International Atomic Energy Agency

INDC(CCP)-211/L

INDC INTERNATIONAL NUCLEAR DATA COMMITTEE

SIMULTANEOUS MULTI-LEVEL ANALYSIS OF THE TOTAL AND FISSION CROSS-SECTIONS OF ^{239}Pu UP TO 160 eV

V.V. Kolesov and A.A. Luk'yanov

Translation from Nuclear Constants 2(46) 3 (1982)

Translated by the IAEA
October 1983

IAEA NUCLEAR DATA SECTION, WAGRAMERSTRASSE 5, A-1400 VIENNA

Reproduced by the IAEA in Austria
October 1983

INDC(CCP)-211/L

SIMULTANEOUS MULTI-LEVEL ANALYSIS OF THE TOTAL AND FISSION CROSS-SECTIONS OF ^{239}Pu UP TO 160 eV

V.V. Kolesov and A.A. Luk'yanov

Translation from Nuclear Constants 2(46) 3 (1982)

Translated by the IAEA
October 1983

83-05939

L83-21666
Translated from Russian

UDC 539 170 013

SIMULTANEOUS MULTI-LEVEL ANALYSIS OF THE TOTAL AND FISSION CROSS-SECTIONS OF ^{239}Pu UP TO 160 eV

V.V. Kolesov and A.A. Luk'yanov

ABSTRACT

In order to describe cross-sections a multi-level scheme based on S-matrix theory was used taking the Doppler effect and the resolution into account. A multi-level analysis of the total cross-section and fission cross-section of ^{239}Pu was made by the least-squares method in the energy region up to 160 eV. The experimental cross-section data used have a good resolution. The multi-level parameters can be used to represent all the features of the detailed energy structure of experimental cross-sections where the regions of interference minima are of the greatest interest.

An exact knowledge of the detailed structure of the cross-section of fissionable nuclei in the resonance energy region is of great importance where practical applications are concerned. Although in recent years considerable progress has been made with measurements, the results of different experiments do not fully agree with each other. The problem of the evaluation of cross-sections and the comparison of different experimental results in a given range is therefore rather complex.

In many cases this problem is best solved by deriving an analytical representation of the cross-sections by introducing a given number of parameters obtained by analysing experiments. This parametrization makes it possible, for example, to solve the following problems:

(1) Intercomparison of the results of different experiments with different resolutions and sample temperatures;

(2) Reconstitution of the true cross-sections at different temperatures;

(3) Comparatively easy calculation of functionals from cross-sections;

(4) Representation of a large number of experimental data with relatively small number of parameters.

At present three basic assumptions have become the most widely used for parametrization and representation of the energy structure of cross-sections: the Brite-Wigner approximation, a scheme based on the R-matrix theory and a scheme based on the S-matrix theory.

The first approximation is easiest to apply but owing to significant interference effects between resonances for many elements, and in particular for ^{239}Pu, it should really not be used. The main advantage of the R-matrix scheme is simplicity of the interpretation of the parameters obtained. Among its disadvantages the main one to be pointed out is the difficulty arising when taking the Doppler effect and resolution into account. This also makes the search for parameters extremely laborious. For fissionable nuclei it is probably better to use the S-matrix scheme. When so doing it is easy to take into account the Doppler effect and the instrumental resolution and it is possible to represent cross-sections with any number of reaction channels and any degree of interference between resonance levels. The representation is in practice just as convenient for use in various calculations as is the Brite-Wigner approximation.

It should be pointed out that most of the literature, Ref. [1] for example, and also many evaluated data libraries give only resonance parameters obtained with the Brite-Wigner approximation. This makes it necessary to use relatively complex sets of programmes for reconstructing the detailed shape of the cross-section.

