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UNSTABLE BARYONS WITHOUT GUTs

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Abstract :

In the rishon model the leptons and the quarks can be classified in either doublets or quadruplets of a SU(2) group. Gauge invariance leads to different charged current interactions in the doublet and the quadruplet cases. Demanding that the neutral currents be the same in the two cases, one obtains relations between the different charged current couplings to leptons and quarks ; moreover, if these transform as linear combinations of doublets and quadruplets, one can estimate the mass of the gauge boson responsible for baryon decay to be not larger than 10^5 GeV. A $SU(2)_L \otimes U(1)$ model is treated in detail.

INTRODUCTION

Among the recent developments in the theory of fundamental interactions the idea of grand unification [1] is one of the most appealing and aesthetically pleasing. Apart from the advantage of explaining all nongravitational interactions as arising from a simple symmetry principle and the differences among them as due to the spontaneous breaking of this symmetry, on the experimental side grand unification leads to such spectacular predictions as the decay of the baryons into leptons and mesons. This particular type of prediction follows from the fact that in grand unified theories (GUTs) the quarks and leptons are put in the same representation and thus transitions among them are possible via the exchange of gauge bosons.

There are however other theories which predict $\Delta B \neq 0$ processes. Most composite models of quarks and leptons lead to unstable baryons via rearrangements of the preons (common constituents of quarks and leptons). The simplest example is that of the rishon model [2] which is based on two constituents, called rishons and labelled T and V. However, these composite models, while quite successful in accounting for the spectroscopy of quarks and leptons do not provide yet a means of getting a better understanding of the interactions among them.

In the present paper, we introduce yet another possibility of obtaining unstable baryons, based in a large measure on the rishon model. While it will be a gauge theory, our proposal is not a GUT : the strong, electromagnetic and weak interactions are not unified, although the last two are mixed. In short, we propose a standard $SU(2) \otimes U(1)$ model but with multiplets given by the rishon model.

We start by introducing the possible bound states of three rishons or anti-rishons and thus constructing the quarks and leptons of the first generation. After classifying them in either doublets or quadruplets of a $SU(2)$ group we introduce the weak and electromagnetic interactions by gauging the group $SU(2) \otimes U(1)$. We do this separately for the doublets and then for the quadruplets. The charged currents are very different in the two cases, but the neutral currents are the same. Requiring the equality of the neutral current couplings in the two cases we obtain a relation between the Weinberg-Salam-like angles and thus between the couplings of the charged currents.

Next, we study a more realistic model, based on the $SU(2)_L \otimes U(1)$ gauge group and show in detail the spontaneous symmetry breaking with several Higgs multiplets. If each quark and lepton transforms as a linear combination of doublet plus quadruplet we get a gauge theory which reproduces the standard model and also predicts the decay of the baryons. We make a short comparison with the results of the SU(5) model.

We end with a discussion of our results and the interpretation and significance of this type of theory.

Details of some calculations are given in the Appendix.

2. The rishon model and the classification of quarks and leptons

The model developed by Harari and Seiberg [2] assumes the existence of two fundamental fermions, denoted T and V which transform as (3,3) and $(\bar{3}, 3)$ under the $SU(3)_C \otimes SU(3)_H$ group. Bound states of 3 rishons or antirishons are all hypercolor singlets and either color singlets (leptons) or triplets (quarks). Neglecting the $SU(3)_C$ content of the quarks, we have the following eight states :

$$\begin{aligned} e_c &= T\,T\,T & \qquad e &= \bar{T}\,\bar{T}\,\bar{T} \\ u &= T\,T\,V & \qquad u_c &= \bar{T}\,\bar{T}\,\bar{V} \\ d_c &= T\,V\,V & \qquad d &= \bar{T}\,\bar{V}\,\bar{V} \\ \nu &= V\,V\,V & \qquad \nu_c &= \bar{V}\,\bar{V}\,\bar{V} \end{aligned}$$

These states form the quarks and leptons (and their antiparticles) of the first generation. There are two conserved quantum numbers, the total number of T rishons $n_T = n(T) - n(\bar{T})$ and the total number of V rishons $n_V = n(V) - n(\bar{V})$. The electric charge $Q = \frac{1}{3} n_T$ and the neutral charge $N = -\frac{1}{3} n_V$ [3]. One can equally well introduce two equivalent quantum numbers $Y = Q + N$ and $G = Q - N$. At the rishon level, all these quantum numbers can be defined in terms of the triality of rishons with respect to the two SU(3) groups. In table I we present the quantum numbers of the above eight states.

