k_2 = \infty$, it leads to some residual displacement.

The solution calculated by TEDEL compares exactly with the analytical solution (the two full lines are identical). It may be noticed that the linear decrease of the maximum displacement is well reproduced.

4. CONCLUSION

An original friction finite element has been developped for the calculation of complete piping systems in statics or dynamics. The examples presented show that the main difficulty is the choice of a good elastic stiffness for these elements, since a too large stiffness may lead to numerical deterioration of the solution. The method used may be seen as a penalty technic. The Lagrange multipliers approach will be investigated as an alternative.

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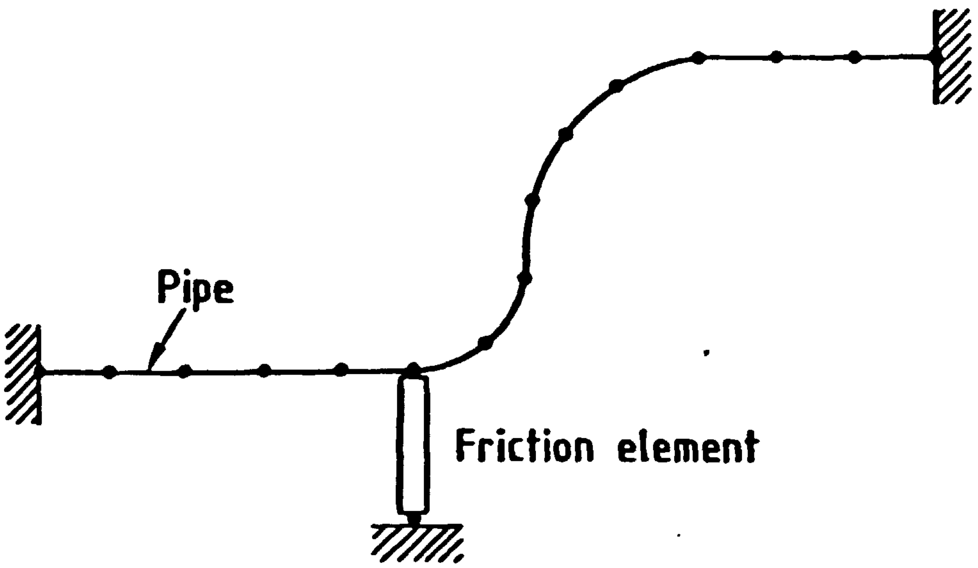