Method of calculation. For representing cross-sections in the present paper use was made of the multi-level scheme based on the S-matrix theory. Taking into account the Doppler effect, in accordance with the data of Ref. [2], general cross-section equations are written in the form

$$\sigma_T = \sigma_p + 0{,}65\cdot 10^6/\sqrt{E}\sum_\lambda 1/\nu_\lambda \left\{G_\lambda^T \Psi\left[(E-\mu_\lambda)/\nu_\lambda;\ \nu_\lambda/\Delta_T\right] + H_\lambda^T \chi\left[(E-\mu_\lambda)/\nu_\lambda;\ \nu_\lambda/\Delta_T\right]\right\};$$

$$\sigma_F = 0{,}65\cdot 10^6/\sqrt{E}\sum_\lambda 1/\nu_\lambda \left\{G_\lambda^F \Psi\left[(E-\mu_\lambda)/\nu_\lambda;\ \nu_\lambda/\Delta_T\right] + H_\lambda^F \chi\left[(E-\mu_\lambda)/\nu_\lambda;\ \nu_\lambda/\Delta_T\right]\right\},$$

where σ_p is the potential scattering, G_λ^T, H_λ^T, G_λ^F, H_λ^F are parameters, μ_λ is the position of the resonance, ν_λ is its width, $\Delta_T = \sqrt{4\ KTE/(A + 1)}$ is the Doppler width, and Ψ and χ are symmetrical and asymmetrical Doppler functions containing the main energy dependence of the cross-section and taking the resonance form. The resolution of the instruments is taken into account here by replacing Δ_T by $\Delta = \sqrt{\Delta_T^2 + \Delta_R^2}$ where Δ_R^2 is the resolution dispersion function.

Analysis of experiments and results. For this analysis use was made of the following sets of experimental data:

For σ_T in the region 0.014-4.5 eV, with a resolution of 2.7-0.52 μs/m [3] and also in the region above 4.45 eV, with a resolution of 0.018-0.001 μs/m [4];

For σ_F in the region 0.019-2 eV, with a resolution of 2.5-0.8 μs/m [5], in the region 2-4.5 eV, with a resolution of 0.2-0.03 μs/m [6], in the region 4.5-37.5 eV, with a resolution of 0.025 μs/m [7], and in the region above 37.5 eV with a resolution of 0.007-0.004 μs/m [8].

Virtually all measurements were performed with the best resolution available at present for the particular energy region. In order to match the positions of resonances in the total and fission cross-sections, it proved necessary to shift the energy scales in the measurements taken from Refs [7,8] so as to bring them into line with the scale in Ref. [4]. The shift was made to obey the law $E' = E - \alpha E + \beta$ where $\alpha = 0.0047729$, $\beta = 0.0152823$ for Ref. [7], and $\alpha = 0$ and $\beta = 0.033$ for Ref. [8]. Analysis of cross-sections was based on the least squares method using the program described in Ref. [9]. The potential scattering was taken to be equal to 10.3 b*/. The resonance parameters obtained in this way are shown in the table, together with single-level parameters from Ref. [1]. In Figs 1-3 the reconstructed cross-sections and experimental data are compared.

The results of the multi-level parametrization illustrated the feasibility of simultaneously representing the total cross-section and the fission cross-section of ^{239}Pu in the resonance region using a single consistent set of resonance parameters. The cross-sections plotted from the parameters found represented all the features of the detailed energy structure of the experimental cross-sections where the region of interference minima is of greatest interest.

---

*/ 1 barn = 10^{-28} m^2

In the future it is planned to test the derived parameters, mainly H_{λ}^{T} and H_{λ}^{F}, against the measurements of neutron spectra and fission cross-sections in filtered beams for relatively thick samples [10].

REFERENCES

[1] ANTSIPOV, G.V., BAKHANOVICH, L.A., ZHARKOV, V.F., et al., Preprint No. 12 Institute of Heat and Mass Exchange (ITMO), Acad. Sci. Byelorus. Sov. Soc. Rep., Minsk (1981).

[2] Adler D.B., Adler F.T. Neutron cross-sections in fissile elements. - In: Proceedings of the Conference on Breeding in Large Fast Reactors. Argonne. ANL-6792, 1963, p.695-708.

[3] Bollinger L.M., Cote R.E., Thomas G.E. The slow neutron cross-sections of plutonium-239. - In: Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy (Geneva, 1958). Geneva, United Nations, 1958, v.15, p.127-137.

