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TOWARDS A SOLUTION TO THE COSMOLOGICAL CONSTANT PROBLEM

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EXTRA SPACE-TIME DIMENSIONS: TOWARDS A SOLUTION TO THE COSMOLOGICAL CONSTANT PROBLEM *

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ABSTRACT

We discuss the possibility that the cosmological constant problem is solved by raising the number of spatial dimensions a la Kaluza-Klein. In (4+2)-dimensional pure gravity theory with the explicit Λ-term we find classical solutions with vanishing physical cosmological constant and compact 2-dimensions. However, there also exist solutions with the non-vanishing physical Λ-term, and the case $\Lambda_{phys} = 0$ is not preferred at the classical level. We conjecture that quantum corrections and/or additional interactions single out the solution with vanishing physical cosmological constant.

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It is wiedely accepted that the cosmological problem [1] is one of the most serious problems in modern particle theory (for a review see, e.g. [2]). While all the fundamental mass scales (the characteristic QCD parameter $\Lambda_{QCD} = 0$ (100 MeV), the weak interaction scale $M_W = 0$ (100 GeV), the Planck mass $M_{Pl} = 0$ $(10^{19}$ GeV) are rather large, the observed absolute value of the vacuum energy density, which gives rise to the cosmological constant (Λ-term), is very small (if any),

$$\varepsilon_{vac} = \Lambda M_{Pl}^2 / 8\pi < (2 \cdot 10^{-9}\ \mathrm{MeV})^4 .$$

The vacuum energy density feels all the complicated physics inherent in elementary particle interactions; it is determined by the Higgs vacuum expectation values, fermion and gauge fields condensates, coupling constants, etc. In particular, it depends on the values of quark and gluon condensates, $\langle \bar{q}q \rangle$ and $\langle G_{\mu\nu}^2 \rangle$ (which in turn can, in principle be expressed through Λ_{QCD} and quark masses) as well as on the vacuum expectation values of the Weinberg-Salam scalars. Of course, one can make the vacuum energy density vanish by adding the constant term to the underlying lagrangian, but this requires the highly unnatural adjustment of this parameter. The vanishing vacuum energy density can be natural in theories with unbroken supersymmetry (for reviews see, e.g., [3,4]); however, our world is non-supersymmetric at energies less than, say, 10 GeV. In any case, it seems unlikely that some symmetry which is exact at high energies but broken at low ones, is alone responsible for vanishing Λ-term, since the (large) contributions to the vacuum energy density come from low-energy physics (such as chiral symmetry breaking in QCD).

How would a solution to the Λ -term problem look like? The problem would be completely solved if the cosmological constant in the Einstein equations naturally vanished irrespectively of what was the particular value of the vacuum energy density, or, in other words, what were the particular values of the parameters of the theory (renormalized coupling constants, masses, constant term in the lagrangian, etc.). At first sight this desire is self-contradictory, since the mere existence of the non-vanishing vacuum energy density would inevitably lead to non-vanishing curvature of our four-dimensional space-time, which is forbidden experimentally. However, this need not be the case if our space-time has more than four dimensions.

In this paper we report an attempt to find a solution of the Λ -term problem starting from the theory in higher-dimensional space-time. The general idea is that in higher dimensions, the presence of the Λ -term can have a significant effect only on the metrics corresponding to extra dimensions, rather than to four physical ones, so that the observed space-time is (almost) flat, while the extra dimensions are curved due to the Λ -term. In this way one can naturally arrive at extra dimensions compactified a la Kaluza-Klein, the necessary condition for the theory to be reliable* (for a recent discussion of Kaluza-Klein type theories see, e.g., [6-9] and references therein). Changes in the vacuum energy density associated, say, with Weinberg-Salam or QCD phase transitions can produce changes in the compactification scale characteristic to extra dimensions, leaving untouched the Einstein equations describing the physical four-dimensional world*. The relevance of Kaluza-Klein type theories for the Λ -term problem has been realized by many authors [10,9]; however, to our knowledge, no solution of the Λ -term problem as stated above has been proposed yet.