		Q	N	Y	G
e_c	T T T	1	0	1	1
ν_c	$\bar{V}\,\bar{V}\,\bar{V}$	0	1	1	-1
u	T T V	2/3	- 1/3	1/3	1
d	$\bar{T}\,\bar{V}\,\bar{V}$	-1/3	2/3	1/3	-1
d_c	T V V	1/3	- 2/3	- 1/3	1
u_c	$\bar{T}\,\bar{T}\,\bar{V}$	- 2/3	1/3	- 1/3	-1
ν	V V V	0	-1	-1	1
e	$\bar{T}\,\bar{T}\,\bar{T}$	-1	0	-1	-1

Table I

It is easy to see that they can be classified as four doublets of a SU(2) group, each doublet being characterized by Y and each member of the doublet by G. Calling this SU(2) group the electroweak isospin group, the third component of the electroweak isospin is given by $I_3 = \frac{1}{2} G$. We then recover the Gell-Mann - Nishijima formula :

$$Q = \frac{1}{2} (G + Y) = I_3 + \frac{1}{2} Y \ . \qquad (1)$$

The four doublets are :

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_{-1} , \quad \begin{pmatrix} u \\ d \end{pmatrix}_{1/3} , \quad \begin{pmatrix} e_c \\ -\nu_c \end{pmatrix}_{1} , \quad \begin{pmatrix} d_c \\ -u_c \end{pmatrix}_{-1/3}$$

where the indices denote the value of Y and the minus signs in the last two doublets insure that antiparticle states transform in the same way as the particle states under the SU(2) group.

Note however that there is an alternative way of classifying the states. One can construct two quadruplets of the same SU(2) group, each quadruplet being characterized by the value of G and each member of the quadruplet by the value of Y . The third component of the electroweak isospin is given by $I_3 = \frac{3}{2}Y$. The Gell-Mann - Nishijima formula is modified to

$$Q = \frac{1}{2}(Y + G) = \frac{1}{3} I_3 + \frac{1}{2} G \tag{2}$$

The two quadruplets are

$$\begin{pmatrix} e_c \\ u \\ d_c \\ \nu \end{pmatrix}_1 \quad , \quad \begin{pmatrix} -\nu_c \\ d \\ -u_c \\ e \end{pmatrix}_{-1}$$

where the index denotes the value of G and the minus signs were introduced such that the two quadruplets (the second is made from the antiparticles of the first) transform in the same way.

We stress that the two quantum numbers Y and G have exactly the same status (no one is more " fundamental " than the other) and that the two choices, of putting the states either in doublets or in quadruplets are completely equivalent. At this stage we have no reason to prefer one form of multiplet over the other.

In the next section, we study first the standard model with doublets and then with quadruplets.

3. The $SU(2) \otimes U(1)$ model

Consider the free Lagrangian for massless fermions

$${}_D = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi \tag{3}$$

where

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \tag{4}$$

and the charges of the members of the doublet are Q_1 and Q_2. Invariance under the gauge transformation :

$$\Psi \longrightarrow e^{i\vec{\alpha}(x)\frac{\vec{\tau}}{2}} e^{i\beta(x)\frac{Y}{2}} \Psi \qquad (5)$$

is guaranted if one replaces ∂_μ by the covariant derivative :

$$D_\mu = \partial_\mu - i g \frac{\vec{\tau}}{2} \vec{W}_\mu - i g' \frac{Y}{2} B_\mu \qquad (6)$$

$\vec{W}_\mu$ and B_μ are the gauge fields corresponding to the SU(2) and the U(1) groups and their transformation properties are fixed by gauge invariance. g and g' are the coupling constants of the two groups. The Lagrangian becomes :

$$\mathcal{L} = i \bar{\Psi} \gamma^\mu D_\mu \Psi - \frac{1}{4} \vec{W}_{\mu\nu} \vec{W}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad (7)$$

with an obvious notation.

The charged current part of $\mathcal{L}$ is

$$\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}} (\bar{\Psi}_1 \not{W} \Psi_2 + \bar{\Psi}_2 \not{W}^+ \Psi_1) \qquad (8)$$

where $W_\mu = \frac{1}{\sqrt{2}} (W_\mu^1 - i W_\mu^2)$ annihilates $\Delta Q = Q_1 - Q_2$ and creates $- \Delta Q$.

In particular, for the doublets under discussion we have :

$$\begin{aligned}\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}} (&\bar{\nu} \not{W} e + \bar{e} \not{W}^+ \nu + \bar{u} \not{W} d + \bar{d} \not{W}^+ u - \\ &- \bar{d}_c \not{W} u_c - \bar{u}_c \not{W}^+ d_c - \bar{e}_c \not{W} \nu_c - \bar{\nu}_c \not{W}^+ e_c)\end{aligned} \qquad (9)$$

Introducing the fields A_μ and Z_μ as linear orthogonal combinations of W_μ^3 and B_μ the neutral current part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_{N.C.} = &\bar{\Psi} (g \frac{\tau_3}{2} \sin\theta_W + g' \frac{Y}{2} \cos\theta_W) \not{A} \Psi + \\ &+ \bar{\Psi} (g \frac{\tau_3}{2} \cos\theta_W - g' \frac{Y}{2} \sin\theta_W) \not{Z} \Psi\end{aligned} \qquad (10)$$

Requiring that the A_μ be the electromagnetic current, the parameters in front of $\bar{\Psi}_i \not{A} \Psi_i$ should be the e.m. charges. These conditions give :

$$\begin{aligned} g \sin \theta_W &= e \, \Delta Q \\ g' \cos \theta_W &= e \\ Y &= Q_1 + Q_2 \end{aligned} \tag{11}$$