[4] DERRIEN, H., BLONS, J., EGGERMAN, C., et al., Total and fission cross-sections of ^{239}Pu, Proc. Conf. Nuclear Data for Reactors (Paris, 1966), IAEA, 2 Vienna (1967) 195-210 [in French].

[5] De Saussure G., Weston L.W., Gwin R. e.a. Measurement of the neutron capture and fission cross-sections and of their ratio alpha for ^{233}U, ^{235}U and ^{239}Pu. - Ibid., p.233-249.

[6] Gwin R., Silver E.G., Ingle R.W. e.a. Measurement of the neutron capture and fission cross-sections of ^{239}Pu and ^{235}U, 0,02 eV to 200 keV, the neutron capture cross-sections of ^{197}Au, 10 to 50 keV, and neutron cross-sections of ^{235}U, 5 to 200 keV. - Nucl. Sci. and Engng, 1976, v. 59, p.79-106.

[7] DE SAUSSURE, G., BLONS, J., IOUSSEAUME, C., et al., Measurement and analysis of the fission cross-section of plutonium-239 from 0 to 5 keV Proc. Symp. Physics and Chemistry of Fission (Salzburg, 1965), IAEA, 1 Vienna (1965) 205-216 [in French].

[8] Blons J. High resolution measurements of neutron - induced fission cross-sections for ^{233}U, ^{235}U, ^{239}Pu and ^{241}Pu below 30 keV. - Nucl. Sci. and Engng, 1973, v.51, p.130-147.

[9] KOLESOV, V.V., A program for multi-level analysis of resonance cross-sections, Voprosy atomnoj nauki i tekhniki, Ser. Yad. konst. 3 (38) (1980) 17 [in Russian].

[10] Bakalov T., Ilchev V., Ukraintsev V.K. e.a. Transmission and self-indication measurements of ^{235}U and ^{239}Pu in 2 eV - 20 keV energy region. - In: Proceedings of the International Conference on Nuclear Cross-Sections for Technology. Knoxville, 1979, p.642-698.