Fig. 1 – Typical layout of the friction element

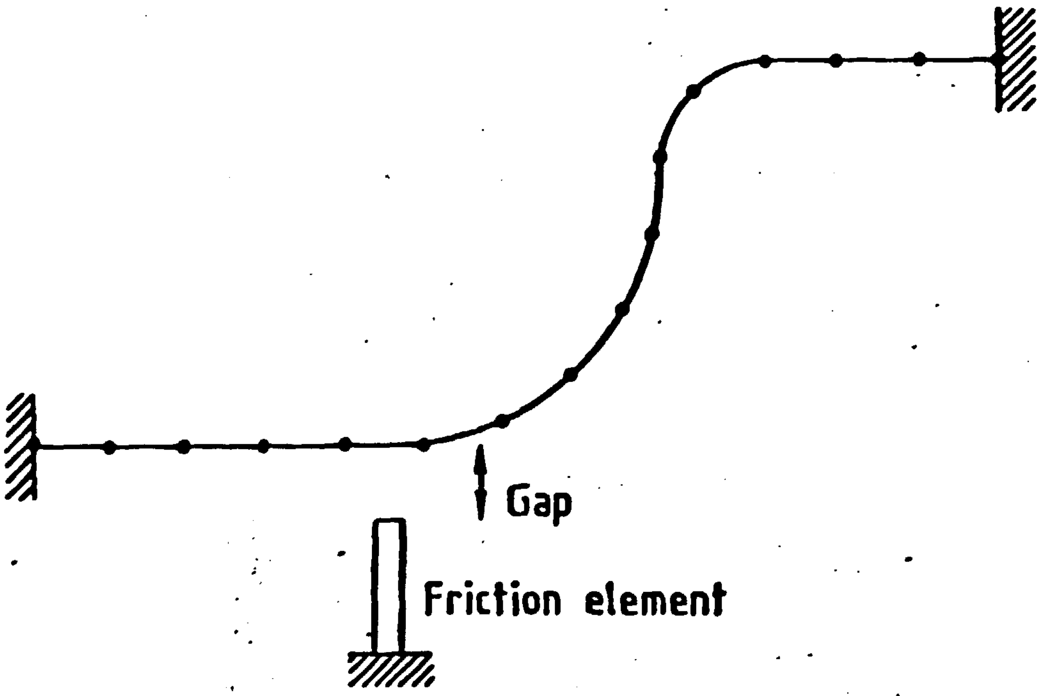


Fig. 4 – Friction element used together with unilateral constraint

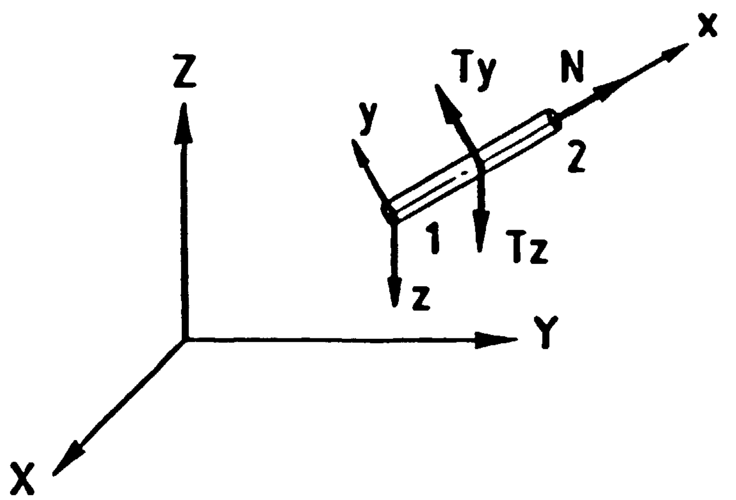