We can now calculate the couplings of the Z neutral current to fermions and we get for the doublets of the first generation :

$$\begin{aligned} \mathcal{L}^Z_{N.C.} = \frac{e}{2} \Big\{ & \left(\frac{\cos\theta_W}{\sin\theta_W} + \frac{\sin\theta_W}{\cos\theta_W}\right) \bar{\nu} \not{Z} \nu + \left(-\frac{\cos\theta_W}{\sin\theta_W} + \frac{\sin\theta_W}{\cos\theta_W}\right) \bar{e} \not{Z} e + \\ & + \left(\frac{\cos\theta_W}{\sin\theta_W} - \frac{1}{3}\frac{\sin\theta_W}{\cos\theta_W}\right) \bar{u} \not{Z} u + \left(-\frac{\cos\theta_W}{\sin\theta_W} - \frac{1}{3}\frac{\sin\theta_W}{\cos\theta_W}\right) \bar{d} \not{Z} d \\ & + \left(\frac{\cos\theta_W}{\sin\theta_W} - \frac{\sin\theta_W}{\cos\theta_W}\right) \bar{e}_c \not{Z} e_c + \left(-\frac{\cos\theta_W}{\sin\theta_W} - \frac{\sin\theta_W}{\cos\theta_W}\right) \bar{\nu}_c \not{Z} \nu_c \\ & + \left(\frac{\cos\theta_W}{\sin\theta_W} + \frac{1}{3}\frac{\sin\theta_W}{\cos\theta_W}\right) \bar{d}_c \not{Z} d_c + \left(-\frac{\cos\theta_W}{\sin\theta_W} + \frac{1}{3}\frac{\sin\theta_W}{\cos\theta_W}\right) \bar{u}_c \not{Z} u_c \Big\} \end{aligned} \tag{12}$$

Consider now the standard model with quadruplets. The free Lagrangian is given by (3) where

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix} \tag{13}$$

with $\widetilde{\Delta Q} = Q_1 - Q_2 = Q_2 - Q_3 = Q_3 - Q_4$ (14)

Under a gauge transformation

$$\Psi \longrightarrow e^{i\vec{\alpha}(x)\vec{T}} \, e^{i\beta(x)\, G/2} \, \Psi \tag{15}$$

$$- \quad [\, T_i \, , \, T_j \,] = i \, \epsilon_{ijk} \, T_k$$

The Lagrangian remains invariant if we substitute ∂_μ by the covariant derivative :

$$D_\mu = \partial_\mu - i\,\tilde{g}\;\vec{T}.\vec{U}_\mu - i\,\tilde{g}'\,\frac{G}{2}\,C_\mu \qquad (16)$$

The charge current Lagrangian is (for details see Appendix) :

$$\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}}\;[\,\sqrt{3}\;\bar{\psi}_1 \not{U} \psi_2 + 2\bar{\psi}_2 \not{U} \psi_3 + \sqrt{3}\;\bar{\psi}_3 \not{U} \psi_4 + h.c.] \qquad (17)$$

where $U_\mu = \frac{1}{\sqrt{2}}\,(U^1_\mu - i\,U^2_\mu)$ annihilates $\widetilde{\Delta Q}$ and creates $-\widetilde{\Delta Q}$. Note that the charged current U_μ is different from the W_μ : in particular they carry different charges.

For the quadruplets of Section 2 we have :

$$\begin{aligned}\mathcal{L}_{c.c.} = \frac{\tilde{g}}{\sqrt{2}}\;[&\sqrt{3}\;\bar{e}_c \not{U} u + 2\bar{u} \not{U} d_c + \sqrt{3}\;\bar{d}_c \not{U} \nu \\ &+ \sqrt{3}\,\bar{u} \not{U}^+ e_c + 2\bar{d}_c \not{U}^+ u + \sqrt{3}\,\bar{\nu} \not{U}^+ d_c \\ &- \sqrt{3}\;\bar{\nu}_c \not{U} d - 2\bar{d} \not{U} u_c - \sqrt{3}\,\bar{u}_c \not{U} e \\ &- \sqrt{3}\;\bar{d} \not{U}^+ \nu_c - 2\,\bar{u}_c \not{U}^+ d - \sqrt{3}\;\bar{e} \not{U}^+ u_c\,]\end{aligned} \qquad (18)$$

Introducing A_μ and Z_μ as linear combinations of U^3_μ and C_μ (see Appendix) we get the neutral current Lagrangian :

$$\begin{aligned}\mathcal{L}_{N.C.} = &\;\bar{\psi}\,(\tilde{g}\;T_3 \sin\tilde{\theta} + \tilde{g}'\,\frac{G}{2}\cos\tilde{\theta})\;\not{A}\,\psi\;+ \\ &+ \bar{\psi}\,(\tilde{g}\,T_3 \cos\tilde{\theta} - \tilde{g}'\,\frac{G}{2}\sin\tilde{\theta})\;\not{Z}\;\psi\end{aligned} \qquad (19)$$

Requiring that A_μ be the e.m. current, we identify its couplings to the e.m. charges of the corresponding fermions. These conditions give :

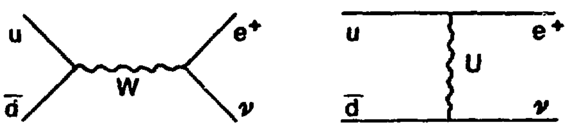
$$\begin{aligned}&\tilde{g}\;\sin\tilde{\theta} = e\;\widetilde{\Delta Q} \\ &\tilde{g}'\;\cos\tilde{\theta} = e \\ &G = Q_1 + Q_4 = Q_2 + Q_3\end{aligned} \qquad (20)$$