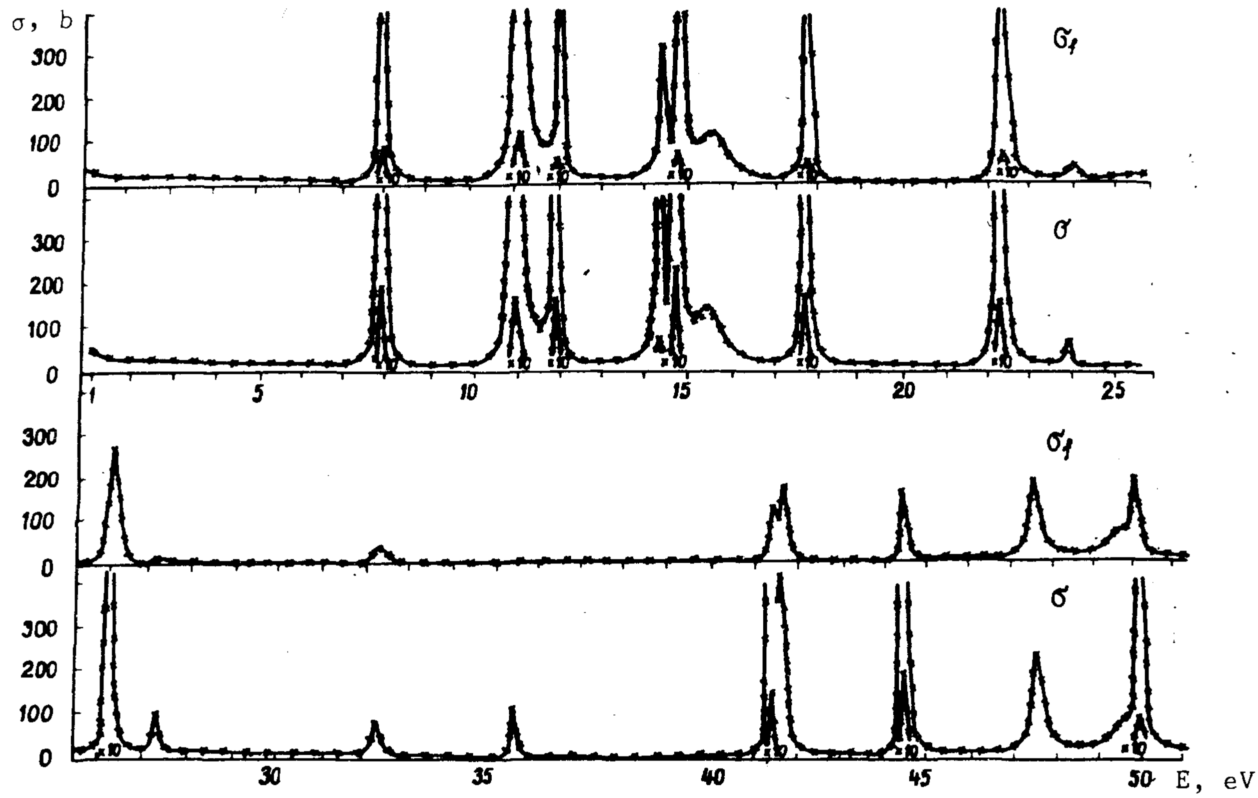

Fig. 1. Calculated and experimental cross-sections in the region 1-50 eV

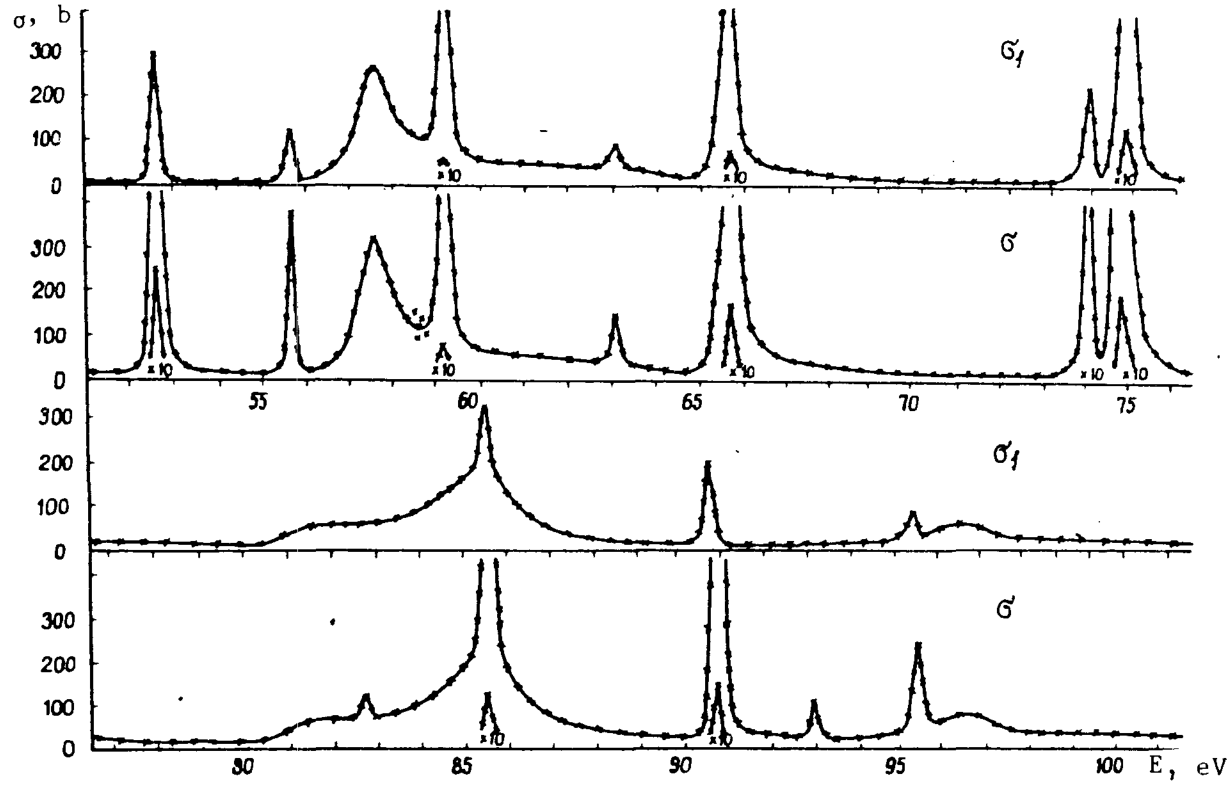

Fig. 2. Calculated and experimental cross-sections in the region 20-100 eV