Before presenting the details, we wish to summarize our main results. We have reached only partial success in realizing the above idea. Namely, we have found that in higher-dimensional pure gravity theory with explicit Λ -term, there does exist a class of solutions with compactified extra dimensions and physical four-dimensional metrics obeying the standard Einstein equations with arbitrary (including zero) values of the observable cosmological constant Λ_{phys}. Therefore, the Λ-term problem can be "solved" by choosing a solution characterized by vanishing Λ_{phys}. However, we have not found convincing arguments in favour of this choice. This is the main question yet to be understood. We hope that quantum corrections and/or additional interactions will single out the solution with $\Lambda_{phys} = 0$, which would be a complete solution to the Λ -term problem. It is not excluded also, that the choice $\Lambda_{phys} = 0$ would arise from stability considerations.

Now, let us consider the pure gravity theory in $(d+N)$-dimensional space-time with metrics $\hat{g}_{AB}$ $(A,B=0,\ldots,d+N-1,$

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* Note that the extra dimensions would also be unobservable directly, if no dimensions were compactified, but known (light) particles were confined inside a potential well, which is sufficiently narrow along N spatial dimensions, but flat along four physical ones. This possibility is discussed in [5].

* Note that changes in the compactification scale can in turn lead to changes in observed coupling constants [7]. Does this mean that the couplings (including Newton constant) vary in the course of the Universe evolution?

signature (+,−,−,...,−)). d dimensions will turn out to be physical (in the real world $d = 4$), while N others will be compactified. In what follows it will be convenient to denote the first d coordinates by x^μ ($\mu = 0,1,...,d-1$) and N others by x^a ($a = d,...,N+d-1$). The Einstein equations with the explicit Λ -term in $(d+N)$ dimensions read

$$\hat R_{AB} - \frac{1}{2}\hat g_{AB}\hat R = \Lambda \hat g_{AB} , \qquad (1)$$

where the Ricci tensor $\hat R_{AB}$ is constructed from $\hat g_{AB}$ in a standard way. We have written the Λ -term so that its sign coincides with the sign of the vacuum energy density. In what follows we assume

$$\Lambda > 0 .$$

Note that this sign may be natural in supersymmetric theories [3].

We search for solutions of eqs. (1) which correspond to d-dimensional metrics $g_{\mu\nu}(x^\mu)$ obeying the d-dimensional Einstein equations, N extra dimensions being compactified. Therefore, the desirable Anzats should not break the group of d-dimensional general coordinate transformations,

$$x'^\mu = x'^\mu(x^\mu) \qquad (2)$$
$$x'^a = x^a .$$

In the standard Kaluza-Klein approach one assumes $\hat g_{\mu\nu} = g_{\mu\nu}(x^\mu)$, $\hat g_{\mu a} = 0$, $\hat g_{ab} = \tilde g_{ab}(x^a)$. It is clear that under this assumption the $\mu\nu$ -components of eqs. (1) cannot transform into the d-dimensional Einstein equations without Λ -term, so the Λ -term problem cannot be solved in this way. We choose another Anzats, which is a slight generalization of the standard Kaluza-Klein one,

$$\hat g_{\mu\nu} = \sigma(x^a)\, g_{\mu\nu}(x^\mu) , \qquad (3)$$
$$\hat g_{\mu a} = 0 , \qquad (4)$$
$$\hat g_{ab} = \tilde g_{ab}(x^a) . \qquad (5)$$

Note that this Anzatz does not break the invariance under the transformations (2).

Substituting (3)-(5) into (1), we obtain after some rearrangements, the following set of equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \Lambda_{phys}\, g_{\mu\nu} , \qquad (6)$$

$$\tilde R_{ab} = -\frac{2}{N+d-2}\Lambda \tilde g_{ab} + d\left(\frac{\tilde\nabla_a\tilde\nabla_b\sigma}{2\sigma} - \frac{\tilde\nabla_a\sigma\tilde\nabla_b\sigma}{4\sigma^2}\right), \qquad (7)$$

$$\frac{1}{2}\tilde\nabla_a\tilde\nabla^a\sigma + \frac{d-2}{4}\frac{\tilde\nabla_a\sigma\tilde\nabla^a\sigma}{\sigma} - \frac{2\Lambda}{N+d-2}\sigma = -\frac{2\Lambda_{phys}}{d-2} . \qquad (8)$$