One can now calculate the couplings of the Z and, for the quadruplets of the first generation this gives ($\widetilde{\Delta Q} = 1/3$) :

$$
\begin{aligned}
\mathcal{L}^{Z}_{N.C.} = \frac{e}{2} \Big\{ & \Big(\frac{\cos\tilde\theta}{\sin\tilde\theta} - \frac{\sin\tilde\theta}{\cos\tilde\theta} \Big) \bar{e}_c \not{Z} e_c + \Big(\frac{1}{3} \frac{\cos\tilde\theta}{\sin\tilde\theta} - \frac{\sin\tilde\theta}{\cos\tilde\theta} \Big) \bar{u} \not{Z} u + \\
& + \Big(-\frac{1}{3} \frac{\cos\tilde\theta}{\sin\tilde\theta} - \frac{\sin\tilde\theta}{\cos\check\theta} \Big) \bar{d}_c \not{Z} d_c + \Big(- \frac{\cos\tilde\theta}{\sin\check\theta} - \frac{\sin\tilde\theta}{\cos\check\theta} \Big) \bar{\nu} \not{Z} \nu + \\
& + \Big(\frac{\cos\check\theta}{\sin\check\theta} + \frac{\sin\tilde\theta}{\cos\tilde\theta} \Big) \bar{\nu}_c \not{Z} \nu_c + \Big(\frac{1}{3} \frac{\cos\tilde\theta}{\sin\check\theta} + \frac{\sin\check\theta}{\cos\check\theta} \Big) \bar{d} \not{Z} d + \\
& + \Big(-\frac{1}{3} \frac{\cos\tilde\theta}{\sin\tilde\theta} + \frac{\sin\check\theta}{\cos\check\theta} \Big) \bar{u}_c \not{Z} u_c + \Big(- \frac{\cos\check\theta}{\sin\check\theta} + \frac{\sin\check\theta}{\cos\check\theta} \Big) \bar{e} \not{Z} e \Big\}
\end{aligned}
\tag{21}
$$

Let us now compare the charged and neutral currents in the doublet and the quadruplet cases. The charged currents are clearly different. The W has integer charge and mediates quark-quark or lepton-lepton transitions. The experimental evidence for its existence is very recent but the standard model has a long history of good phenomenological descriptions of charged current data. The U on the other hand carries charge 1/3 and mediates quark-lepton and quark - antiquark transitions. The Lagrangian (18) leads trivially to the decay of baryons.

In fact, U behaves exactly like the Y gauge bosons from the GUT based on SU(5) . Finally, note that some transitions can be equally well described by either W or U exchanges. An example is shown in Fig. 1 :



It is very likely that these diagrams should be added, i.e. that there is no " duality " between W and U .

Thus, W and U are different, since they couple to different objects. The A and Z however, couple to the same quarks or leptons in both the doublet and the quadruplet cases. Are they identical in the two cases ? For the e.m. current, the answer is positive by construction. For the Z, its coupling to fermions can be made equal in the two cases. Indeed, the choice of multiplet (doublet or quadruplet) was arbitrary. There was no physical reason to prefer one choice over the other. Therefore, the final result should not depend on what choice we made. The Z couplings will be equal if we choose (compare eqs.(12) and (21)) :

$$\tan \tilde{\theta} = - \tan^{-1} \theta_W \tag{22}$$

or

$$\tilde{\theta} = \theta_W \pm \frac{\pi}{2} \tag{23}$$

This implies that the charged current couplings in the doublet and the quadruplet cases are not independent. We get :

$$\begin{aligned} \tilde{g} &= \frac{1}{3} \frac{e}{\sin \tilde{\theta}} = \pm \frac{1}{3} \frac{e}{\cos \theta_W} = \pm \frac{1}{3} g' \\ \tilde{g}' &= \frac{e}{\cos \tilde{\theta}} = \mp \frac{e}{\sin \theta_W} = \mp g \end{aligned} \tag{24}$$

We can , in principle, predict the rate of baryon decay from the known rates of electroweak processes.

Due to the absence of axial currents, the above discussion is of essentially academic interest. In order to get closer to the real world, we separate the left and right components and study in the next section a model based on the $SU(2)_L \otimes U(1)$ gauge group.

4. The $SU(2)_L \otimes U(1)$ model [4]

We treat separately the left and the right components, with the left components in doublets (or quadruplets) and all right components being singlets.