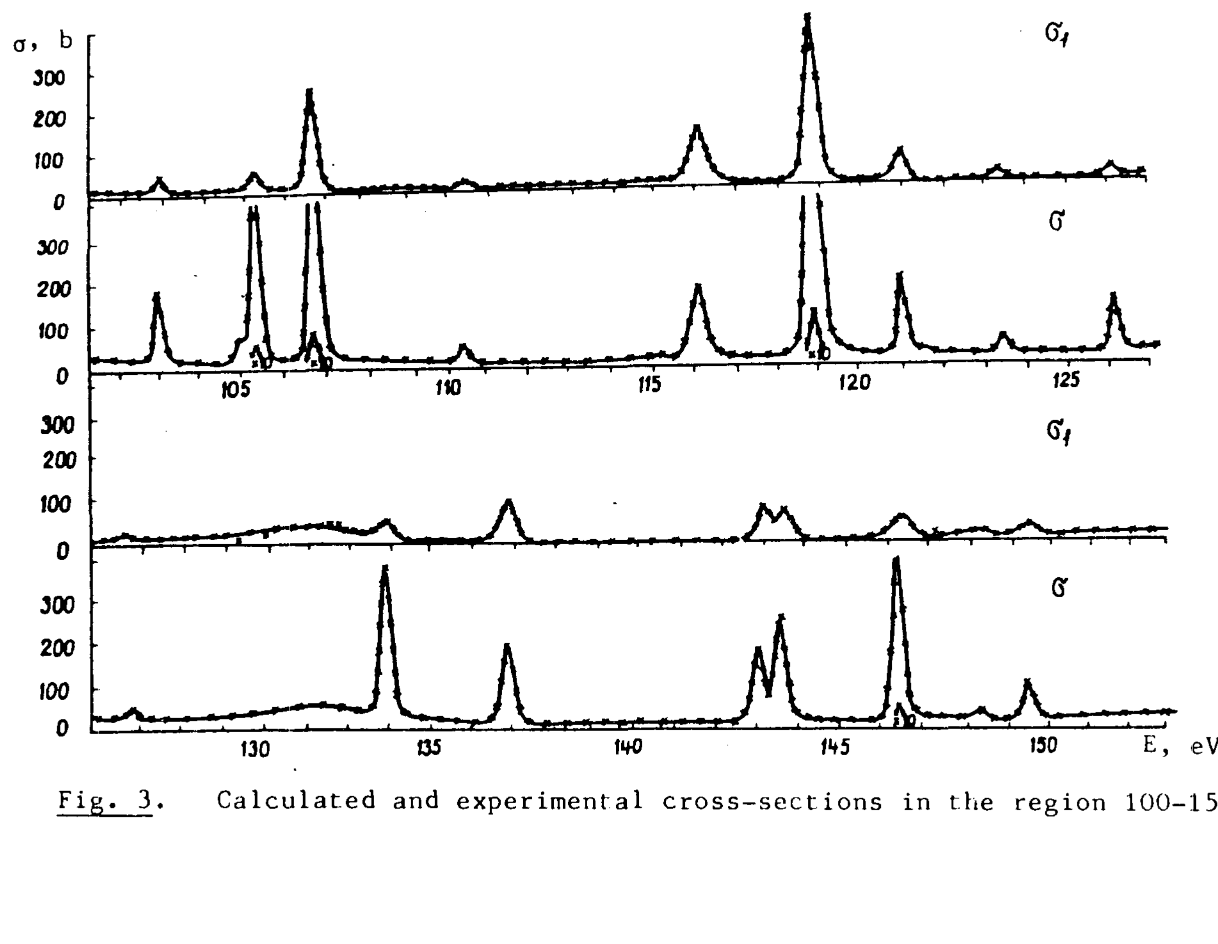

Fig. 3. Calculated and experimental cross-sections in the region 100-150 eV

Resonance parameters for ^{239}Pu (single-level parameters of Ref. [1] are shown in brackets)

