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POISSON's ratio correction in elastic analysis of low cycle fatigue

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Attention K_ν NU

SUMMARY

During the operation of nuclear plants, components experience low-cycle fatigue due to thermal transients. As is well-known, low-cycle fatigue is the condition in which inelastic strain predominates. A good knowledge of the local strain range is therefore highly desirable for low-cycle fatigue analysis. Since current practice is to perform a linear elastic analysis, results thus obtained must be corrected to assess accurately the actual strain range.

There are two basic discrepancies between the plastic behaviour of the material and the linear elastic model. The first is the non-linear stress/strain relationship, which mainly affects the strain range in the vicinity of stress raisers. The second is that plastic deformation shows no change in volume. In other words, the plastic POISSON's ratio (P.R.) is equal to 0.5 in the plastic range, i.e., it has a higher value than the elastic P.R.. This paper covers the corrective action related to this absence of volume variation.

Current practice is to require that stresses be evaluated on an elastic basis, although with a P.R. value different from the elastic P.R. This procedure is rather inconvenient, and seldom used. It appears preferable to use an alternative rule requiring only multiplying the computed equivalent elastic strain range by a corrective factor which may de designated as K_ν.

Under plane stress conditions (e.g., close to a wall), the actual equivalent strain range can be computed using an equivalent P.R., the value of which is obtained from simple considerations.

$$\bar{\nu} = \frac{E_s}{E}\nu + 0.5\left(1 - \frac{E_s}{E}\right)$$

where E_s is the secant modulus of the cyclic stress/strain curve (at the actual strain range value) and E, ν are the elastic constants.

This permits deriving the corrective factor K_ν and plotting curves showing K_ν as a function of the elastic strain range (and the ratio of principal strains ~~stresses~~) for a given material.

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1. <u>Elastic analysis is generally used to evaluate low-cycle fatigue. Since the latter is dependent on actual deformation conditions, two correction types are required</u>.

During the operation of nuclear plants, components experience low-cycle fatigue due to thermal transients. As is well-known, low-cycle fatigue is the condition in which inelastic strain predominates. A good knowledge of the local strain range is therefore highly desirable for low-cycle fatigue analysis. Since current practice is to perform a linear elastic analysis, results thus obtained must be corrected to assess accurately the actual strain range.

Calculations are based on the assumption that material behaviour is fully elastic and linear. This method has the advantages of simplicity and economy. It is also justified by the fact that selecting a suitable constitutive ~~characteristic~~ equation is difficult and often unreliable.

Unfortunately, strain variations obtained through elastic analysis are not identical to the actual variations which are required for evaluating low-cycle fatigue behaviour. Differences between these two values of strain variations have two distinct origins.

The first is the non-linear stress/strain relationship in the event of plastic behaviour. Because of this, the actual strain variation is often larger than the value computed assuming the material to be elastic and linear. The magnification due to this non-linearity is particularly noticeable in the vicinity of stress raisers. Practical rules have been proposed in a number of codes, e.g., ASME Section III (2), where elastic analysis results must be multiplied by a factor Ke. Although this effect will not be discussed here, readers are invited to refer to the article by P. Petrequin et al. (1) for more detailed information.

The other origin is the fact that only elastic strain causes volume changes in sound material. Plastic strain produces no volume changes. Such changes are directly related to the value of Poisson's ratio (P.R.). If this value is 0.5, volume variation is nil, which is the case of non-elastic deformation. For linear, elastic deformation in most metals, P.R. is close to 0.30, representing an appreciable change in volume. A change in P.R. when plastic deformation takes place may also amplify the strain variation. Unlike the phenomenon discussed above, amplification does not occur in the stress raisers, but rather where the stress or strain field is bi-axial. Such is the shocks case in a wall subjected to thermal ~~stress~~. The object of this paper is the P.R. correction through a factor designated as K_{ν} to make a distinction between this phenomenon and the former. Such a correction is actually prescribed by the construction codes (1), (2) in certain cases of fatigue analysis. It is generally required that computations be performed using a nominal P.R. (different from the physical value) which accounts for stress cycle amplitude. Unfortunately, applying this correction is not straightforward and it is frequently omitted. This paper therefore aims at presenting a more convenient method.

2. If fatigue resistance is dependent on the equivalent variation, a P.R. correction factor must be applied to elastic analysis data.

Although fatigue testing is performed using mono-axially loaded specimens, practical fatigue analysis often covers multi-axial load cases. The multi-axial nature of the load is the reason for the P.R. correction.