In (6) Λ_{phys} is an arbitrary constant, $R_{\mu\nu}(x^\mu)$ is constructed from $g_{\mu\nu}(x^\mu)$ in a standard way. In (7), (8) $\tilde R_{ab}(x^a)$ and $\tilde\nabla_a$ are the Ricci tensor and the covariant derivative in N-dimensional space-time with metrics $\tilde g_{ab}(x^a)$, the indices are lowered with the use of $\tilde g_{ab}$. Eqs. (6) coincide with the Einstein equations in d-dimensions with the cosmological constant Λ_{phys}. For $\Lambda_{phys} = 0$, the Λ-term present in the initial equations (1) does not give rise to the cosmological constant in d dimensions, provided that $g_{\mu\nu}(x^\mu)$ indeed can be interpreted as the metrics of the physical d-dimensional world. Moreover, if eqs. (7),(8) admit solutions corresponding to the compact N-dimensional space, the extra dimensions would be unobservable at low energies.

Let us first show that eqs. (7), (8) do have solutions with the desired properties, at least for $N=2$. For $N=2$, let us denote the fourth and fifth spatial coordinates as follows,

$$x^4 = \rho, \quad x^5 = \theta$$

and assume θ to be an angular variable, $\theta \in [0, 2\pi]$. Let us further assume

$$\tilde{g}_{44} = -1, \quad \tilde{g}_{45} = 0, \quad \tilde{g}_{55} = \tilde{g}_{55}(\rho) \quad . \tag{9}$$

Then the set of equations (7),(8) reduces to a single equation

$$z'' = -\frac{dU(z)}{dz}, \tag{10}$$

where prime denotes the derivative with respect to ρ, the "potential" is

$$U(z) = \frac{d+1}{4d}\Lambda z^2 - \frac{(d+1)^2}{8(d-1)(d-2)}\Lambda_{phys}\, z^{2\frac{d-1}{d+1}} \tag{11}$$

(see Fig.1) and the functions $\sigma(\rho)$ and $g_{55}(\rho)$ are expressed in terms of $z(\rho)$ as follows:

$$\tilde{g}_{55} = -R^4 z'^2 z^{-2\frac{d-1}{d+1}} \qquad\qquad \sigma = z^{4/d+1}, \tag{12}$$

R being an arbitrary length scale (we choose $R^2 \sim \Lambda^{-1}$ for definiteness).

For $\Lambda_{phys} \leqslant 0$, any solution of eq. (10) corresponds to the "motion" of a "classical particle" in the "potential" (11) with vanishing initial "velocity" $z'(0)$, from some point z_0 to $z = 0$, the coordinate ρ playing the role of "time". This solution describes the compactified extra spatial dimensions in the sense that:

i) The coordinates ρ and θ run from 0 to ρ_{max} and from 0 to 2π, respectively, where ρ_{max} is the (finite) "time" necessary for the "particle" to reach $z = 0$ from $z(0) = z_0$;

ii) The volume of the two-dimensional space with metrics $\tilde{g}_{ab}$ is finite,

$$\int_0^{2\pi} d\theta \int_0^{\rho_{max}} \sqrt{\tilde{g}}\, d\rho < \infty ,$$

where $\tilde{g} = \det \tilde{g}_{ab}$.

iii) The volume element $d\Omega = \sqrt{-\hat{g}}\, d^{d+2}x$ in (d+2)-dimensional space-time is everywhere finite, provided that $g = \det g_{\mu\nu}$ is finite, so the volume of the two-dimensional slice between the hypersurfaces $x^\mu = x_0^\mu$ and $x^\mu = x_0^\mu + dx^\mu$ is finite,

$$d^4x \int_{x^\mu = x_0^\mu} \sqrt{-\hat{g}}\, d\rho\, d\theta < \infty .$$

Note, however, that the length of a circle $x^\mu =$ const, $\rho =$ const, $\left(\ell = 2\pi\sqrt{-\tilde{g}_{55}}\right)$ can be arbitrarily large (since $-\tilde{g}_{55} \to \infty$ as $\rho \to \rho_{max}$), so the notion of compactness is not standard here.