Consider first the doublet case :

$$\mathcal{L}_D = i \bar{\Psi}_L \not{D} \Psi_L + i \Sigma \bar{\Psi}_R \not{\partial} \Psi_R \tag{25}$$

where

$$\Psi_L = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}_L \qquad (26)$$

$$\Psi_R = \Psi_{1R} \text{ or } \Psi_{2R}$$

Invariance under gauge transformations requires the substitution of normal derivatives with covariant ones. This leads to the following interaction Lagrangians (see Appendix) :

$$\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}} \left(\bar{\Psi}_{1L} \not{W} \Psi_{2L} + \bar{\Psi}_{2L} \not{W}^{+} \Psi_{1L} \right) \qquad (27)$$

(the W's coupleonly to the left components of the fermions)

$$\begin{aligned} \mathcal{L}_{N.C.} = {} & \bar{\Psi}_L \left(g \frac{\tau_3}{2} \sin\theta_W + g' \frac{Y_L}{2} \cos\theta_W \right) \not{A} \Psi_L \\ & + \bar{\Psi}_L \left(g \frac{\tau_3}{2} \cos\theta_W - g' \frac{Y_L}{2} \sin\theta_W \right) \not{Z} \Psi_L \\ & + g' \left(\bar{\Psi}_{1R} \frac{Y_{1R}}{2} \cos\theta_W \not{A} \Psi_{1R} + \bar{\Psi}_{2R} \frac{Y_{2R}}{2} \cos\theta_W \not{A} \Psi_{2R} \right. \\ & \left. - \bar{\Psi}_{1R} \frac{Y_{1R}}{2} \sin\theta_W \not{Z} \Psi_{1R} - \bar{\Psi}_{2R} \frac{Y_{2R}}{2} \sin\theta_W \not{Z} \Psi_{2R} \right) \end{aligned} \qquad (28)$$

Identifying A with the e.m. current, its couplings must be equal to the fermion charges and this leads to

$$\begin{aligned} g \sin\theta_W &= e\,\Delta Q \\ g' \cos\theta_W &= e \\ Y_L &= Q_1 + Q_2 \\ Y_{1R} &= 2Q_1, \quad Y_{2R} = 2Q_2 \end{aligned} \qquad (29)$$

The Z part of the N.C. Lagrangian becomes :

$$\mathcal{L}^{Z}_{N.C.} = \frac{e}{2}\,\overline{\Psi}_L \left(\tau_3\, \Delta Q\, \frac{\cos\theta_W}{\sin\theta_W} - Y_L \frac{\sin\theta_W}{\cos\theta_W} \right) \not{Z}\, \Psi_L - e Q_1 \frac{\sin\theta_W}{\cos\theta_W}\, \overline{\Psi}_{1R} \not{Z}\, \Psi_{1R} - e Q_2 \frac{\sin\theta_W}{\cos\theta_W}\, \overline{\Psi}_{2R} \not{Z}\, \Psi_{2R} \tag{30}$$

The couplings of Z to fermions are not $V-A$ type.

Consider now

$$\mathcal{L}_Q = i\,\overline{\Psi}_L \not{D}\, \Psi_L + i\,\Sigma\, \overline{\Psi}_R \not{D}\, \Psi_R \tag{31}$$

where

$$\Psi_L = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix}_L , \quad \Psi_R = \Psi_{1R} ; \Psi_{2R} ; \Psi_{3R} ; \Psi_{4R} \tag{32}$$

Imposing gauge invariance, the covariant derivative leads to :

$$\mathcal{L}_{c.c.} = \frac{\tilde{g}}{\sqrt{2}} \left[\sqrt{3}\,\overline{\Psi}_{1L} \not{V} \Psi_{2L} + 2\,\overline{\Psi}_{2L} \not{V} \Psi_{3L} + \sqrt{3}\,\overline{\Psi}_{3L} \not{V} \Psi_{4L} + \sqrt{3}\,\overline{\Psi}_{2L} \not{V}^{+} \Psi_{1L} + 2\,\overline{\Psi}_{3L} \not{V}^{+} \Psi_{2L} + \sqrt{3}\,\overline{\Psi}_{4L} \not{V}^{+} \Psi_{3L} \right] \tag{33}$$

and

$$\begin{aligned} \mathcal{L}_{N.C.} = {} & \overline{\Psi}_L \left(\tilde{g}\, T_3 \sin\tilde{\theta} + \tilde{g}' \frac{G_L}{2} \cos\tilde{\theta} \right) \not{A}\, \Psi_L + \\ & + \overline{\Psi}_L \left(\tilde{g}\, T_3 \cos\tilde{\theta} - \tilde{g}' \frac{G_L}{2} \sin\tilde{\theta} \right) \not{Z}\, \Psi_L + \\ & + \tilde{g}'\, \Sigma\, \overline{\Psi}_{iR} \frac{G_{iR}}{2} \cos\tilde{\theta}\, \not{A}\, \Psi_{iR} - \\ & - \tilde{g}'\, \Sigma\, \overline{\Psi}_{iR} \frac{G_{iR}}{2} \sin\tilde{\theta}\, \not{Z}\, \Psi_{iR} \end{aligned} \tag{34}$$

Identifying A_μ with the e.m. current we get :

$$\begin{aligned}
\tilde{g}\ \sin\tilde{\theta} &= e\ \widetilde{\Delta Q} \\
\tilde{g}'\ \cos\tilde{\theta} &= e \\
G_L = Q_1 + Q_4 &= Q_2 + Q_3 \\
G_{i\,R} &= 2Q_i
\end{aligned} \tag{35}$$