μ, eV	ν, eV	$G^T \cdot 10^4$, $eV^{1/2}$	$H^T \cdot 10^4$, $eV^{1/2}$	$G^F \cdot 10^4$, $eV^{1/2}$	$H^F \cdot 10^4$, $eV^{1/2}$
-0,26	0,I00	0	0,4274	0	0,3I00
(-I,8I)	(I,650)	(3,I998)	–	(2,83I4)	–
(-0,08)	(0,035)	(0,05I8)	–	(0,0489)	–
0,30	0,047	2,9322	0,0482	I,3247	0,034I
(0,29)	(0,047)	(2,23I5)	–	(I,3534)	–
3,I5	2,250	0,4700	0,0830	0,4500	-0,0532
(5,89)	(I,65I)	(0,3870)	–	(0,38I9)	–
7,8I	0,043	4,I733	-0,0922	2,359I	0,04I2
(7,8I)	(0,044)	(4,I0I7)	–	(2,2526)	–
I0,92	0,089	8,385I	0,5II4	6,4547	0,70I5
(I0,92)	(0,I00)	(8,04I6)	–	(6,2786)	–
(II,49)	(0,026)	(0,2508)	–	(0,0505)	–
II,88	0,033	4,2233	-0,3242	I,6253	-0,2832
(II,88)	(0,038)	(3,8830)	–	(I,4645)	–
I4.30	0,053	2,6068	-0,3247	I,5852	-0,3I54
(I4,30)	(0,05I)	(2,285I)	–	(I,5072)	–
I4,66	0,036	7,4337	0,5076	3,2002	0,36I4
(I4,67)	(0,035)	(7,4II3)	–	(3,0963)	–

μ, eV	ν, eV	$G^T \cdot 10^4$, $eV^{1/2}$	$H^T \cdot 10^4$, $eV^{1/2}$	$G^F \cdot 10^4$, $eV^{1/2}$	$H^F \cdot 10^4$, $eV^{1/2}$
I5,42	0,405	2,8498	-0,I8I4	2,6II4	-0,0598
(I5,45)	(0,350)	(2,3758)	–	(2,I999)	–
I7,63	0,038	6,4463	0,0I90	2,9I45	-0,II93
(I7,65)	(0,037)	(5,8296)	–	(2,5307)	–
22,24	0,052	8,2352	0,2296	4,734I	0,2243
(22,28)	(0,054)	(7,8679)	–	(4,4822)	–
23,88	0,046	0,2852	-0,0430	0,I806	-0,0485
(23,92)	(0,035)	(0,26I0)	–	(0,I490)	–
26,23	0,043	4,533I	0,0262	2,2577	-0,065I
(26,22)	(0,042)	(3,5III)	–	(I,9244)	–
27,24	0,025	0,3962	0,02I7	0,0558	-0,0048
(27,22)	(0,02I)	(0,4II4)	–	(0,0586)	–
32,29	0,083	0,7340	0,0207	0,5092	0,04I3
(32,29)	(0,076)	(0,6596)	–	(0,485I)	–
(34,58)	(0,046)	(0,03II)	–	(0,0I68)	–
35,43	0,0I9	0,5888	0,0006	0,0568	0,00I4
(35,48)	(0,024)	(0,686I)	–	(0,058I)	–
4I,38	0,023	8,9349	0,3422	0,7I49	0,0664
(4I,40)	(0,026)	(9,8778)	–	(0,9455)	–
4I,63	0,05I	3,0294	-0,I235	I,3963	-0,0887
(4I,64)	(0,052)	(3,4550)	–	(I,5393)	–