Fig. 2 - Generalized stresses for friction element

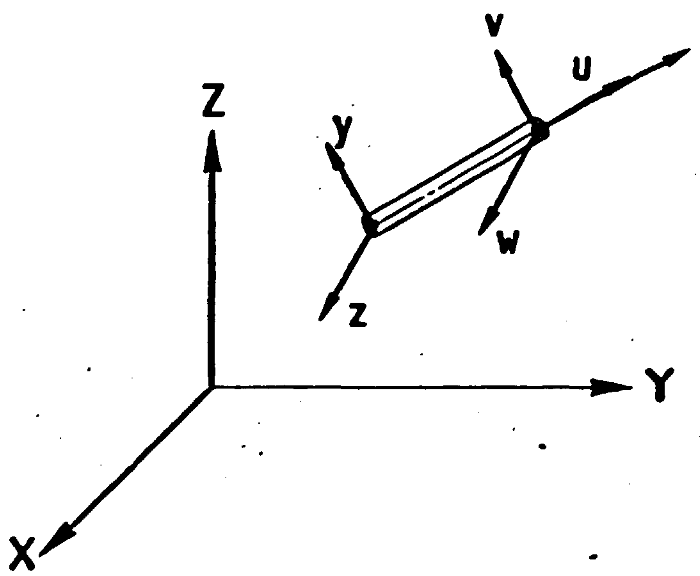


Fig. 3 - Local displacements for friction element

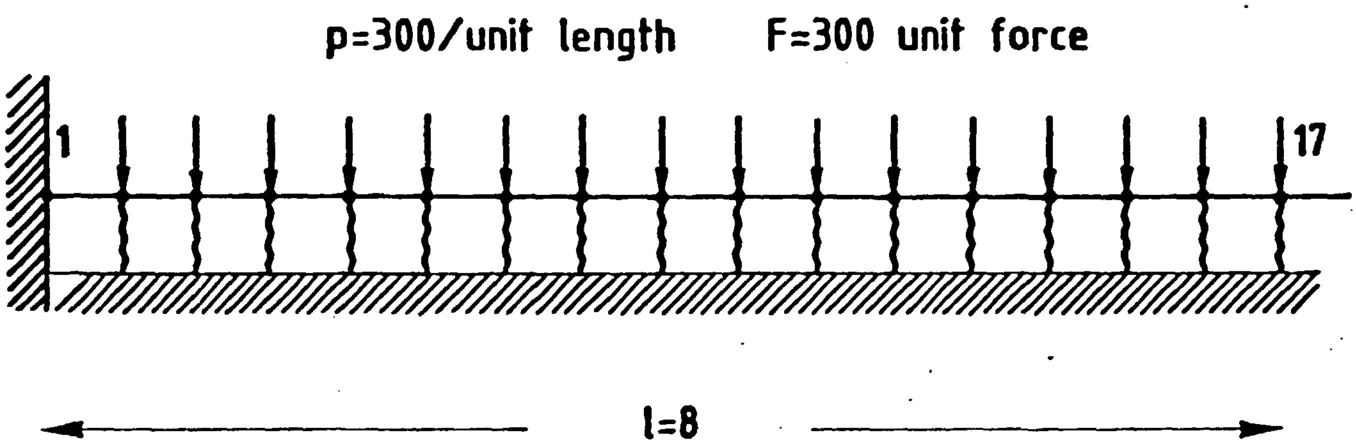
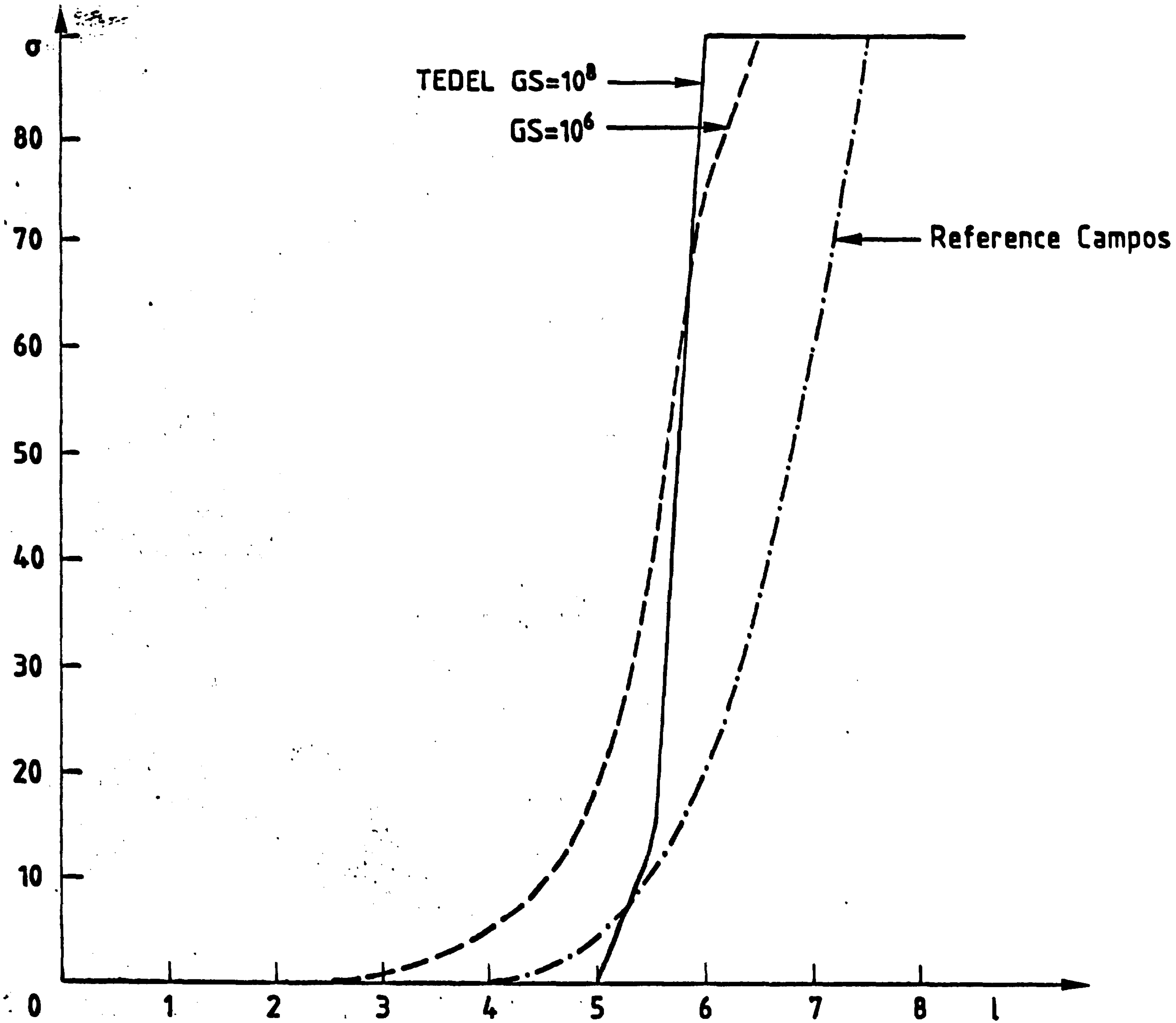