Thus, the Z part of the N.C. Lagrangian becomes :

$$\begin{aligned}
\mathcal{L}^{Z}_{N.C.} = \frac{e}{2}\ \bar{\Psi}_L\ \Big(2T_3\ \widetilde{\Delta Q}\ \frac{\cos\tilde{\theta}}{\sin\tilde{\theta}} - G_L\ \frac{\sin\tilde{\theta}}{\cos\tilde{\theta}}\Big)\ \not{Z}\ \Psi_L \\
- e\ \frac{\sin\tilde{\theta}}{\cos\tilde{\theta}}\ \ \Sigma\ Q_i\ \ \bar{\Psi}_{i\,R}\ \not{Z}\ \Psi_{i\,R}
\end{aligned} \tag{36}$$

Comparing eqs. (30) and (36) we see that it is no longer possible to have equal all couplings of the Z in the doublet and the quadruplet cases. We can however make equal either the left couplings or the right ones. The condition that the left couplings be equal is again given by (22) and (23) ; the right couplings are equal for $\tilde{\theta} = \theta_W$. For reasons which will become clear at the end of this section we choose the couplings of the Z to the left components of the fermions to be equal.

We now give masses to the W's , U's and Z by spontaneously breaking the symmetry with the help of Higgs multiplets. In the doublet case the SSB is realized by a single Higgs doublet depending on four real parameters (three of which can be absorbed in a SU(2) gauge transformation).

Denoting by $\frac{1}{\sqrt{2}}\lambda$ the nonvanishing V.E.V. of the Higgs field, one gets the following masses :

$$\begin{aligned}
m_W^2 &= \frac{1}{4}\ g^2\ \lambda^2 \\
m_Z^2 &= \frac{1}{4}\ \frac{g^2\,g'^2}{e^2}\ \lambda^2 = \frac{m_W^2}{\cos^2\theta_W}
\end{aligned} \tag{37}$$

while the $\underset{\to}{A}_\mu$ field remains massless. The fermions can be made massive through Yukawa-type interaction Lagrangians.

In the quadruplet case, the SSB can be realized with two Higgs quadruplets :

$$\Phi = \begin{pmatrix} \Phi^{0} \\ \Phi^{\widetilde{\Delta Q}} \\ \Phi^{2\widetilde{\Delta Q}} \\ \Phi^{3\widetilde{\Delta Q}} \end{pmatrix} \qquad \Phi' = \begin{pmatrix} \Phi'^{-\widetilde{\Delta Q}} \\ \Phi'^{0} \\ \Phi'^{\widetilde{\Delta Q}} \\ \Phi'^{2\widetilde{\Delta Q}} \end{pmatrix} \tag{38}$$

with V.E.V.'s :

$$< \Phi > = \begin{pmatrix} v_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad <\Phi'> = \begin{pmatrix} 0 \\ v_2 \\ 0 \\ 0 \end{pmatrix} \tag{39}$$

We need both Higgses if we want to get fermion masses for all fermions from Yukawa-type couplings : for the U and Z masses one of the two Higgses is sufficient.

Summing the two contributions we get :

$$m_U^2 = \tilde{g}^2 \left(\frac{3}{2} v_1^2 + \frac{7}{2} v_2^2 \right) \tag{40}$$

$$m_Z^2 = \frac{\tilde{g}^2}{\cos^2\tilde{\theta}} \left(\frac{9}{2} v_1^2 + \frac{1}{2} v_2^2 \right) \tag{41}$$

As in the doublet case, m_Z is proportional to m_U . Thus, in the quadruplet case, making m_U very large in order to keep the baryons fairly stable will also push m_Z to values where neutral current events become as rare as the proton decay.

Nevertheless, by combining $\mathcal{L}_D$ and $\mathcal{L}_Q$ one would have a theory which reproduces all the results of the standard model and allows the baryons to be unstable. We therefore propose the following model :

$$\mathcal{L} = (1 - \varepsilon)\mathcal{L}_D + \varepsilon\mathcal{L}_Q \tag{42}$$

For the free Lagrangians, (25) and (31), the above relation is trivial and the ε dependence disappears. Imposing gauge invariance however will introduce all the gauge interactions from both the doublet and the quadruplet cases. Note that every fermion field appears twice : once in $\mathcal{L}_D$ and once in $\mathcal{L}_Q$. Therefore, each fermion transforms under the electroweak SU(2) group as a linear combination of doublet plus quadruplet. This allows us to interpret ε as a measure of the quadruplet content of each fermion. The idea of an elementary particle transforming as a linear combination of two multiplets is not new. It has long been known that the photon couples to both ω and ρ and thus that it transforms as singlet plus triplet under the isospin group.

Consider now the interactions which follow from (42) . The charged currents mediated by W will have their couplings multiplied by $(1 - \varepsilon)$ while the ones mediated by U have their couplings multiplied by ε . The e.m. couplings, like the couplings of the Z to the left components of the fermions, being equal in $\mathcal{L}_D$ and $\mathcal{L}_Q$ will add in (42) and thus become independent of ε . Moreover, the decay of the proton, which in this model requires a U exchange, can be inhibited in two ways : one is by making the mass of U very large, the other is by making ε very small. By choosing ε small enough one can obtain a U mass many orders of magnitude smaller than the value obtained in GUTs .

To illustrate this we estimate m_U and compare it with the zero order value obtained in SU(5) for the Y (and X) gauge boson.