For $\Lambda_{phys} > 0$, there also exist another type of solutions of eq. (10), which corresponds to a half-period of the "negative energy motion" inside the "potential well" of Fig.1. In this case the topology of the extra two-dimensional space coincides with that of a sphere S^2.

Though the extra dimensions are **not** compact in the usual sense, their existence leads to no contradiction with everyday experience, in a perfect analogy to the standard Kaluza-Klein approach. To see this and to show that the set of functions $g_{\mu\nu}(x^\mu)$ can indeed be interpreted as the metrics of the physical space-time, we analyse the matter fields in the background metrics (3-5), (9), (12) (cf. [7]). Consider, for example, the massless scalar field Φ minimally coupled gravity. The equation of motion and the energy-momentum tensor are, respectively,

$$\hat{\nabla}_A \hat{\nabla}^A \Phi(x^A) = 0, \quad \hat{T}_{AB} = \hat{\nabla}_A \Phi \hat{\nabla}_B \Phi - \hat{g}_{AB}(\hat{\nabla}^C \Phi \hat{\nabla}_C \Phi) \tag{13}$$

where $\hat{\nabla}_A$ is the covariant derivative in (d+2)-dimensional space-time and indices are lowered by $\hat{g}_{AB}$. The Anzatz

$$\Phi(x^A) = f(x^a)\,\varphi(x^\mu) \tag{14}$$

transforms eq. (13) into

$$-\nabla_\mu \nabla^\mu \varphi = m^2 \varphi \tag{15}$$

$$\sigma \tilde{\nabla}_a \tilde{\nabla}^a f = m^2 f \tag{16}$$

Eq. (15) is just the Klein-Gordon equation for the particle of mass m propagating in the d-dimensional space-time with metrics $g_{\mu\nu}$, while eq. (16) determines the allowed values of m. In order that the particle with the wave function (14) have finite energy, $\oint \int_{x^0=const} \hat{T}_0^{\ A}\, d\Sigma_A < \infty$, the function $f(x^a)$ should obey the following conditions,

$$\int f^2 \frac{\sqrt{\hat{g}}}{\sigma}\, d\rho\, d\theta < \infty, \quad \int \tilde{\nabla}_a f\, \tilde{\nabla}^a f \sqrt{\hat{g}}\, d\rho\, d\theta < \infty. \tag{17}$$

One can show that under these conditions the operator $\sigma \tilde{\nabla}_a \tilde{\nabla}^a$ has a discrete set of eigenvalues. The lowest eigenvalue, $m^2 = 0$, corresponds to f = const, others are of order Λ by dimensional arguments. Therefore, at low energies only particles with the wave functions independent of extra coordinates survive and, according to (15), they live in d-dimensional space-time with metrics $g_{\mu\nu}$, which is the desired result.

In conclusion we wish to point out the main difference between our approach to the compactification of extra dimensions, and the standard Kaluza-Klein one. Within our approach the metrics of the observed four-dimensional world is identified with $g_{\mu\nu} = \sigma^{-1} \hat{g}_{\mu\nu}$ rather than with the μ, ν -components of the 4+N dimensional metrics themselves. Therefore, the structure of the 4+N dimensional space-time is <u>not</u> $M^4 \times R^N$, where M^4 is the ordinary Minkowski space and R^N is some compact manifold. Nevertheless, the observed world is described by the Minkowski space-time (for $\Lambda_{phys}=0$). An extension of our approach to the supergravity theories in higher dimensions [11] presumably would lead to new patterns of spontaneous compactification of extra dimensions in these theories. We hope that in this way one would be able to obtain reliable higher-dimensional supergravity theories with vanishing cosmological constant. It might well occur that these new patterns would be of significance for constructing realistic models based on supergravity plus Kaluza-Klein.

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