Fig. 5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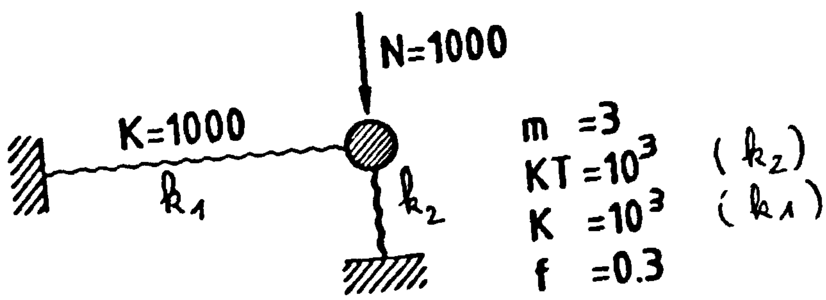

Fig. 7

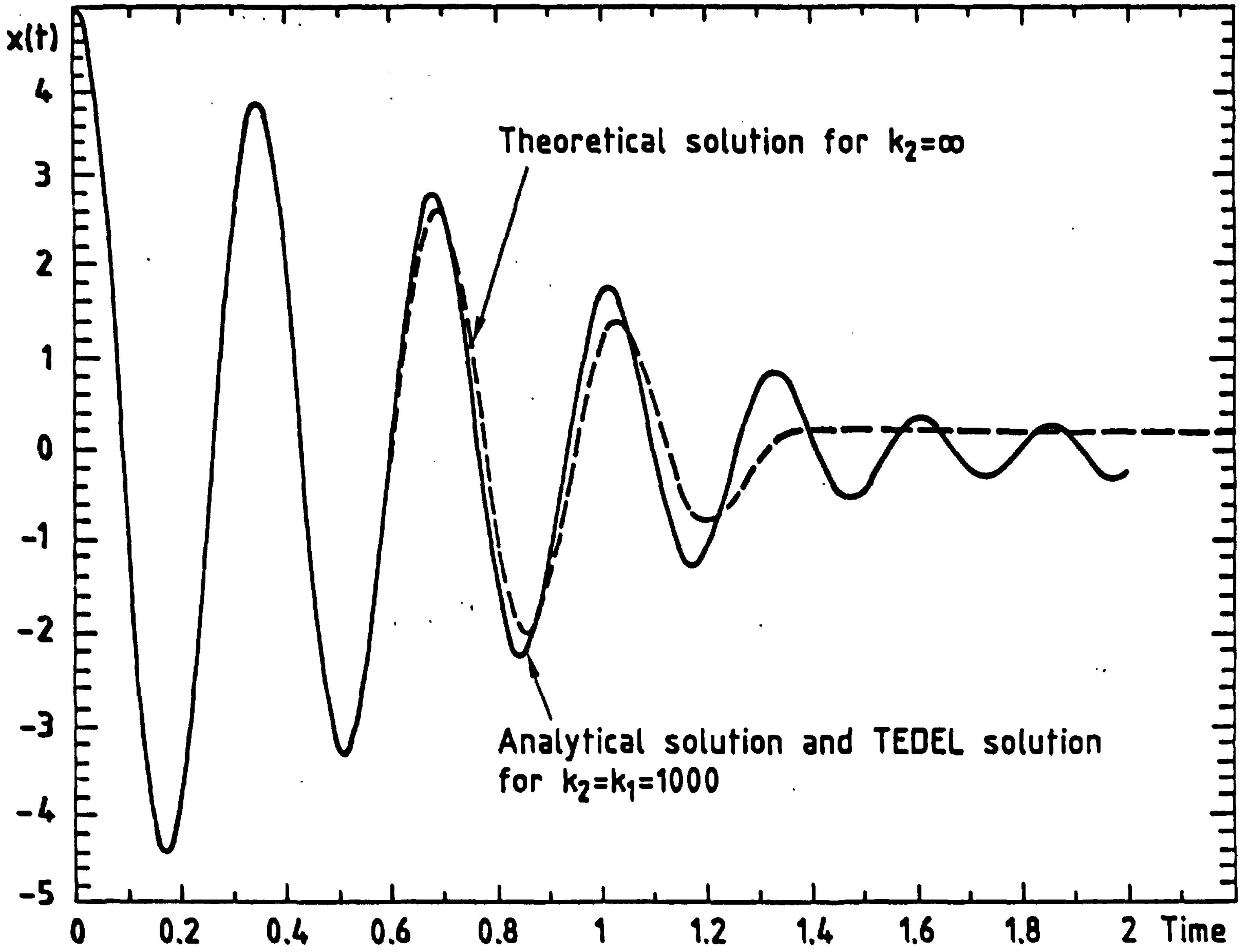

Fig. 8 - Dynamic example