First, from (42) , after SSB the mass squared of Z will be :

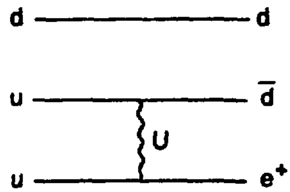
$$m_Z^2 = (1 - \varepsilon)\frac{m_W^2}{\cos^2\theta_W} + \varepsilon\frac{m_U^2}{\sin^2\theta_W} \cdot k \qquad (43)$$

where k , of order unity can be obtained from (40) and (41). The experimentally very successful standard Weinberg - Salam model gives :

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W} \simeq 1 \qquad (44)$$

This implies that $\varepsilon \cdot m_U^2$ can be at most a few percent of m_W^2 . We shall generously write $\varepsilon \cdot m_U^2 = 10^{-P} m_W^2$ with $P > 2$. Consider now a diagram which leads to proton decay. The amplitude for the process in Fig. 2 , for m_U large enough gives roughly :

$$A \sim \varepsilon^2 \tilde{g}^2 \frac{1}{m_U^2} \qquad (45)$$



Comparing this with the SU(5) expression (Y exchange) we get, up to factors of order unity :

$$M^2_{SU(5)} \simeq \frac{m_U^2}{\epsilon^2} \qquad (46)$$

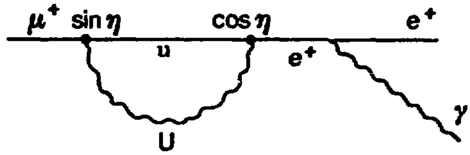
Using the estimate for ϵ . m_U^2 we obtain :

$$m_U^3 \simeq 10^{-P} M_{SU(5)} \; m_W^2 \qquad (47)$$

For $M_{SU(5)} \sim 10^{14-16}$ GeV and $m_W \sim 10^2$ GeV we find $m_U \sim 10^5$ GeV . In general, the larger P is (smaller ϵ) the smaller will be the value of m_U which gives a proton lifetime of about $10^{31\pm}$ years.

Obviously, for such a small value of ϵ , all the interactions mediated by W , which have couplings proportional to $1 - \epsilon$, will reproduce very well all the results of the standard model with doublets.

We end this section with a short comment on generation mixing. So far we have worked with only one generation of leptons and quarks. In the doublet case, generation mixing can be taken into account via Cabibbo or Kobayashi-Maskawa matrices. If the neutrinos are massless, there will be no mixing in the lepton sector. In the quadruplet case, if the neutrinos are massless, one can arrange things so that only two other states are mixed ; there is no natural way of having only the charge - 1/3 quarks mixed as in the doublet case. If the charged leptons are mixed, one expects to see $\mu \longrightarrow e\gamma$ at a rate may be even smaller than that for proton decay, via the diagram in Fig. 3 .



5. Conclusions

We have presented in this paper a model which reproduces all the results of the standard Weinberg-Salam model and also predicts baryon decays. The model assumes that every left component of a fermion transforms as a linear combination of doublet plus quadruplet under the electroweak isospin group while all right components transform as singlets. The choice of doublets and quadruplets was suggested by the rishon model. However, our results do not really depend on the rishon model. Any preon model which leads to the same states and the possibility of constructing different multiplets will give rise to the same theory.

There is a more fundamental feature of our model which has to do with preon models : the interactions do not depend on the constituents. Take again as example the rishon model : the rishons transform like the fundamental representations of $SU(3)_C \otimes SU(3)_H$. Gauging this group we obtain the hypercolor interactions used to form bound states of three rishons and the color interactions which will be responsible for the strong interactions among the quarks, which are color triplets. The electroweak interactions however know nothing about rishons and their interactions : they depend solely on the choice of multiplets in which we put our states. From the point of view of the $SU(2) \otimes U(1)$ theory, W and U exchanges are on equal footing ; from the rishon theory point of view they are very different, U being a $(T\bar{V})$ state while W is a $(TTTVVV)$ state. Thus, the rishons and their interactions are useful only as far as the spectroscopy is concerned ; the electroweak dynamics is independent of them. Only in strong interactions do the rishon interactions spill over at the quark level : the quarks inherit their color quantum numbers from the rishon color interactions. In short, strong and electroweak interactions are decoupled, unlike what is happening in grand unified theories. This interpretation goes directly against our most cherished philosophycal and aesthetical prejudices [5] but might, in spite of everything, be not entirely nonsense. The main idea is that the dynamics at one level does not necessarily reduce to the dynamics at the inferior level. The dynamics of rishons fixes the spectroscopy of quarks and leptons but not their dynamics. The electroweak interactions arise from gauging the $SU(2) \otimes U(1)$ group and there is nothing at the rishon level which tells us that we should do so. Maybe this explains why the reductionist paradigm has met with so little success in nuclear and particle physics.

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Appendix

We give here the conventions used in defining A_μ and Z_μ and some details of the calculations for the $SU(2)_L \otimes U(1)$ model.