μ, eV	ν, eV	$G^T\cdot10^4$, $eV^{1/2}$	$H^T\cdot10^4$, $eV^{1/2}$	$G^F\cdot10^4$, $eV^{1/2}$	$H^F\cdot10^4$, $eV^{1/2}$
44,44	0,026	13,5942	0,3824	1,2069	-0,0266
(44,46)	(0,029)	(14,1207)	–	(1,3084)	–
47,56	0,141	3,7368	0,2674	3,2502	0,1571
(47,57)	(0,156)	(4,0877)	–	(3,2136)	–
49,65	0,367	2,4500	0,1974	2,3500	0,1177
(49,68)	(0,401)	(2,8772)	–	(2,6869)	–
50,04	0,027	6,3568	0,1099	1,3829	-0,0474
(50,05)	(0,029)	(6,8441)	–	(1,5547)	–
52,54	0,029	20,2067	0,7271	2,7112	0,0467
(52,57)	(0,034)	(19,8633)	–	(2,4681)	–
55,58	0,029	3,0807	-0,0041	1,1610	-0,1479
(55,60)	(0,029)	(3,6128)	–	(1,3211)	–
57,42	0,466	15,0716	6,2855	13,7402	5,2303
(57,41)	(0,255)	(10,6615)	–	(9,3002)	–
(58,81)	(0,551)	(7,6472)	–	(7,4624)	–
59,16	0,069	9,1128	0,8103	6,5670	-0,[illegible]249
(59,19)	(0,090)	(10,5124)	–	(7,0514)	–
63,60	3,511	10,0000	-11,9197	9,8000	-10,9950
(60,91)	(3,399)	(12,9[illegible])	–	(12,7871)	–
63,03	0,049	1,1[illegible]	0,0007	0,7143	-0,0[illegible]89
(63,05)	(0,076)	(1,5228)	–	(1,0962)	
65,45	0,196	[illegible],22[illegible]3	2,1266	5,3894	3,[illegible]32
(65,33)	(0,046)	(0,63[illegible])	–	(0,3435)	
65,70	0,336	16,73[illegible]	1,9750	5,9393	-0,[illegible]254
(65,68)	(0,068)	(20,5695)	–	(11,0313)	–

μ, eV	ν, eV	$G^T\cdot10^4$, $eV^{1/2}$	$H^T\cdot10^4$, $eV^{1/2}$	$G^F\cdot10^4$, $eV^{1/2}$	$H^F\cdot10^4$, $eV^{1/2}$
74,03	0,039	5,7519	-0,7261	2,6380	-0,8253
(74,01)	(0,036)	(5,7009)	–	(2,5197)	–
74,90	0,093	34,4847	2,3414	22,3753	1,0277
(74,91)	(0,073)	(36,3043)	–	(21,1433)	–
78,94	0,063	0,1079	0,0231	0,0022	-0,0037
(78,91)	(0,046)	(0,2303)	–	(0,1219)	–
81,13	0,835	2,5826	4,0203	2,3074	3,7796
(81,72)	(1,023)	(4,7818)	–	(4,6613)	–
82,66	0,024	0,5693	0,0167	0,0338	-0,0287
(82,64)	(0,035)	(0,8259)	–	(0,3456)	–
(83,48)	(0,875)	(1,3404)	–	(1,3057)	–
85,42	1,165	28,6130	-3,4829	25,9597	-4,7143
(85,28)	(1,049)	(27,8232)	–	(26,5710)	–
85,49	0,038	12,5233	0,4069	2,3643	-0,1621
(85,44)	(0,037)	(12,3335)	–	(2,8105)	–
90,72	0,030	17,6433	0,8753	2,6941	-0,0635
(90,70)	(0,030)	(17,8340)	–	(2,6829)	–
92,97	0,021	1,0150	-0,0112	0,0930	-0,0685
(92,92)	(0,029)	(1,3749)	–	(0,2013)	–
95,37	0,039	2,9964	0,2744	0,8838	-0,0540
(95,31)	(0,049)	(3,2589)	–	(0,9632)	–
96,65	0,732	5,3388	-1,1699	4,9017	-1,0593
(96,44)	(0,850)	(6,9614)	–	(6,7293)	–
98,87	4,652	14,5392	1,5010	14,0253	-0,8727
(100,20)	(3,001)	(6,0530)	–	(5,9975)	–