Such an investigation requires determining the incidence of multi-axial strains on fatigue behaviour. This is a complex problem which has been extensively covered by Brown (4) and Marloff (5). However, it should be noted that workers in the field do not seem to agree unanimously on a single general law. The authors do not intend to discuss this point; the octahedral strain amplitude law will therefore be accepted as a good representation of physical reality as regards low cycle fatigue. This statement is in agreement with common practice as mentioned in code case N47 (3).

In other terms, if we know the variation $\Delta\varepsilon_{ij}$ of every actual strain tensor component, we must subsequently agree that the following relationship should be used to enter the "design fatigue curves":

$$\Delta\varepsilon_{eq.} = \sqrt{\left(\frac{2}{3} \Delta \varepsilon_{ij}^{2} + \frac{2}{9} (\Delta\varepsilon_{kk})^{2} \right)} \qquad (1)$$

Unfortunately, these actual variations remain unknown if computations have been based on the assumption that material behaviour is elastic. The only known quantity is the equivalent strain variation $\Delta\varepsilon\, eq_e$, calculated under elastic conditions.

It can be readily be shown that, for an elastic, isotropic material (with Young's modulus E and Poisson's ratio ν, the following relationship holds:

$$\Delta\varepsilon\, eq_e = \frac{2}{3} \frac{1+\nu}{E} \Delta\sigma\, eq_e \qquad (2)$$

where $\Delta\sigma eq._e$ is the equivalent stress ~~strain~~ variation based on the law of octahedral shear and given by:

$$\Delta\sigma\, eq._e = \sqrt{\left(\frac{3}{2} \Delta\sigma_{ij} - \frac{1}{2} (\sigma_{kk})^{2} \right)} \qquad (3)$$

(σ_{ij} is the elastic component of the stress tensor).

loading path

However, the above relationship is no longer valid if the material, although isotropic, is not fully elastic and linear. It remains possible to use a similar relationship if the ~~load considered~~ remains ~~in~~ the proportional range. To this end, one should define two quantities E_b and $\bar{\nu}$, such that the following relationship holds at both ends of the loading cycle:

E_s

$$\varepsilon_{ij} = \frac{1+\bar{\nu}}{E_s} \sigma_{ij} - \frac{\bar{\nu}}{E_s} \sigma_{kk} \delta_{ij} \qquad (4)$$

We may then write relationship (5), similar to (2), in cases where the material is not fully elastic and linear.

$$\boxed{\Delta\varepsilon_{eq} = \frac{2}{3} \cdot \frac{1+\bar{\nu}}{E_s} \Delta\sigma_{eq}} \qquad (5)$$

It is apparent that Es must be the secant modul~~e~~us taken from the material cyclic curve at the considered loading point. $\bar{\nu}$ is the effective P.R., which is equal to ν in the elastic case, and tends toward 0.5 when behaviour is totally plastic (plastic deformation entails no change in volume).

The value of effective P.R. $\bar{\nu}$ has a strong incidence on fatigue analysis results, since for a given cyclic strain, the value of $\Delta\varepsilon_{eq}$ (entered in the design fatigue curves) may be highly dependent on it. It is sufficient to consider a point where $\Delta\varepsilon_{11} = \Delta\varepsilon_{22} = \alpha\Delta T$, $\Delta\sigma_{33} = 0$ and $\Delta\varepsilon_{12} = \Delta\varepsilon_{23} = \Delta\sigma_{13} = \Delta\sigma_{32} = 0$ (thermal stress with principal directions 1, 2 and 3). Then:

$$\Delta\varepsilon_{eq} = \frac{2}{3} \; \frac{1+\bar{\nu}}{1-\bar{\nu}} \alpha \Delta T \qquad (6)$$

It can readily be seen that, if the material exhibits a strongly plastic behaviour (with $\bar{\nu}$ close to 0.5) the computed value of $\Delta\varepsilon_{eq}$ is significantly greater than in the elastic case ($\bar{\nu} = \nu$). Maximum magnification is

$\frac{3(1-\nu)}{1+\nu}$, i.e., approximately 1.61.

3. When a wall experiences thermal shock, the strain condition is dependent on the effective P.R., i.e., on the magnitude of plastic deformation. Actual equivalent strain is K_ν times that calculated under elastic conditions. Factor K_ν can be determined as a function of thermal deformation.

The value of maximum amplification (1.61) derived through the above calculation is consistent with published data. Finite element calculations performed under inelastic conditions using the ANSYS code and quoted by Severud (6) give similar results. The same subject has been investigated by Gonyea (7) and Houtman (8). A diagram as in figure 4 of (6) gives a correction factor solely due to the incidence of the effective P.R.. This therefore suggests that the relevant correction should be applied through increasing by a factor K_ν the value of equivalent strain variation as computed under elastic conditions. Further, it will now be shown that the diagram which gives K_ν can be derived analytically from material properties and loading characteristics.