In the doublet case, gauge invariance

$$
\begin{aligned}
\Psi_L &\longrightarrow e^{i\vec{\alpha}(x)\frac{\vec{\tau}}{2}}\, e^{i\beta(x)\frac{Y_L}{2}}\, \Psi_L \\
\Psi_{iR} &\longrightarrow e^{i\beta(x)\frac{Y_{iR}}{2}}\, \Psi_{iR}
\end{aligned}
\qquad (A.1)
$$

necessitates the covariant derivative

$$
D_\mu = \partial_\mu - i g \frac{\vec{\tau}}{2} \vec{W}_\mu - i g' \frac{Y}{2} B_\mu \qquad (A.2)
$$

with $\vec{\tau}\Psi_{iR} = 0$.

The charged current Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{c.c.} &= g \bar{\Psi}_L \left(\frac{\tau_1}{2} \not{W}^1 + \frac{\tau_2}{2} \not{W}^2 \right) \Psi_L \\
&= \frac{g}{\sqrt{2}} \bar{\Psi}_L \left(\tau^{(+)} \not{W} + \tau^{(-)} \not{W}^+ \right) \Psi_L
\end{aligned}
\qquad (A.3)
$$

where $\tau^{(\pm)} = \frac{1}{2} (\tau_1 \pm i \tau_2)$ and W_μ was defined in section 3 .

The neutral current Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{N.C.} &= \bar{\Psi}_L \left(g \frac{\tau_3}{2} \not{W}^3 + g' \frac{Y_L}{2} \not{B} \right) \Psi_L + \\
&+ g' \left(\bar{\Psi}_{1R} \frac{Y_{1R}}{2} \not{B} \Psi_{1R} + \bar{\Psi}_{2R} \frac{Y_{2R}}{2} \not{B} \Psi_{2R} \right)
\end{aligned}
\qquad (A.4)
$$

Introducing the fields A_μ and Z_μ via

$$\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \qquad (A.5)$$

one gets eq. (28).

The conditions that the couplings of A_μ to fermions are the e.m. couplings give :

$$\frac{1}{2} g \sin\theta_W + \frac{1}{2} g' Y_L \cos\theta_W = \frac{1}{2} g' Y_{1R} \cos\theta_W = e Q_1$$

$$-\frac{1}{2} g \sin\theta_W + \frac{1}{2} g' Y_L \cos\theta_W = \frac{1}{2} g' Y_{2R} \cos\theta_W = e Q_2 \qquad (A.6)$$

These relations lead to eq. (29).

In the quadruplet case, gauge invariance

$$\Psi_L \longrightarrow e^{i\vec{\alpha}(x)\,\vec{T}} \, e^{i\beta(x)\frac{G_L}{2}} \, \Psi_L \qquad (A.7)$$

$$\Psi_{iR} \longrightarrow e^{i\beta(x)\frac{G_{iR}}{2}} \, \Psi_{iR}$$

requires $\partial_\mu \longrightarrow D_\mu$.

$$D_\mu = \partial_\mu - i\tilde{g}\,\vec{T}\,\vec{U}_\mu - i\tilde{g}'\,\frac{G}{2}\,C_\mu \qquad (A.8)$$

with $\vec{T}\,\Psi_{iR} = 0$.

The charged current Lagrangian is given by

$$\mathcal{L}_{c.c.} = \tilde{g}\,\bar{\Psi}_L\,(T_1 U^1_\mu + T_2 U^2_\mu)\,\gamma^\mu\,\Psi_L$$

$$= \frac{\tilde{g}}{\sqrt{2}}\,\bar{\Psi}_L\,(T^{(+)}\,\not{U} + T^{(-)}\,\not{U}^+)\,\Psi_L \qquad (A.9)$$

with $T^{(\pm)} = T_1 \pm i\, T_2$ and U_μ given in section 3 .

The explicit form of $T^{(\pm)}$ is

$$T^{(+)} = \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad T^{(-)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \tag{A.10}$$

This leads to eq. (33).

For the neutral current part, we introduce A_μ and Z_μ via

$$\begin{pmatrix} C_\mu \\ U^3_\mu \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta} & -\sin\tilde{\theta} \\ \sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \tag{A.11}$$

and obtain eq. (34) .

Identifying A with the e.m. current we get :

$$\begin{aligned}
\frac{3}{2}\,\tilde{g}\,\sin\tilde{\theta} + \tilde{g}'\,\frac{G_L}{2}\,\cos\tilde{\theta} &= \tilde{g}'\,\frac{G_{1R}}{2}\,\cos\tilde{\theta} = e\,Q_1 \\
\frac{1}{2}\,\tilde{g}\,\sin\tilde{\theta} + \tilde{g}'\,\frac{G_L}{2}\,\cos\tilde{\theta} &= \tilde{g}'\,\frac{G_{2R}}{2}\,\cos\tilde{\theta} = e\,Q_2 \\
-\frac{1}{2}\,\tilde{g}\,\sin\tilde{\theta} + \tilde{g}'\,\frac{G_L}{2}\,\cos\tilde{\theta} &= \tilde{g}'\,\frac{G_{3R}}{2}\,\cos\tilde{\theta} = e\,Q_3 \\
-\frac{3}{2}\,\tilde{g}\,\sin\tilde{\theta} + \tilde{g}'\,\frac{G_L}{2}\,\cos\tilde{\theta} &= \tilde{g}'\,\frac{G_{4R}}{2}\,\cos\tilde{\theta} = e\,Q_4
\end{aligned} \tag{A.12}$$

which lead to eqs. (35) .