Окончание таблицы

μ, eV	ν, eV	$G^{T}\cdot IO^{4}$, $eV^{I/2}$	$H^{T}\cdot IO^{4}$, $eV^{I/2}$	$G^{F}\cdot IO^{4}$, $eV^{I/2}$	$H^{F}\cdot IO^{4}$, $eV^{I/2}$
I03,0I	0,025	2,307I	0,I037	0,4703	-0,0383
(I02,94)	(0,023)	(2,5535)	-	(0,4908)	-
I05,3I	0,030	6,5878	0,2973	0,6896	-0,0I68
(I05,25)	(0,024)	(6,2304)	-	(0,7794)	-
I06,69	0,036	I2,6625	0,0342	4,6437	-0,3053
(I06,62)	(0,038)	(I3,9489)	-	(4,7907)	-
II0,42	0,030	0,6378	0,0206	0,2580	-0,0395
(II0,33)	(0,022)	(0,6807)	-	(0,2035)	-
II3,96	0,9I9	0,2II5	0,2II5	0,2I00	0,9800
(II4,38)	(0,749)	(0,7848)	-	(0,76I3)	-
II5,28	0,086	0,I852	-0,I980	0	-0,I90I
(II5,04)	(0,I03)	(0,32I0)	-	(0,2560)	-
II6,06	0,I22	4,8072	0,5479	4,28I3	-0,0809
(II5,97)	(0,I34)	(5,4545)	-	(4,4244)	-
II8,84	0,04I	22,350I	0,9402	9,094I	-0,4038
(II8,77)	(0,05I)	(25,7776)	-	(I0,3383)	-
II9,22	0,405	0,6I96	-0,2233	0,I546	0,2436
I2I,02	0,024	3,0593	-0,0709	I,4482	-0,2362
(I20,93)	(0,039)	(3,7027)	-	(I,8I68)	-
I23,48	0,040	0,6926	-0,0036	0,3583	-0,I555
(I23,38)	(0,032)	(0,63I8)	-	(0,3783)	-
I26,23	0,020	2,I035	0,I266	0,45I7	-0,0093
(I26,I4)	(0,046)	(2,745I)	-	(0,5728)	-
I27,56	0,0I9	0,6344	-0,0I60	0,I599	-0,0804
(I27,45)	(0,032)	(0,6654)	-	(0,2579)	-
I32,04	I,575	II,I373	-I,0295	I0,6627	-2,390I
(I3I,69)	(I,900)	(I6,5727)	-	(I6,2I95)	-
I33,6I	0,022	6,4I09	0,I869	0,8250	-0,0349
(I33,72)	(0,028)	(6,4892)	-	(0,7599)	-
I35,23	7,506	I0,6572	-6,79I9	6,8I96	-I,6538
I36,79	0,050	4,I424	0,IIII	2,797I	-0,2782
(I36,68)	(0,063)	(4,2II2)	-	(2,7774)	-
I39,2I	0,00002	0,0444	0,0303	0	0,006I
(I39,2I)	(0,I6I)	(0,I363)	-	(0,II83)	-
I42,96	0,04I	3,7593	-0,0849	2,4286	-0,3I29
(I42,85)	(0,069)	(4,2409)	-	(2,4696)	-
I43,48	0,042	5,I322	0,5257	2,2537	-0,0I55
(I43,40)	(0,042)	(5,I962)	-	(I,8748)	-
I46,I4	0,406	I,4373	0,527I	I,25I4	-0,2300
(I47,37)	(0,50I)	(I,0995)	-	(I,0494)	-
I46,27	0,002	6,5670	0,7200	I,0224	0,0888
(I46,I8)	(0,035)	(8,92I9)	-	(I,5253)	-
I48,29	0,047	0,4334	-0,0964	0,3032	-0,3I38
(I48,I4)	(0,075)	(0,5I48)	-	(0,3583)	-
I48,93	2,402	2,95I8	-I,0449	2,9360	3,0953
I49,45	0,026	I,6839	0,I667	0,5990	-0,2506
(I49,35)	(0,059)	(I,9555)	-	(0,8740)	-
I56,22	0,I62	0,I900	-0,3589	0,I800	0,32I3
I57,08	0,342	I3,9589	I,2623	9,9080	-0,4II7
(I57,0I)	(0,6I3)	(I3,8339)	-	(6,I005)	-