It is advisable to begin with the simplest case, i.e., thermal shock at a wall not subjected to pressure. This is the case of the example in the above paragraph. Along the main directions, $\Delta\varepsilon_{11} = \Delta\varepsilon_{22} = \alpha\Delta T$ and $\Delta\sigma_{33} = 0$; the value of the third ~~main~~ principal deformation $\Delta\varepsilon_{33}$ results from the relationship:

$$0 = \Delta\sigma_{33} = \frac{E_s}{(1+\bar{\nu})(1-2\bar{\nu})}\left(\bar{\nu}(\Delta\varepsilon_{11} + \Delta\varepsilon_{22}) + (1-\bar{\nu})\Delta\varepsilon_{33}\right) \qquad (6)\ \text{Bis}$$

which gives $$\Delta\varepsilon_{33} = \frac{-2\bar{\nu}}{1-\bar{\nu}}\,\alpha\Delta T \qquad (7)$$

a value highly dependent on effective P.R. $\bar{\nu}$; equivalent strain variation is:

$$\Delta \varepsilon_{eq} = \frac{2}{3} \frac{1+\bar{\nu}}{1-\bar{\nu}} \alpha \Delta T \tag{8}$$

It is found again that, in the case of an elastic material, the result would have been, as above:

$$\Delta \varepsilon_{eq_e} = \frac{2}{3} \frac{1+\nu}{1-\nu} \alpha \Delta T \tag{9}$$

a value which is too low and must be multiplied by factor

$$\boxed{K_\nu = \frac{\Delta \varepsilon_{eq} \quad \text{(actual)}}{\Delta \varepsilon_{eq_e} \quad \text{(computed under elastic conditions)}}} \tag{10}$$

which in this case is:

$$\boxed{K_\nu = \frac{1+\bar{\nu}}{1+\nu} \cdot \frac{1-\nu}{1-\bar{\nu}}} \tag{11}$$

The effective P.R. value, designated as $\bar{\nu}$, is given by relationship (12), established by Nadaï (9):

$$\bar{\nu} = \nu \frac{E_\Delta}{E} + 0.5 \left(1 - \frac{E_s}{E}\right) \tag{12}$$

Derivation of this relationship is given in the Appendix.

In practical cases, the secant modulus E_s associated with equivalent strain amplitude $\alpha \Delta T = \Delta \varepsilon$ can be found on the material cyclic curve (figure 1). Effective P.R. $\bar{\nu}$ can be computed using formula (12) and elastic values (E, ν). K_ν is then found from formula (11), and associated with $\alpha \Delta T$ (9) and $\Delta \varepsilon_{eqe}$.

It is thus possible to plot a diagram giving K_ν as a function of $\Delta \varepsilon_{eqe}$. This has been done (figure 2) in the case of a type 316 austenitic steel (at 20°C), the cyclic curve of which is shown by figure 1.

The above example illustrates the effect of P.R. . The specific case investigated is that of a point in a wall with no pressure applied, and where the only known deformations are those in the plane of the wall. However, deformation $\Delta \varepsilon_{33}$ in the perpendicular direction is derived from condition $\Delta\sigma_{33} = 0$. Its value is therefore directly dependent on material compressibility; it is different if material behaviour is strongly plastic (low compressibility) or fully elastic. In such cases, the value of $\Delta \varepsilon_{33}$ is dependent on material behaviour. Hence correcting factor K_ν.

4. <u>When a free wall is subjected in its own plane to cyclic bi-axial deformation, the actual equivalent cyclic strain to be used in fatigure calculations is equal to K_ν times the equivalent strain variation computed under elastic conditions. Factor K_ν is easily derived from the material cyclic curve.</u>

principal

In the example given earlier, strain values were the same in all directions (thermal shock case). One might consider more general cases in which the two ~~main~~ actual deformations $\Delta\varepsilon_{11}$ and $\Delta\varepsilon_{22}$ are not equal. The value of deformation in the direction perpendicular to the plane results from condition $\Delta\sigma_{33} = 0$, leading to

$$\bar{\nu}(\Delta\varepsilon_{11} + \Delta\varepsilon_{22}) + (1-\bar{\nu})\,\Delta\varepsilon_{33} = 0 \text{ , hence}$$

$$\Delta\varepsilon_{33} = \frac{-\bar{\nu}}{1-\bar{\nu}}\left(\Delta\varepsilon_{11} + \Delta\varepsilon_{22}\right) \qquad (14)$$

Introducing a change in notation defined by:

$$\Delta \varepsilon_{11} = \frac{e+d}{2}, \quad \Delta \varepsilon_{22} = \frac{e-d}{2} \text{ and } \bar{\mu} = \frac{1-\bar{\nu}}{1+\bar{\nu}}$$

where e and d are values of actual expansion and distortion respectively.

Actual strain equivalent variation is

$$\Delta \varepsilon_{eq} = \frac{1}{3} \sqrt{\left(3d^2 + \frac{e^2}{\bar{\mu}^2}\right)} \tag{15}$$

As above, if the material had been regarded as elastic, identical results would have been obtained with $\bar{\nu} = \nu$, giving a low value which must be multiplied by factor K_ν to derive the actual value.

$$K_\nu = \frac{\Delta \varepsilon \text{eq (actual)}}{\Delta \varepsilon \text{eqe (computed under elastic conditions)}}$$

therefore:

$$\boxed{K_\nu = \frac{\mu}{\bar{\mu}} \sqrt{\left(\frac{1 + 3\,\delta^2\,\bar{\mu}^2}{1 + 3\,\delta^2\,\mu^2}\right)}}$$

where

$$\delta = \frac{d}{e} = \frac{\Delta \varepsilon_{11} - \Delta \varepsilon_{22}}{\Delta \varepsilon_{11} + \Delta \varepsilon_{22}} \tag{13}$$

Effective values of P.R. $\bar{\nu}$ and $\bar{\mu}$ are derived from Nadai's relationship through a procedure similar to that in the previous paragraph. In the above formula, it will be noted that ratio $\mu/\bar{\mu}$ is the value of factor K_ν in the case of a fully equi-axial deformation where $d = \delta = 0$.

Based on the above considerations, a diagram can be plotted, giving K_ν as a function of $e = \Delta \varepsilon 11 + \Delta \varepsilon 22$,

$\delta = \frac{d}{e}$ for a material with known cyclic curve. This requires making the assumption that, under multi-axial conditions, the cyclic curve can be plotted in the $\Delta\varepsilon_{eq}$ - $\Delta\sigma_{eq}$ plane. The procedure is then the following: for a given value of $\Delta\varepsilon_{eq}$, one finds the secant module of the cyclic curve, hence $\bar{\nu}$ from Nadaï's formula, and $\bar{\mu}$. From values of ν and δ, K_ν is found by formula (13) and e by formula (15), written as:

$$e = \frac{3\bar{\mu}\,\Delta\varepsilon_{eq}}{\sqrt{(1+3\delta^2\bar{\mu}^2)}} \qquad (16)$$

As in the above example, curves giving K_ν as a function of $\Delta\varepsilon_{eq_e}$ and δ can be derived from the cyclic curve of a type 316 austenitic steel at 20°C; they are shown by figures 4 and 5 respectively.
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5. Conclusion. Proposed rule

Since plastic deformation takes place with no change in volume, the equivalent variation in deformation relevant to fatigue analysis is greater than in the elastic case.

Although certain codes ((2), (3)) recommend using a nominal P.R. value in calculations, it appears more convenient to apply a multiplying factor K_ν to the equivalent variation in deformation computed under elastic conditions.

It has been shown that this factor K_ν can be derived analytically from the material cyclic curve. Diagrams can be plotted, giving K_ν as a function of changes in deformation taking place in the plane of the wall.

This correcting factor K_ν is independent from correction Ke which applies to strain amplification in stress raisers.

APPENDIX - Determination of effective P.R. $\bar{\nu}$

By definition, effective P.R. $\bar{\nu}$ is given by formula:

$$\varepsilon_{ij} = \frac{1+\bar{\nu}}{E_s}\sigma_{ij} - \frac{\bar{\nu}}{E_s}\sigma_{kk}\,\delta_{ij} \tag{17}$$

for an isotropic material. In this relationship, E_s is the secant modules taken from the equivalent stress/ equivalent deformation curve.

Since it is well known that plastic deformation occurs with no change in volume, the only change in volume is due to elastic deformation, and written as:

$$\frac{dV}{V} = \varepsilon_{kk} = \frac{1-2\nu}{E}\sigma_{kk} \tag{18}$$

where E is Young's modulus and ν is the elastic P.R. of the material. This therefore requires:

$\frac{1-2\bar{\nu}}{E_s} = \frac{1-2\nu}{E}$, hence the value of effective P.R.

$$\bar{\nu} = \nu\,\frac{\cancel{\sigma_D}\;E_s}{E} + 0.5\left(1 - \frac{E_{\cancel{D}s}}{E}\right) \tag{19}$$

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